# ArtiSynth Modeling Guide

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# Preface

This guide describes how to create mechanical and biomechanical models in ArtiSynth using its Java API. Detailed information on how to use the ArtiSynth GUI for model visualization, navigation and simulation control is given in the ArtiSynth User Interface Guide. It is also possible to interface ArtiSynth with, or run it under, MATLAB. For information on this, see the guide Interfacing ArtiSynth to MATLAB.

Information on how to install and configure ArtiSynth is given in the installation guides for Windows, MacOS, and Linux.

It is assumed that the reader is familiar with basic Java programming, including variable assignment, control flow, exceptions, functions and methods, object construction, inheritance, and method overloading. Some familiarity with the basic I/O classes defined in `java.io.*`, including input and output streams and the specification of file paths using `File`, as well as the collection classes `ArrayList` and `LinkedList` defined in `java.util.*`, is also assumed.

## How to read this guide

Section 1 offers a general overview of ArtiSynth's software design, and briefly describes the algorithms used for physical simulation (Section 1.2). The latter section may be skipped on first reading. A more comprehensive overview paper is available online.

The remainder of the manual gives details instructions on how to build various types of mechanical and biomechanical models. Sections 3 and 4 give detailed information about building general mechanical models, involving particles, springs, rigid bodies, joints, constraints, and contact. Section 5 describes how to add control panels, controllers, and input and output data streams to a simulation. Section 6 describes how to incorporate finite element models. The required mathematics is reviewed in Section A.

If time permits, the reader will profit from a top-to-bottom read. However, this may not always be necessary. Many of the sections contain detailed examples, all of which are available in the package `artisynth.demos.tutorial` and which may be run from ArtiSynth using Models > All demos > tutorials. More experienced readers may wish to find an appropriate example and then work backwards into the text and preceding sections for any needed explanatory detail.

# Chapter 1

# ArtiSynth Overview

ArtiSynth is an open-source, Java-based system for creating and simulating mechanical and biomechanical models, with specific capabilities for the combined simulation of rigid and deformable bodies, together with contact and constraints. It is presently directed at application domains in biomechanics, medicine, physiology, and dentistry, but it can also be applied to other areas such as traditional mechanical simulation, ergonomic design, and graphical and visual effects.

## 1.1 System structure

An ArtiSynth model is composed of a hierarchy of models and model components which are implemented by various Java classes. These may include sub-models (including finite element models), particles, rigid bodies, springs, connectors, and constraints. The component hierarchy may be in turn connected to various *agent* components, such as control panels, controllers and monitors, and input and output data streams (i.e., *probes*), which have the ability to control and record the simulation as it advances in time. Agents are presented in more detail in Section 5.

The models and agents are collected together within a top-level component known as a *root model*. Simulation proceeds under the control of a *scheduler*, which advances the models through time using a physics simulator. A rich graphical user interface (GUI) allows users to view and edit the model hierarchy, modify component properties, and edit and temporally arrange the input and output probes using a *timeline* display.

### 1.1.1 Model components

Every ArtiSynth component is an instance of ModelComponent. When connected to the hierarchy, it is assigned a unique number relative to its parent; the parent and number can be obtained using the methods getParent() and getNumber(), respectively. Components may also be assigned a name (using setName()) which is then returned using getName().

> A component's number is not the same as its *index*. The index gives the component's sequential list position within the parent, and is always in the range $0 \ldots n-1$, where $n$ is the parent's number of child components. While indices and numbers frequently are the same, they sometimes are not. For example, a component's number is guaranteed to remain unchanged as long as it remains attached to its parent; this is different from its index, which will change if any preceding components are removed from the parent. For example, if we have a set of components with numbers
>
> `0 1 2 3 4 5`
>
> and components 2 and 4 are then removed, the remaining components will have numbers
>
> `0 1 3 5`
>
> whereas the indices will be `0 1 2 3`. This consistency of numbers is why they are used to identify components.

A sub-interface of `ModelComponent` includes CompositeComponent, which contains child components. A ComponentList is a `CompositeComponent` which simply contains a list of other components (such as particles, rigid bodies, sub-models, etc.).

Components which contain state information (such as position and velocity) should extend HasState, which provides the methods getState() and setState() for saving and restoring state.

A Model is a sub-interface of `CompositeComponent` and `HasState` that contains the notion of advancing through time and which implements this with the methods `initialize(t0)` and `advance(t0, t1, flags)`, as discussed further in Section 1.1.4. The most common instance of `Model` used in ArtiSynth is `MechModel` (Section 1.1.5), which is the top-level container for a mechanical or biomechanical model.

### 1.1.2 The RootModel

The top-level component in the hierarchy is the *root model*, which is a subclass of RootModel and which contains a list of models along with lists of agents used to control and interact with these models. The component lists in `RootModel` include:

| | |
|---|---|
| `models` | top-level models of the component hierarchy |
| `inputProbes` | input data streams for controlling the simulation |
| `controllers` | functions for controlling the simulation |
| `monitors` | functions for observing the simulation |
| `outputProbes` | output data streams for observing the simulation |

Each agent may be associated with a specific top-level model.

### 1.1.3 Component path names

The names and/or numbers of a component and its ancestors can be used to form a component path name. This path has a construction analogous to Unix file path names, with the '/' character acting as a separator. Absolute paths start with '/', which indicates the root model. Relative paths omit the leading '/' and can begin lower down in the hierarchy. A typical path name might be

```
/models/JawHyoidModel/axialSsprings/lad
```

For nameless components in the path, their numbers can be used instead. Numbers can also be used for components that have names. Hence the path above could also be represented using only numbers, as in

```
/0/0/1/5
```

although this would most likely appear only in machine-generated output.

### 1.1.4 Model advancement

ArtiSynth simulation proceeds by advancing all of the root model's top-level models through a sequence of time steps. Every time step is achieved by calling each model's advance() method:

```
public StepAdjustment advance (double t0, double t1) {
   ... perform simulation ...
}
```

This method advances the model from time `t0` to time `t1`, performing whatever physical simulation is required (see Section 1.2). The method may optionally return a `StepAdjustment` indicating that the step size (`t1 - t0`) was too large and that the advance should be redone with a smaller step size.

The root model has it's own advance(), which in turn calls the advance method for all of the top-level models, in sequence. The advance of each model is surrounded by the application of whatever agents are associated with that model. This is done by calling the agent's `apply()` method:

```
model.preadvance (t0, t1);
for (each input probe p) {
   p.apply (t1);
}
for (each controller c) {
   c.apply (t0, t1);
}
model.advance (t0, t1);
for (each monitor m) {
   m.apply (t0, t1);
}
for (each output probe p) {
   p.apply (t1);
}
```

Agents not associated with a specific model are applied before (or after) the advance of all other models.

More precise details about model advancement are given in the ArtiSynth Reference Manual.

### 1.1.5 MechModel

Most ArtiSynth applications contain a single top-level model which is an instance of MechModel. This is a CompositeComponent that may (recursively) contain an arbitrary number of mechanical components, including finite element models, other `MechModel`s, particles, rigid bodies, constraints, attachments, and various force effectors. The `MechModel` `advance()` method invokes a physics simulator that advances these components forward in time (Section 1.2).

For convenience each `MechModel` contains a number of predefined containers for different component types, including:

| | |
|---|---|
| `particles` | 3 DOF particles |
| `points` | other 3 DOF points |
| `rigidBodies` | 6 DOF rigid bodies |
| `frames` | other 6 DOF frames |
| `axialSprings` | point-to-point springs |
| `connectors` | joint-type connectors between bodies |
| `constrainers` | general constraints |
| `forceEffectors` | general force-effectors |
| `attachments` | attachments between dynamic components |
| `renderables` | renderable components (for visualization only) |

Each of these is a child component of `MechModel` and is implemented as a ComponentList. Special methods are provided for adding and removing items from them. However, applications are not required to use these containers, and may instead create any component containment structure that is appropriate. If not used, the containers will simply remain empty.

## 1.2 Physics simulation

Only a brief summary of ArtiSynth physics simulation is described here. Full details are given in [11] and in the related overview paper.

For purposes of physics simulation, the components of a `MechModel` are grouped as follows:

**Dynamic components**
: Components, such as a particles and rigid bodies, that contain position and velocity state, as well as mass. All dynamic components are instances of the Java interface DynamicComponent.

**Force effectors**
: Components, such as springs or finite elements, that exert forces between dynamic components. All force effectors are instances of the Java interface ForceEffector.

**Constrainers**
: Components that enforce constraints between dynamic components. All constrainers are instances of the Java interface Constrainer.

**Attachments**
: Attachments between dynamic components. While technically these are constraints, they are implemented using a different approach. All attachment components are instances of DynamicAttachment.

The positions, velocities, and forces associated with all the dynamic components are denoted by the composite vectors $\mathbf{q}$, $\mathbf{u}$, and $\mathbf{f}$. In addition, the composite mass matrix is given by $\mathbf{M}$. Newton's second law then gives

$$\mathbf{f} = \frac{d\mathbf{M}\mathbf{u}}{dt} = \mathbf{M}\dot{\mathbf{u}} + \dot{\mathbf{M}}\mathbf{u}, \tag{1.1}$$

where the $\dot{\mathbf{M}}\mathbf{u}$ accounts for various "fictitious" forces.

Each integration step involves solving for the velocities $\mathbf{u}^{k+1}$ at time step $k+1$ given the velocities and forces at step $k$. One way to do this is to solve the expression

$$\mathbf{M}\mathbf{u}^{k+1} = \mathbf{M}\mathbf{u}^{k} + h\bar{\mathbf{f}} \tag{1.2}$$

for $\mathbf{u}^{k+1}$, where $h$ is the step size and $\bar{\mathbf{f}} \equiv \mathbf{f} - \dot{\mathbf{M}}\mathbf{u}$. Given the updated velocities $\mathbf{u}^{k+1}$, one can determine $\dot{\mathbf{q}}^{k+1}$ from

$$\dot{\mathbf{q}}^{k+1} = \mathbf{Q}\mathbf{u}^{k+1}, \tag{1.3}$$

where $\mathbf{Q}$ accounts for situations (like rigid bodies) where $\dot{\mathbf{q}} \neq \mathbf{u}$, and then solve for the updated positions using

$$\mathbf{q}^{k+1} = \mathbf{q}^{k} + h\dot{\mathbf{q}}^{k+1}. \tag{1.4}$$

(1.2) and (1.4) together comprise a simple *symplectic Euler* integrator.

In addition to forces, bilateral and unilateral constraints give rise to locally linear constraints on $\mathbf{u}$ of the form

$$\mathbf{G}(\mathbf{q})\mathbf{u} = 0, \qquad \mathbf{N}(\mathbf{q})\mathbf{u} \geq 0. \tag{1.5}$$

Bilateral constraints may include rigid body joints, FEM incompressibility, and point-surface constraints, while unilateral constraints include contact and joint limits. Constraints give rise to constraint forces (in the directions $\mathbf{G}(\mathbf{q})^T$ and $\mathbf{N}(\mathbf{q})^T$) which supplement the forces of (1.1) in order to enforce the constraint conditions. In addition, for unilateral constraints, we have a complementarity condition in which $\mathbf{N}\mathbf{u} > 0$ implies no constraint force, and a constraint force implies $\mathbf{N}\mathbf{u} = 0$. Any given constraint usually involves only a few dynamic components and so $\mathbf{G}$ and $\mathbf{N}$ are generally sparse.

Adding constraints to the velocity solve (1.2) leads to a mixed linear complementarity problem (MLCP) of the form

$$\begin{pmatrix} \hat{\mathbf{M}}^k & -\mathbf{G}^T & -\mathbf{N}^T \\ \mathbf{G} & 0 & 0 \\ \mathbf{N} & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{u}^{k+1} \\ \tilde{\lambda} \\ \tilde{\theta} \end{pmatrix} + \begin{pmatrix} -\mathbf{M}\mathbf{u}^k - h\hat{\mathbf{f}}^k \\ -\mathbf{g} \\ -\mathbf{n} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \mathbf{w} \end{pmatrix},$$
$$0 \leq \theta \perp \mathbf{w} \geq 0, \tag{1.6}$$

where $\mathbf{w}$ is a slack variable, $\tilde{\lambda}$ and $\tilde{\theta}$ give the force constraint impulses over the time step, and $\mathbf{g}$ and $\mathbf{n}$ are derivative terms defined by

$$\mathbf{g} \equiv -h\dot{\mathbf{G}}\mathbf{u}^k, \quad \mathbf{n} \equiv -h\dot{\mathbf{N}}\mathbf{u}^k, \tag{1.7}$$

to account for time variations in $\mathbf{G}$ and $\mathbf{N}$. In addition, $\hat{\mathbf{M}}$ and $\hat{\mathbf{f}}$ are $\mathbf{M}$ and $\bar{\mathbf{f}}$ augmented with stiffness and damping terms terms to accommodate implicit integration, which is often required for problems involving deformable bodies. The actual constraint forces $\lambda$ and $\theta$ can be determined by dividing the impulses by the time step $h$:

$$\lambda = \tilde{\lambda}/h, \quad \theta = \tilde{\theta}/h. \tag{1.8}$$

We note here that ArtiSynth uses a *full coordinate* formulation, in which the position of each dynamic body is solved using full, or unconstrained, coordinates, with constraint relationships acting to restrict these coordinates. In contrast, some other simulation systems, including OpenSim [7], use *reduced* coordinates, in which the system dynamics are

formulated using a smaller set of coordinates (such as joint angles) that implicitly take the system's constraints into account. Each methodology has its own advantages. Reduced formulations yield systems with fewer degrees of freedom and no constraint errors. On the other hand, full coordinates make it easier to combine and connect a wide range of components, including rigid bodies and FEM models.

Attachments between components can be implemented by constraining the velocities of the attached components using special constraints of the form

$$\mathbf{u}_j = -\mathbf{G}_{j\alpha}\mathbf{u}_\alpha \tag{1.9}$$

where $\mathbf{u}_j$ and $\mathbf{u}_\alpha$ denote the velocities of the attached and non-attached components. The constraint matrix $\mathbf{G}_{j\alpha}$ is sparse, with a non-zero block entry for each *master* component to which the attached component is connected. The simplest case involves attaching a point $j$ to another point $k$, with the simple velocity relationship

$$\mathbf{u}_j = \mathbf{u}_k \tag{1.10}$$

That means that $\mathbf{G}_{j\alpha}$ has a single entry of $-\mathbf{I}$ (where $\mathbf{I}$ is the $3 \times 3$ identity matrix) in the $k$-th block column. Another common case involves connecting a point $j$ to a rigid frame $k$. The velocity relationship for this is

$$\mathbf{u}_j = \mathbf{u}_k - \mathbf{l}_j \times \omega_k \tag{1.11}$$

where $\mathbf{u}_k$ and $\omega_k$ are the translational and rotational velocity of the frame and $l_j$ is the location of the point relative to the frame's origin (as seen in world coordinates). The corresponding $\mathbf{G}_{j\alpha}$ contains a single $3 \times 6$ block entry of the form

$$\begin{pmatrix} \mathbf{I} & [l_j] \end{pmatrix} \tag{1.12}$$

in the $k-th$ block column, where

$$[l] \equiv \begin{pmatrix} 0 & -l_z & l_y \\ l_z & 0 & -l_x \\ -l_y & l_x & 0 \end{pmatrix} \tag{1.13}$$

is a skew-symmetric *cross product matrix*. The attachment constraints $\mathbf{G}_{j\alpha}$ could be added directly to (1.6), but their special form allows us to explicitly solve for $\mathbf{u}_j$, and hence reduce the size of (1.6), by factoring out the attached velocities before solution.

The MLCP (1.6) corresponds to a single step integrator. However, higher order integrators, such as Newmark methods, usually give rise to MLCPs with an equivalent form. Most ArtiSynth integrators use some variation of (1.6) to determine the system velocity at each time step.

To set up (1.6), the `MechModel` component hierarchy is traversed and the methods of the different component types are queried for the required values. Dynamic components (type `DynamicComponent`) provide $\mathbf{q}$, $\mathbf{u}$, and $\mathbf{M}$; force effectors (`ForceEffector`) determine $\hat{\mathbf{f}}$ and the stiffness/damping augmentation used to produce $\hat{\mathbf{M}}$; constrainers (`Constrainer`) supply $\mathbf{G}$, $\mathbf{N}$, $\mathbf{g}$ and $\mathbf{n}$, and attachments (`DynamicAttachment`) provide the information needed to factor out attached velocities.

## 1.3 Basic packages

The core code of the ArtiSynth project is divided into three main packages, each with a number of sub-packages.

### 1.3.1 maspack

The packages under `maspack` contain general computational utilities that are independent of ArtiSynth and could be used in a variety of other contexts. The main packages are:

```
maspack.util              // general utilities
maspack.matrix            // matrix and linear algebra
maspack.graph             // graph algorithms
maspack.fileutil          // remote file access
maspack.properties        // property implementation
maspack.spatialmotion     // 3D spatial motion and dynamics
maspack.solvers           // LCP solvers and linear solver interfaces
maspack.render            // viewer and rendering classes
maspack.geometry          // 3D geometry and meshes
maspack.collision         // collision detection
maspack.widgets           // Java swing widgets for maspack data types
maspack.apps              // stand-alone programs based only on maspack
```

### 1.3.2 artisynth.core

The packages under `artisynth.core` contain the core code for ArtiSynth model components and its GUI infrastructure.

```
artisynth.core.util         // general ArtiSynth utilities
artisynth.core.modelbase    // base classes for model components
artisynth.core.materials    // materials for springs and finite elements
artisynth.core.mechmodels   // basic mechanical models
artisynth.core.femmodels    // finite element models
artisynth.core.probes       // input and output probes
artisynth.core.workspace    // RootModel and associated components
artisynth.core.driver       // start ArtiSynth and drive the simulation
artisynth.core.gui          // graphical interface
artisynth.core.inverse      // inverse tracking controller
artisynth.core.moviemaker   // used for making movies
artisynth.core.renderables  // components that are strictly visual
artisynth.core.opensim      // OpenSim model parser (under development)
artisynth.core.mfreemodels  // mesh free models (experimental, not supported)
```

### 1.3.3 artisynth.demos

These packages contain demonstration models that illustrate ArtiSynth's modeling capabilities:

```
artisynth.demos.mech        // mechanical model demos
artisynth.demos.fem         // demos involving finite elements
artisynth.demos.inverse     // demos involving inverse control
artisynth.demos.tutorial    // demos in this manual
```

## 1.4 Properties

ArtiSynth components expose *properties*, which provide a uniform interface for accessing their internal parameters and state. Properties vary from component to component; those for `RigidBody` include `position`, `orientation`, `mass`, and `density`, while those for `AxialSpring` include `restLength` and `material`. Properties are particularly useful for automatically creating control panels and probes, as described in Section 5. They are also used for automating component serialization.

Properties are described only briefly in this section; more detailed descriptions are available in the Maspack Reference Manual and the overview paper.

The set of properties defined for a component is fixed for that component's class; while property values may vary between component instances, their definitions are class-specific. Properties are exported by a class through code contained in the class definition, as described in Section 5.2.

### 1.4.1 Querying and setting property values

Each property has a unique name that can be used to access its value interactively in the GUI. This can be done either by using a custom control panel (Section 5.1) or by selecting the component and choosing Edit properties ... from the right-click context menu).

Properties can also be accessed in code using their set/get accessor methods. Unless otherwise specified, the names for these are formed by simply prepending `set` or `get` to the property's name. More specifically, a property with the name foo and a value type of `Bar` will usually have accessor signatures of

```
Bar getFoo()

void setFoo (Bar value)
```

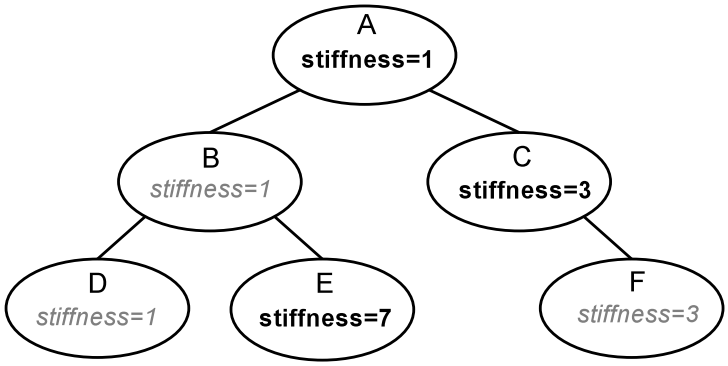



Figure 1.1: Inheritance of a property named *stiffness* among a component hierarchy. Explicit settings are in bold; inherited settings are in gray italic.

### 1.4.2 Property handles and paths

A property's name can also be used to obtain a *property handle* through which its value may be queried or set generically. Property handles are implemented by the class Property and are returned by the component's getProperty() method. `getProperty()` takes a property's name and returns the corresponding handle. For example, components of type `Muscle` have a property `excitation`, for which a handle may be obtained using a code fragment such as

```
Muscle muscle;
...
Property prop = muscle.getProperty ("excitation");
```

Property handles can also be obtained for subcomponents, using a *property path* that consists of a path to the subcomponent followed by a colon '`:`' and the property name. For example, to obtain the `excitation` property for a subcomponent located by `axialSprings/lad` relative to a `MechModel`, one could use a call of the form

```
MechModel mech;
...
Property prop = mech.getProperty ("axialSprings/lad:excitation");
```

### 1.4.3 Composite and inheritable properties

Composite properties are possible, in which a property value is a composite object that in turn has subproperties. A good example of this is the `RenderProps` class, which is associated with the property `renderProps` for renderable objects and which itself can have a number of subproperties such as `visible`, `faceStyle`, `faceColor`, `lineStyle`, `lineColor`, etc.

Properties can be declared to be `inheritable`, so that their values can be inherited from the same properties hosted by ancestor components further up the component hierarchy. Inheritable properties require a more elaborate declaration and are associated with a *mode* which may be either `Explicit` or `Inherited`. If a property's mode is inherited, then its value is obtained from the closest ancestor exposing the same property whose mode is explicit. In Figure (1.1), the property *stiffness* is explicitly set in components A, C, and E, and inherited in B and D (which inherit from A) and F (which inherits from C).

## 1.5 Creating an application model

ArtiSynth applications are created by writing and compiling an *application model* that is a subclass of `RootModel`. This application-specific root model is then loaded and run by the ArtiSynth program.

The code for the application model should:

- Declare a no-args constructor

- Override the `RootModel` build() method to construct the application.

ArtiSynth can load a model either using the build method or by reading it from a file:

**Build method**

ArtiSynth creates an instance of the model using the no-args constructor, assigns it a name (which is either user-specified or the simple name of the class), and then calls the `build()` method to perform the actual construction.

**Reading from a file**

ArtiSynth creates an instance of the model using the no-args constructor, and then the model is named and constructed by reading the file.

The no-args constructor should perform whatever initialization is required in both cases, while the `build()` method takes the place of the file specification. Unless a model is originally created using a file specification (which is very tedious), the first time creation of a model will almost always entail using the `build()` method.

The general template for application model code looks like this:

```
package artisynth.models.experimental; // package where the model resides
import artisynth.core.workspace.RootModel;
... other imports ...

public class MyModel extends RootModel {

   // no-args constructor
   public MyModel() {
      ... basic initialization ...
   }

   // build method to do model construction
   public void build (String[] args) {
      ... code to build the model ....
   }
}
```

Here, the model itself is called `MyModel`, and is defined in the (hypothetical) package `artisynth.models.experimental` (placing models in the super package `artisynth.models` is common practice but not necessary).

> Note: The `build()` method was only introduced in ArtiSynth 3.1. Prior to that, application models were constructed using a constructor taking a `String` argument supplying the name of the model. This method of model construction still works but is deprecated.

### 1.5.1 Implementing the build() method

As mentioned above, the `build()` method is responsible for actual model construction. Many applications are built using a single top-level `MechModel`. Build methods for these may look like the following:

```
   public void build (String[] args) {
      MechModel mech = new MechModel("mech");
      addModel (mech);

      ... create and add components to the mech model ...
      ... create and add any needed agents to the root model ...

   }
```

First, a MechModel is created (with the name `"mech"` in this example, although any name, or no name, may be given) and added to the list of models in the root model using the `addModel()` method. Subsequent code then creates and adds the components required by the `MechModel`, as described in Sections 3, 4 and 6. The `build()` method also creates and adds to the root model any agents required by the application (controllers, probes, etc.), as described in Section 5.

When constructing a model, there is no fixed order in which components need to be added. For instance, in the above example, `addModel(mech)` could be called near the end of the `build()` method rather than at the beginning. The only restriction is that when a component is added to the hierarchy, all other components that it refers to should already have been added to the hierarchy. For instance, an axial spring (Section 3.1) refers to two points. When it is added to the hierarchy, those two points should already be present in the hierarchy.

The `build()` method supplies a `String` array as an argument, which can be used to transmit application arguments in a manner analogous to the `args` argument passed to static `main()` methods. Build arguments can be specified when a model is loaded directly from a class using Models > Load from class ..., or when the *startup model* is set to automatically load a model when ArtiSynth is first started (Settings > Startup model). Details are given in the "Loading, Simulating and Saving Models" section of the User Interface Guide.

Build arguments can also be listed directly on the ArtiSynth command line when specifying a model to load using the `-model <classname>` option. This is done by enclosing the desired arguments within square brackets `[ ]` immediately following the `-model` option. So, for example,

```
> artisynth -model projects.MyModel [ -size 50 ]
```

would cause the strings `"-size"` and `"50"` to be passed to the `build()` method of `MyModel`.

### 1.5.2 Making models visible to ArtiSynth

In order to load an application model into ArtiSynth, the classes associated with its implementation must be made visible to ArtiSynth. This usually involves adding the top-level class folder associated with the application code to the classpath used by ArtiSynth.

> The demonstration models referred to in this guide belong to the package `artisynth.demos.tutorial` and are already visible to ArtiSynth.

In most current ArtiSynth projects, classes are stored in a folder tree separate from the source code, with the top-level class folder named `classes`, located one level below the project root folder. A typical top-level class folder might be stored in a location like this:

```
/home/joeuser/artisynthProjects/classes
```

In the example shown in Section 1.5, the model was created in the package `artisynth.models.experimental`. Since Java classes are arranged in a folder structure that mirrors package names, with respect to the sample project folder shown above, the model class would be located in

```
/home/joeuser/artisynthProjects/classes/artisynth/models/experimental
```

At present there are three ways to make top-level class folders known to ArtiSynth:

**Add projects to your Eclipse launch configuration**

If you are using the Eclipse IDE, then you can add the project in which are developing your model code to the launch configuration that you use to run ArtiSynth. Other IDEs will presumably provide similar functionality.

**Add the folders to the external classpath**

You can explicitly add the class folders to ArtiSynth's external classpath. The easiest way to do this is to select "Settings > External classpath ..." from the Settings menu, which will open an external classpath editor which lists all the classpath entries in a large panel on the left. (When ArtiSynth is first installed, the external classpath has no entries, and so this panel will be blank.) Class folders can then by added via the "Add class folder" button, and the classpath is saved using the Save button.

**Add the folders to your CLASSPATH environment variable**

If you are running ArtiSynth from the command line, using the `artisynth` command (or `artisynth.bat` on Windows), then you can define a CLASSPATH environment variable in your environment and add the needed folders to this.

All of these methods are described in more detail in the "Installing External Models and Packages" section of the ArtiSynth Installation Guide (available for Linux, Windows, and MacOS).

### 1.5.3 Loading and running a model

If a model's classes are visible to ArtiSynth, then it may be loaded into ArtiSynth in several ways:

**Loading from the Model menu**

If the root model is contained in a package located under `artisynth.demos` or `artisynth.models`, then it will appear in the default model menu (Models in the main menu bar) under the submenu All demos or All models.

**Loading by class path**

A model may also be loaded by choosing "Load from class ..." from the Models menu and specifying its package name and then choosing its root model class. It is also possible to use the `-model <classname>` command line argument to have a model loaded directly into ArtiSynth when it starts up.

**Loading from a file**

If a model has been saved to a `.art` file, it may be loaded from that file by choosing File > Load model ....

These methods are described in detail in the section "Loading and Simulating Models" of the ArtiSynth User Interface Guide.

> The demonstration models referred to in this guide should already be present in the model menu and may be loaded from the submenu Models > All demos > tutorial.

Once a model is loaded, it can be simulated, or *run*. Simulation of the model can then be started, paused, single-stepped, or reset using the play controls (Figure 1.2) located at the upper right of the ArtiSynth window frame. Starting and stopping a simulation is done by clicking play/pause, while reset resets the simulation to time 0. The single-step button advances the simulation by one time step. The stop-all button will also stop the simulation, along with any Jython commands or scripts that are running.


Figure 1.2: The ArtiSynth play controls. From left to right: step size control, current simulation time, and the reset, skip-back, play/pause, single-step, skip-forward and stop-all buttons.

Comprehensive information on exploring and interacting with models is given in the ArtiSynth User Interface Guide.

# Chapter 2

# Supporting classes

ArtiSynth uses a large number of supporting classes, mostly defined in the super package `maspack`, for handling mathematical and geometric quantities. Those that are referred to in this manual are summarized in this section.

## 2.1 Vectors and matrices

Among the most basic classes are those used to implement vectors and matrices, defined in `maspack.matrix`. All vector classes implement the interface Vector and all matrix classes implement Matrix, which provide a number of standard methods for setting and accessing values and reading and writing from I/O streams.

General sized vectors and matrices are implemented by VectorNd and MatrixNd. These provide all the usual methods for linear algebra operations such as addition, scaling, and multiplication:

```
VectorNd v1 = new VectorNd (5);         // create a 5 element vector
VectorNd v2 = new VectorNd (5);
VectorNd vr = new VectorNd (5);
MatrixNd M = new MatrixNd (5, 5);       // create a 5 x 5 matrix

M.setIdentity();                        // M = I
M.scale (4);                            // M = 4*M

v1.set (new double[] {1, 2, 3, 4, 5}); // set values
v2.set (new double[] {0, 1, 0, 2, 0});
v1.add (v2);                            // v1 += v2
M.mul (vr, v1);                         // vr = M*v1

System.out.println ("result=" + vr.toString ("%8.3f"));
```

As illustrated in the above example, vectors and matrices both provide a `toString()` method that allows their elements to be formatted using a C-printf style format string. This is useful for providing concise and uniformly formatted output, particularly for diagnostics. The output from the above example is

```
result=   4.000   12.000   12.000   24.000   20.000
```

Detailed specifications for the format string are provided in the documentation for NumberFormat.set(String). If either no format string, or the string `"%g"`, is specified, `toString()` formats all numbers using the full-precision output provided by `Double.toString(value)`.

For computational efficiency, a number of fixed-size vectors and matrices are also provided. The most commonly used are those defined for three dimensions, including Vector3d and Matrix3d:

```
Vector3d v1 = new Vector3d (1, 2, 3);
Vector3d v2 = new Vector3d (3, 4, 5);
Vector3d vr = new Vector3d ();
Matrix3d M = new Matrix3d();
```

```
M.set (1, 2, 3,  4, 5, 6,  7, 8, 9);

M.mul (vr, v1);         // vr = M * v1
vr.scaledAdd (2, v2);   // vr += 2*v2;
vr.normalize();         // normalize vr
System.out.println ("result=" + vr.toString ("%8.3f"));
```

## 2.2 Rotations and transformations

`maspack.matrix` contains a number classes that implement rotation matrices, rigid transforms, and affine transforms.

Rotations (Section A.1) are commonly described using a RotationMatrix3d, which implements a rotation matrix and contains numerous methods for setting rotation values and transforming other quantities. Some of the more commonly used methods are:

```
RotationMatrix3d();          // create and set to the identity
RotationMatrix3d(u, angle); // create and set using an axis-angle

setAxisAngle (u, ang);       // set using an axis-angle
setRpy (roll, pitch, yaw);  // set using roll-pitch-yaw angles
setEuler (phi, theta, psi); // set using Euler angles
invert ();                   // invert this rotation
mul (R)                      // post multiply this rotation by R
mul (R1, R2);                // set this rotation to R1*R2
mul (vr, v1);                // vr = R*v1, where R is this rotation
```

Rotations can also be described by AxisAngle, which characterizes a rotation as a single rotation about a specific axis.

Rigid transforms (Section A.2) are used by ArtiSynth to describe a rigid body's pose, as well as its relative position and orientation with respect to other bodies and coordinate frames. They are implemented by RigidTransform3d, which exposes its rotational and translational components directly through the fields `R` (a `RotationMatrix3d`) and `p` (a `Vector3d`). Rotational and translational values can be set and accessed directly through these fields. In addition, `RigidTransform3d` provides numerous methods, some of the more commonly used of which include:

```
RigidTransform3d();          // create and set to the identity
RigidTransform3d(x, y, z);  // create and set translation to x, y, z

// create and set translation to x, y, z and rotation to roll-pitch-yaw
RigidTransform3d(x, y, z, roll, pitch, yaw);

invert ();                   // invert this transform
mul (T)                      // post multiply this transform by T
mul (T1, T2);                // set this transform to T1*T2
mulLeftInverse (T1, T2);     // set this transform to inv(T1)*T2
```

Affine transforms (Section A.3) are used by ArtiSynth to effect scaling and shearing transformations on components. They are implemented by AffineTransform3d.

Rigid transformations are actually a specialized form of affine transformation in which the basic transform matrix equals a rotation. `RigidTransform3d` and `AffineTransform3d` hence both derive from the same base class AffineTransform3dBase.

## 2.3 Points and Vectors

The rotations and transforms described above can be used to transform both vectors and points in space.

Vectors are most commonly implemented using Vector3d, while points can be implemented using the subclass Point3d. The only difference between `Vector3d` and `Point3d` is that the former ignores the translational component of rigid and

affine transforms; i.e., as described in Sections A.2 and A.3, a vector `v` has an implied homogeneous representation of

$$\mathbf{v}^* \equiv \begin{pmatrix} \mathbf{v} \\ 0 \end{pmatrix}, \tag{2.1}$$

while the representation for a point `p` is

$$\mathbf{p}^* \equiv \begin{pmatrix} \mathbf{p} \\ 1 \end{pmatrix}. \tag{2.2}$$

Both classes provide a number of methods for applying rotational and affine transforms. Those used for rotations are

```
void transform (R);             // this = R * this
void transform (R, v1);         // this = R * v1
void inverseTransform (R);      // this = inverse(R) * this
void inverseTransform (R, v1);  // this = inverse(R) * v1
```

where `R` is a rotation matrix and `v1` is a vector (or a point in the case of `Point3d`).

The methods for applying rigid or affine transforms include:

```
void transform (X);             // transforms this by X
void transform (X, v1);         // sets this to v1 transformed by X
void inverseTransform (X);      // transforms this by the inverse of X
void inverseTransform (X, v1);  // sets this to v1 transformed by inverse of X
```

where `X` is a rigid or affine transform. As described above, in the case of `Vector3d`, these methods ignore the translational part of the transform and apply only the matrix component (`R` for a `RigidTransform3d` and `A` for an `AffineTransform3d`). In particular, that means that for a `RigidTransform3d` given by `X` and a `Vector3d` given by `v`, the method calls

```
v.transform (X.R)
v.transform (X)
```

produce the same result.

## 2.4 Spatial vectors and inertias

The velocities, forces and inertias associated with 3D coordinate frames and rigid bodies are represented using the 6 DOF spatial quantities described in Sections A.5 and A.6. These are implemented by classes in the package `maspack.spatialmotion`.

Spatial velocities (or twists) are implemented by Twist, which exposes its translational and angular velocity components through the publicly accessible fields `v` and `w`, while spatial forces (or wrenches) are implemented by Wrench, which exposes its translational force and moment components through the publicly accessible fields `f` and `m`.

Both `Twist` and `Wrench` contain methods for algebraic operations such as addition and scaling. They also contain `transform()` methods for applying rotational and rigid transforms. The rotation methods simply transform each component by the supplied rotation matrix. The rigid transform methods, on the other hand, assume that the supplied argument represents a transform between two frames fixed within a rigid body, and transform the twist or wrench accordingly, using either (A.27) or (A.29).

The spatial inertia for a rigid body is implemented by SpatialInertia, which contains a number of methods for setting its value given various mass, center of mass, and inertia values, and querying the values of its components. It also contains methods for scaling and adding, transforming between coordinate systems, inversion, and multiplying by spatial vectors.

## 2.5 Meshes

ArtiSynth makes extensive use of 3D meshes, which are defined in `maspack.geometry`. They are used for a variety of purposes, including visualization, collision detection, and computing physical properties (such as inertia or stiffness variation within a finite element model).

A mesh is essentially a collection of vertices (i.e., points) that are topologically connected in some way. All meshes extend the abstract base class MeshBase, which supports the vertex definitions, while subclasses provide the topology.

Through `MeshBase`, all meshes provide methods for adding and accessing vertices. Some of these include:

```
int numVertices();                  // return the number of vertices
Vertex3d getVertex (int idx);       // return the idx-th vertex
void addVertex (Vertex3d vtx);      // add vertex vtx to the mesh
Vertex3d addVertex (Point3d p);     // create and return a vertex at position p
void removeVertex (Vertex3d vtx);   // remove vertex vtx for the mesh
ArrayList<Vertex3d> getVertices(); // return the list of vertices
```

Vertices are implemented by Vertex3d, which defines the position of the vertex (returned by the method getPosition()), and also contains support for topological connections. In addition, each vertex maintains an index, obtainable via getIndex(), that equals the index of its location within the mesh's vertex list. This makes it easy to set up parallel array structures for augmenting mesh vertex properties.

Mesh subclasses currently include:

**PolygonalMesh**

Implements a 2D surface mesh containing faces implemented using half-edges.

**PolylineMesh**

Implements a mesh consisting of connected line-segments (polylines).

**PointMesh**

Implements a point cloud with no topological connectivity.

PolygonalMesh is used quite extensively and provides a number of methods for implementing faces, including:

```
int numFaces();                 // return the number of faces
Face getFace (int idx);         // return the idx-th face
Face addFace (int[] vidxs);     // create and add a face using vertex indices
void removeFace (Face f);       // remove the face f
ArrayList<Face> getFaces();     // return the list of faces
```

The class Face implements a face as a counter-clockwise arrangement of vertices linked together by half-edges (class HalfEdge). `Face` also supplies a face's (outward facing) normal via getNormal().

Some mesh uses within ArtiSynth, such as collision detection, require a *triangular* mesh; i.e., one where all faces have three vertices. The method isTriangular() can be used to check for this. Meshes that are not triangular can be made triangular using triangulate().

### 2.5.1 Mesh creation

Meshes are most commonly created using either one of the factory methods supplied by MeshFactory, or by reading a definition from a file (Section 2.5.5). However, it is possible to create a mesh by direct construction. For example, the following code fragment creates a simple closed tetrahedral surface:

```
// a simple four-faced tetrahedral mesh
PolygonalMesh mesh = new PolygonalMesh();
mesh.addVertex (0, 0, 0);
mesh.addVertex (1, 0, 0);
mesh.addVertex (0, 1, 0);
mesh.addVertex (0, 0, 1);
mesh.addFace (new int[] { 0, 2, 1 });
mesh.addFace (new int[] { 0, 3, 2 });
mesh.addFace (new int[] { 0, 1, 3 });
mesh.addFace (new int[] { 1, 2, 3 });
```

Some of the more commonly used factory methods for creating polyhedral meshes include:

```
MeshFactory.createSphere (radius, nslices, nlevels);
MeshFactory.createIcosahedralSphere (radius, divisons);
MeshFactory.createBox (widthx, widthy, widthz);
MeshFactory.createCylinder (radius, height, nslices);
MeshFactory.createPrism (double[] xycoords, height);
MeshFactory.createTorus (rmajor, rminor, nmajor, nminor);
```

Each factory method creates a mesh in some standard coordinate frame. After creation, the mesh can be transformed using the transform(X) method, where `X` is either a rigid transform ( RigidTransform3d) or a more general affine transform (AffineTransform3d). For example, to create a rotated box centered on $(5,6,7)$, one could do:

```
// create a box centered at the origin with widths 10, 20, 30:
PolygonalMesh box = MeshFactory.createBox (10, 20, 20);

// move the origin to 5, 6, 7 and rotate using roll-pitch-yaw
// angles 0, 0, 45 degrees:
box.transform (
   new RigidTransform3d (5, 6, 7,  0, 0, Math.toRadians(45)));
```

One can also scale a mesh using scale(s), where `s` is a single scale factor, or scale(sx,sy,sz), where `sx`, `sy`, and `sz` are separate scale factors for the x, y and z axes. This provides a useful way to create an ellipsoid:

```
 // start with a unit sphere with 12 slices and 6 levels ...
PolygonalMesh ellipsoid = MeshFactory.createSphere (1.0, 12, 6);

// and then turn it into an ellipsoid by scaling about the axes:
ellipsoid.scale (1.0, 2.0, 3.0);
```

MeshFactory can also be used to create new meshes by performing Boolean operations on existing ones:

```
MeshFactory.getIntersection (mesh1, mesh2);
MeshFactory.getUnion (mesh1, mesh2);
MeshFactory.getSubtraction (mesh1, mesh2);
```

### 2.5.2 Setting normals, colors, and textures

Meshes provide support for adding normal, color, and texture information, with the exact interpretation of these quantities depending upon the particular mesh subclass. Most commonly this information is used simply for rendering, but in some cases normal information might also be used for physical simulation.

> For polygonal meshes, the normal information described here is used only for smooth shading. When flat shading is requested, normals are determined directly from the faces themselves.

Normal information can be set and queried using the following methods:

```
setNormals (
   List<Vector3d> nrmls, int[] indices);  // set all normals and indices

ArrayList<Vector3d> getNormals();         // get all normals
int[] getNormalIndices();                 // get all normal indices
int numNormals();                         // return the number of normals
Vector3d getNormal (int idx);             // get the normal at index idx

setNormal (int idx, Vector3d nrml);       // set the normal at index idx
clearNormals();                           // clear all normals and indices
```

The method setNormals() takes two arguments: a set of normal vectors (`nrmls`), along with a set of index values (`indices`) that map these normals onto the vertices of each of the mesh's geometric features. Often, there will be one unique normal per vertex, in which case `nrmls` will have a size equal to the number of vertices, but this is not always the case, as described below. Features for the different mesh subclasses are: faces for `PolygonalMesh`, polylines for

PolylineMesh, and vertices for PointMesh. If indices is specified as null, then normals is assumed to have a size equal to the number of vertices, and an appropriate index set is created automatically using createVertexIndices() (described below). Otherwise, indices should have a size of equal to the number of features times the number of vertices per feature. For example, consider a PolygonalMesh consisting of two triangles formed from vertex indices (0, 1, 2) and (2, 1, 3), respectively. If normals are specified and there is one unique normal per vertex, then the normal indices are likely to be

```
[ 0 1 2  2 1 3 ]
```

As mentioned above, sometimes there may be *more* than one normal per vertex. This happens in cases when the same vertex uses different normals for different faces. In such situations, the size of the nrmls argument will exceed the number of vertices.

The method setNormals() makes internal copies of the specified normal and index information, and this information can be later read back using getNormals() and getNormalIndices(). The number of normals can be queried using numNormals(), and individual normals can be queried or set using getNormal(idx) and setNormal(idx,nrml). All normals and indices can be explicitly cleared using clearNormals().

Color and texture information can be set using analogous methods. For colors, we have

```
setColors (
   List<float[]> colors, int[] indices);  // set all colors and indices

ArrayList<float[]> getColors();           // get all colors
int[] getColorIndices();                  // get all color indices
int numColors();                          // return the number of colors
float[] getColor (int idx);               // get the color at index idx

setColor (int idx, float[] color);        // set the color at index idx
setColor (int idx, Color color);          // set the color at index idx
setColor (
   int idx, float r, float g, float b, float a); // set the color at index idx
clearColors();                            // clear all colors and indices
```

When specified as float[], colors are given as RGB or RGBA values, in the range $[0, 1]$, with array lengths of 3 and 4, respectively. The colors returned by getColors() are always RGBA values.

With colors, there may often be *fewer* colors than the number of vertices. For instance, we may have only two colors, indexed by 0 and 1, and want to use these to alternately color the mesh faces. Using the two-triangle example above, the color indices might then look like this:

```
[ 0 0 0 1 1 1 ]
```

Finally, for texture coordinates, we have

```
setTextureCoords (
   List<Vector3d> coords, int[] indices); // set all texture coords and indices

ArrayList<Vector3d> getTextureCoords();   // get all texture coords
int[] getTextureIndices();                // get all texture indices
int numTextureCoords();                   // return the number of texture coords
Vector3d getTextureCoords (int idx);      // get texture coords at index idx

setTextureCoords (int idx, Vector3d coords);// set texture coords at index idx
clearTextureCoords();                     // clear all texture coords and indices
```

When specifying indices using setNormals, setColors, or setTextureCoords, it is common to use the same index set as that which associates vertices with features. For convenience, this index set can be created automatically using

```
int[] createVertexIndices();
```

Alternatively, we may sometimes want to create a index set that assigns the same attribute to each feature vertex. If there is one attribute per feature, the resulting index set is called a *feature index* set, and can be created using

```
int[] createFeatureIndices();
```

If we have a mesh with three triangles and one color per triangle, the resulting feature index set would be

```
[ 0 0 0 1 1 1 2 2 2 ]
```

> Note: when a mesh is modified by the *addition* of new features (such as faces for PolygonalMesh), all normal, color and texture information is cleared by default (with normal information being automatically recomputed on demand if automatic normal creation is enabled; see Section 2.5.3). When a mesh is modified by the *removal* of features, the index sets for normals, colors and textures are adjusted to account for the removal.
>
> For colors, it is possible to request that a mesh explicitly maintain colors for either its vertices or features (Section 2.5.4). When this is done, colors will persist when vertices or features are added or removed, with default colors being automatically created as necessary.

Once normals, colors, or textures have been set, one may want to know which of these attributes are associated with the vertices of a specific feature. To know this, it is necessary to find that feature's offset into the attribute's index set. This offset information can be found using the array returned by

```
int[] getFeatureIndexOffsets()
```

For example, the three normals associated with a triangle at index `ti` can be obtained using

```
int[] indexOffs = mesh.getFeatureIndexOffsets();
ArrayList<Vector3d> nrmls = mesh.getNormals();
// get the three normals associated with the triangle at index ti:
Vector3d n0 = nrmls.get (indexOffs[ti]);
Vector3d n1 = nrmls.get (indexOffs[ti]+1);
Vector3d n2 = nrmls.get (indexOffs[ti]+2);
```

Alternatively, one may use the convenience methods

```
Vector3d getFeatureNormal (int fidx, int k);
float[] getFeatureColor (int fidx, int k);
Vector3d getFeatureTextureCoords (int fidx, int k);
```

which return the attribute values for the *k*-th vertex of the feature indexed by `fidx`.

In general, the various `get` methods return references to internal storage information and so should **not** be modified. However, specific values within the lists returned by getNormals(), getColors(), or getTextureCoords() may be modified by the application. This may be necessary when attribute information changes as the simulation proceeds. Alternatively, one may use methods such as setNormal(idx,nrml) setColor(idx,color), or setTextureCoords(idx,coords).

Also, in some situations, particularly with colors and textures, it may be desirable to *not* have color or texture information defined for certain features. In such cases, the corresponding index information can be specified as -1, and the `getNormal()`, `getColor()` and `getTexture()` methods will return `null` for the features in question.

### 2.5.3 Automatic creation of normals and hard edges

For some mesh subclasses, if normals are not explicitly set, they are computed automatically whenever `getNormals()` or `getNormalIndices()` is called. Whether or not this is true for a particular mesh can be queried by the method

```
boolean hasAutoNormalCreation();
```

Setting normals explicitly, using a call to `setNormals(nrmls,indices)`, will overwrite any existing normal information, automatically computed or otherwise. The method

```
boolean hasExplicitNormals();
```

will return `true` if normals have been explicitly set, and `false` if they have been automatically computed or if there is currently no normal information. To explicitly remove normals from a mesh which has automatic normal generation, one may call `setNormals()` with the `nrmls` argument set to `null`.

More detailed control over how normals are automatically created may be available for specific mesh subclasses. For example, `PolygonalMesh` allows normals to be created with multiple normals per vertex, for vertices that are associated with either open or hard edges. This ability can be controlled using the methods

```
boolean getMultipleAutoNormals();
setMultipleAutoNormals (boolean enable);
```

Having multiple normals means that even with smooth shading, open or hard edges will still appear sharp. To make an edge hard within a `PolygonalMesh`, one may use the methods

```
boolean setHardEdge (Vertex3d v0, Vertex3d v1);
boolean setHardEdge (int vidx0, int vidx1);
boolean hasHardEdge (Vertex3d v0, Vertex3d v1);
boolean hasHardEdge (int vidx0, int vidx1);
int numHardEdges();
int clearHardEdges();
```

which control the hardness of edges between individual vertices, specified either directly or using their indices.

### 2.5.4 Vertex and feature coloring

The method setColors() makes it possible to assign any desired coloring scheme to a mesh. However, it does require that the user explicitly reset the color information whenever new features are added.

For convenience, an application can also request that a mesh explicitly maintain colors for either its vertices or features. These colors will then be maintained when vertices or features are added or removed, with default colors being automatically created as necessary.

Vertex-based coloring can be requested with the method

```
setVertexColoringEnabled();
```

This will create a separate (default) color for each of the mesh's vertices, and set the color indices to be equal to the vertex indices, which is equivalent to the call

```
setColors (colors, createVertexIndices());
```

where `colors` contains a default color for each vertex. However, once vertex coloring is enabled, the color and index sets will be updated whenever vertices or features are added or removed. Meanwhile, applications can query or set the colors for any vertex using `getColor(idx)`, or any of the various `setColor` methods. Whether or not vertex coloring is enabled can be queried using

```
getVertexColoringEnabled();
```

Once vertex coloring is established, the application will typically want to set the colors for all vertices, perhaps using a code fragment like this:

```
mesh.setVertexColoringEnabled();
for (int i=0; i<mesh.numVertices(); i++) {
   ... compute color for the vertex ...
   mesh.setColor (i, color);
}
```

Similarly, feature-based coloring can be requested using the method

```
setFeatureColoringEnabled();
```

This will create a separate (default) color for each of the mesh's features (faces for PolygonalMesh, polylines for PolylineMesh, etc.), and set the color indices to equal the feature index set, which is equivalent to the call

```
setColors (colors, createFeatureIndices());
```

where `colors` contains a default color for each feature. Applications can query or set the colors for any vertex using `getColor(idx)`, or any of the various `setColor` methods. Whether or not feature coloring is enabled can be queried using

```
getFeatureColoringEnabled();
```

### 2.5.5 Reading and writing mesh files

`PolygonalMesh`, `PolylineMesh`, and `PointMesh` all provide constructors that allow them to be created from a definition file, with the file format being inferred from the file name suffix:

```
PolygonalMesh (String fileName) throws IOException
PolygonalMesh (File file) throws IOException

PolylineMesh (String fileName) throws IOException
PolylineMesh (File file) throws IOException

PointMesh (String fileName) throws IOException
PointMesh (File file) throws IOException
```

| Suffix | Format | `PolygonalMesh` | `PolylineMesh` | `PointMesh` |
|---|---|---|---|---|
| .obj | Alias Wavefront | X | X | X |
| .ply | Polygon file format | X | | X |
| .stl | STereoLithography | X | | |
| .gts | GNU triangulated surface | X | | |
| .off | Object file format | X | | |
| .vtk | VTK ascii format | X | | |
| .vtp | VTK XML format | X | X | |

Table 2.1: Mesh file formats which are supported for different mesh types

The currently supported file formats, and their applicability to the different mesh types, are given in Table 2.1. For example, a `PolygonalMesh` can be read from either an Alias Wavefront `.obj` file or an `.stl` file, as show in the following example:

```
PolygonalMesh mesh0 = null;
PolygonalMesh mesh1 = null;
try {
   mesh0 = new PolygonalMesh ("meshes/torus.obj");
}
catch (IOException e) {
   System.err.println ("Can't read mesh:");
   e.printStackTrace();
}
try {
   mesh1 = new PolygonalMesh ("meshes/cylinder.stl");
}
catch (IOException e) {
   System.err.println ("Can't read mesh:");
   e.printStackTrace();
}
```

The file-based mesh constructors may throw an I/O exception if an I/O error occurs or if the indicated format does not support the mesh type. This exception must either be caught, as in the example above, or thrown out of the calling routine.

In addition to file-based constructors, all mesh types implement read and write methods that allow a mesh to be read from or written to a file, with the file format again inferred from the file name suffix:

```
read (File file) throws IOException
write (File file) throws IOException

read (File file, boolean zeroIndexed) throws IOException
write (File file, String fmtStr, boolean zeroIndexed) throws IOException
```

For the latter methods, the argument `zeroIndexed` specifies zero-based vertex indexing in the case of Alias Wavefront `.obj` files, while `fmtStr` is a C-style format string specifying the precision and style with which the vertex coordinates should be written. (In the former methods, zero-based indexing is false and vertices are written using full precision.)

As an example, the following code fragment writes a mesh as an `.stl` file:

```
PolygonalMesh mesh;

... initialize ...

try {
   mesh.write (new File ("data/mymesh.obj"));
}
catch (IOException e) {
   System.err.println ("Can't write mesh:");
   e.printStackTrace();
}
```

Sometimes, more explicit control is needed when reading or writing a mesh from/to a given file format. The constructors and read/write methods described above make use of a specific set of reader and writer classes located in the package `maspack.geometry.io`. These can be used directly to provide more explicit read/write control. The readers and writers (if implemented) associated with the different formats are given in Table 2.2.

| Suffix | Format | Reader class | Writer class |
|---|---|---|---|
| .obj | Alias Wavefront | `WavefrontReader` | `WavefrontWriter` |
| .ply | Polygon file format | `PlyReader` | `PlyWriter` |
| .stl | STereoLithography | `StlReader` | `StlWriter` |
| .gts | GNU triangulated surface | `GtsReader` | `GtsWriter` |
| .off | Object file format | `OffReader` | `OffWriter` |
| .vtk | VTK ascii format | `VtkAsciiReader` | |
| .vtp | VTK XML format | `VtkXmlReader` | |

Table 2.2: Reader and writer classes associated with the different mesh file formats

The general usage pattern for these classes is to construct the desired reader or writer with a path to the desired file, and then call `readMesh()` or `writeMesh()` as appropriate:

```
// read a mesh from a .obj file:
WavefrontReader reader = new WavefrontReader ("meshes/torus.obj");
PolygonalMesh mesh = null;
try {
   mesh = reader.readMesh();
}
catch (IOException e) {
   System.err.println ("Can't read mesh:");
   e.printStackTrace();
}
```

Both `readMesh()` and `writeMesh()` may throw I/O exceptions, which must be either caught, as in the example above, or thrown out of the calling routine.

For convenience, one can also use the classes GenericMeshReader or GenericMeshWriter, which internally create an appropriate reader or writer based on the file extension. This enables the writing of code that does not depend on the file format:

```
String fileName;
...
PolygonalMesh mesh = null;
```

```
try {
   mesh = (PolygonalMesh)GenericMeshReader.readMesh(fileName);
}
catch (IOException e) {
   System.err.println ("Can't read mesh:");
   e.printStackTrace();
}
```

Here, `fileName` can refer to a mesh of any format supported by `GenericMeshReader`. Note that the mesh returned by `readMesh()` is explicitly cast to `PolygonalMesh`. This is because `readMesh()` returns the superclass `MeshBase`, since the default mesh created for some file formats may be different from `PolygonalMesh`.

### 2.5.6 Reading and writing normal and texture information

When writing a mesh out to a file, normal and texture information are also written if they have been explicitly set and the file format supports it. In addition, by default, automatically generated normal information will also be written if it relies on information (such as hard edges) that can't be reconstructed from the stored file information.

Whether or not normal information will be written is returned by the method

```
boolean getWriteNormals();
```

This will always return `true` if any of the conditions described above have been met. So for example, if a `PolygonalMesh` contains hard edges, and multiple automatic normals are enabled (i.e., `getMultipleAutoNormals()` returns `true`), then `getWriteNormals()` will return `true`.

Default normal writing behavior can be overridden within the MeshWriter classes using the following methods:

```
int getWriteNormals()
setWriteNormals (enable)
```

where `enable` should be one of the following values:

**0** normals will *never* be written;

**1** normals will *always* be written;

**-1** normals will written according to the default behavior described above.

When reading a `PolygonalMesh` from a file, if the file contains normal information with multiple normals per vertex that suggests the existence of hard edges, then the corresponding edges are set to be hard within the mesh.

### 2.5.7 Constructive solid geometry

ArtiSynth contains primitives for performing constructive solid geometry (CSG) operations on volumes bounded by triangular meshes. The class that performs these operations is maspack.collision.SurfaceMeshIntersector, and it works by robustly determining the intersection contour(s) between a pair of meshes, and then using these to compute the triangles that need to be added or removed to produce the necessary CSG surface.

The CSG operations include union, intersection, and difference, and are implemented by the following methods of `SurfaceMeshIntersector`:

```
findUnion (mesh0, mesh1);          // volume0 U volume1
findIntersection (mesh0, mesh1);   // volume0 ^ volume1
findDifference01 (mesh0, mesh1);   // volume0 - volume1
findDifference10 (mesh0, mesh1);   // volume1 - volume0
```

Each takes two `PolyhedralMesh` objects, `mesh0` and `mesh1`, and creates and returns another `PolyhedralMesh` which represents the boundary surface of the requested operation. If the result of the operation is `null`, the returned mesh will be empty.

The example below uses `findUnion` to create a dumbbell shaped mesh from two balls and a cylinder:

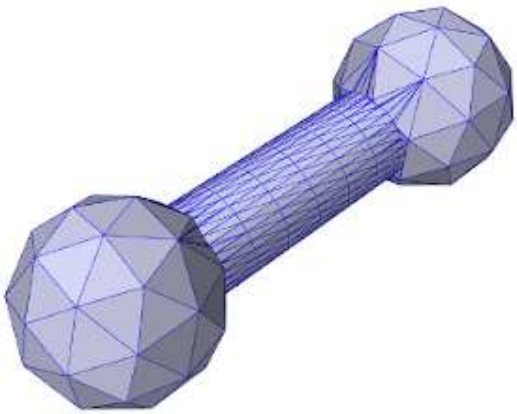


Figure 2.1: Dumbbell shaped mesh produced from the CSG union of two balls and a cylinder.

```
// first create two ball meshes and a bar mesh
double radius = 1.0;
int division = 1; // number of divisons for icosahedral sphere
PolygonalMesh ball0 = MeshFactory.createIcosahedralSphere (radius, division);
ball0.transform (new RigidTransform3d (0, -2*radius, 0));
PolygonalMesh ball1 = MeshFactory.createIcosahedralSphere (radius, division);
ball1.transform (new RigidTransform3d (0, 2*radius, 0));
PolygonalMesh bar = MeshFactory.createCylinder (
   radius/2, radius*4, /*ns=*/32, /*nr=*/1, /*nh*/10);
bar.transform (new RigidTransform3d (0, 0, 0, 0, 0, Math.PI/2));

// use a SurfaceMeshIntersector to create a CSG union of these meshes
SurfaceMeshIntersector smi = new SurfaceMeshIntersector();
PolygonalMesh balls = smi.findUnion (ball0, ball1);
PolygonalMesh mesh = smi.findUnion (balls, bar);
```

The balls and cylinder are created using the MeshFactory methods createIcosahedralSphere() and createCylinder(), where the latter takes arguments `ns`, `nr`, and `nh` giving the number of slices along the circumference, end-cap radius, and length. The final resulting mesh is shown in Figure 2.1.

## 2.6 Reading source relative files

ArtiSynth applications frequently need to read in various kinds of data files, including mesh files (as discussed in Section 2.5.5), FEM mesh geometry (Section 6.2.2), probe data (Section 5.4.4), and custom application data.

Often these data files do not reside in an absolute location but instead in a location relative to the application's class or source files. For example, it is common for applications to store geometric data in a subdirectory `"geometry"` located beneath the source directory. In order to access such files in a robust way, and ensure that the code does not break when the source tree is moved, it is useful to determine the application's source (or class) directory at run time. ArtiSynth supplies several ways to conveniently handle this situation. First, the `RootModel` itself supplies the following methods:

```
// find path to the root model's source directory
String findSourceDir ();

// get path to a file specified relative to the root model's source directory
String getSourceRelativePath (String relpath);
```

The first method returns the path to the source directory of the root model, while the second returns the path to a file specified relative to the root model source directory. If the root model source directory cannot be found (see discussion at the end of this section) both methods return `null`. As a specific usage example, assume that we have an application model whose `build()` method needs to load in a mesh `torus.obj` from a subdirectory `meshes` located beneath the source directory. This could be done as follows:

```
String pathToMesh = getSourceRelativePath ("meshes/torus.obj");
// read the mesh from a .obj file :
WavefrontReader reader = new WavefrontReader (pathToMesh);
PolygonalMesh mesh = null;
try {
   mesh = reader.readMesh () ;
}
catch (IOException e) {
   System.err.println ("Can't read mesh:");
   e.printStackTrace () ;
}
```

A more general path finding utility is provided by maspack.util.PathFinder, which provides several static methods for locating source and class directories:

```
// find path to the source directory associated with classObj
String findSourceDir (Object classObj);

// get path to a file specified relative to classObj source directory
String getSourceRelativePath (Object classObj, String relpath);

// find path to the class directory associated with classObj
String findClassDir (Object classObj);

// get path to a file specified relative to classObj class directory
String getClassRelativePath (Object classObj, String relpath);
```

The "find" methods return a string path to the indicated class or source directory, while the "relative path" methods locate the class or source directory and append the additional path `relpath`. For all of these, the class is determined from `classObj`, either directly (if it is an instance of `Class`), by name (if it is a `String`), or otherwise by calling `classObj.getClass()`. When identifying a package by name, the name should be either a fully qualified class name, or a simple name that can be located with respect to the packages obtained via `Package.getPackages()`. For example, if we have a class whose fully qualified name is `artisynth.models.test.Foo`, then the following calls should all return the same result:

```
Foo foo = new Foo();

PathFinder.findSourceDir (foo);
PathFinder.findSourceDir (Foo.class);
PathFinder.findSourceDir ("artisynth.models.test.Foo");
PathFinder.findSourceDir ("Foo");
```

If the source directory for `Foo` happens to be `/home/projects/src/artisynth/models/test`, then

```
PathFinder.getSourceRelativePath (foo, "geometry/mesh.obj");
```

will return `/home/projects/src/artisynth/models/test/geometry/mesh.obj`.

> When calling `PathFinder` methods from *within* the relevant class, one can specify `this` as the `classObj` argument.

With respect to the above example locating the file `"meshes/torus.obj"`, the call to the root model method `getSourceRelativePath()` could be replaced with

```
String pathToMesh = PathFinder.getSourceRelativePath (
   this, "meshes/torus.obj");
```

Since this is assumed to be called from the root model's build method, the "class" can be indicated by simply passing `this` to `getSourceRelativePath()`.

> As an alternative to placing data files in the source directory, one could place them in the class directory, and then use `findClassDir()` and `getClassRelativePath()`. If the data files were originally defined in the source directory, it will be necessary to copy them to the class directory. Some Java IDEs will perform this automatically.

The `PathFinder` methods work by climbing the class's resource hierarchy. Source directories are assumed to be located relative to the parent of the root class directory, via one of the paths specified by getSourceRootPaths(). By default, this list includes `"src"`, `"source"`, and `"bin"`. Additional paths can be added using addSourceRootPath(path), or the entire list can be set using setSourceRootPaths(paths).

At preset, source directories will not be found if the reference class is contained in a jar file.

## 2.7 Reading and caching remote files

ArtiSynth applications often require the use of large data files to specify items such as FEM mesh geometry, surface mesh geometry, or medical imaging data. The size of these files may make it inconvenient to store them in any version control system that is used to store the application source code. As an alternative, ArtiSynth provides a *file manager* utility that allows such files to be stored on a separate server, and then downloaded on-demand and cached locally. To use this, one starts by creating an instance of a FileManager, using the constructor

```
FileManager (String downloadPath, String remoteSourceName)
```

where `downloadPath` is a path to the local directory where the downloaded file should be placed, and `remoteSourceName` is a URI indicating the remote server location of the files. After the file manager has been created, it can be used to fetch remote files and cache them locally, using various *get* methods:

```
File get (String destName);

File get (String destName, String sourceName);
```

Both of these look for the file `destName` specified relative to the local directory, and return a `File` handle for it if it is present. Otherwise, they attempt to download the file from the remote source location, place it in the local directory, and return a `File` handle for it. The location of the remote file is given relative to the remote source URI by `destName` for the first method and `sourceName` for the second.

A simple example of using a file manager within a `RootModel build()` method is given by the following fragment:

```
// create the file manager ...
FileManager fm = new FileManager (
  getSourceRelativePath("geometry"),
  "http://myserver.org/artisynth/data/geometry");
// ... and use it to get a bone mesh file
File meshFile = fm.get ("tibia.obj");
```

Here, a file manager is created that uses a local directory `"geometry"`, located relative to the `RootModel` source directory (see Section 2.6), and looks for missing files relative to the URI

```
http://myserver.org/artisynth/data/geometry
```

The `get()` method is then used to obtain the file `"tibia.obj"` from the local directory. If it is not already present, it is downloaded from the remote location.

The FileManager contains other features and functionality, and one should consult its API documentation for more information.

# Chapter 3

# Mechanical Models I

This section details how to build basic multibody-type mechanical models consisting of particles, springs, rigid bodies, joints, and other constraints.

## 3.1 Springs and particles

The most basic type of mechanical model consists simply of particles connected together by axial springs. Particles are implemented by the class Particle, which is a dynamic component containing a three-dimensional position state, a corresponding velocity state, and a mass. It is an instance of the more general base class Point, which is used to also implement spatial points such as `markers` which do not have a mass.

### 3.1.1 Axial springs and materials

An axial spring is a simple spring that connects two points and is implemented by the class AxialSpring. This is a *force effector* component that exerts equal and opposite forces on the two points, along the line separating them, with a magnitude $f$ that is a function $f(l,\dot{l})$ of the distance $l$ between the points, and the distance derivative $\dot{l}$.

Each axial spring is associated with an *axial material*, implemented by a subclass of AxialMaterial, that specifies the function $f(l,\dot{l})$. The most basic type of axial material is a LinearAxialMaterial, which determines $f$ according to the linear relationship

$$f(l,\dot{l}) = k(l-l_0)+d\dot{l} \tag{3.1}$$

where $l_0$ is the rest length and $k$ and $d$ are the stiffness and damping terms. Both $k$ and $d$ are properties of the material, while $l_0$ is a property of the spring.

Axial springs are assigned a linear axial material by default. More complex, nonlinear axial materials may be defined in the package `artisynth.core.materials`. Setting or querying a spring's material may be done with the methods `setMaterial()` and `getMaterial()`.

### 3.1.2 Example: a simple particle-spring model

An complete application model that implements a simple particle-spring model is given below.

```
1  package artisynth.demos.tutorial;
2
3  import java.awt.Color;
4  import maspack.matrix.*;
5  import maspack.render.*;
6  import artisynth.core.mechmodels.*;
7  import artisynth.core.materials.*;
8  import artisynth.core.workspace.RootModel;
9
10 /**
```

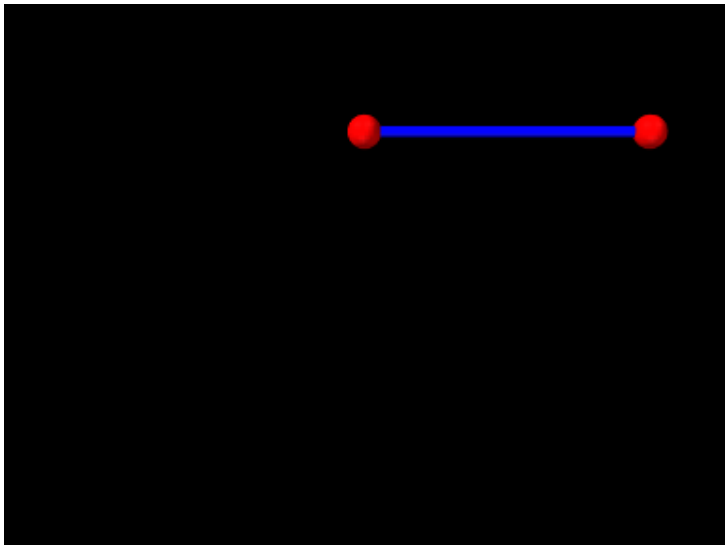


Figure 3.1: ParticleSpring model loaded into ArtiSynth.

```
11  * Demo of two particles connected by a spring
12  */
13 public class ParticleSpring extends RootModel {
14
15    public void build (String[] args) {
16
17       // create MechModel and add to RootModel
18       MechModel mech = new MechModel ("mech");
19       addModel (mech);
20
21       // create the components
22       Particle p1 = new Particle ("p1", /*mass=*/2, /*x,y,z=*/0, 0, 0);
23       Particle p2 = new Particle ("p2", /*mass=*/2, /*x,y,z=*/1, 0, 0);
24       AxialSpring spring = new AxialSpring ("spr", /*restLength=*/0);
25       spring.setPoints (p1, p2);
26       spring.setMaterial (
27          new LinearAxialMaterial (/*stiffness=*/20, /*damping=*/10));
28
29       // add components to the mech model
30       mech.addParticle (p1);
31       mech.addParticle (p2);
32       mech.addAxialSpring (spring);
33
34       p1.setDynamic (false);                // first particle set to be fixed
35
36       // increase model bounding box for the viewer
37       mech.setBounds (/*min=*/-1, 0, -1, /*max=*/1, 0, 0);
38       // set render properties for the components
39       RenderProps.setSphericalPoints (p1, 0.06, Color.RED);
40       RenderProps.setSphericalPoints (p2, 0.06, Color.RED);
41       RenderProps.setCylindricalLines (spring, 0.02, Color.BLUE);
42    }
43 }
```

Line 1 of the source defines the package in which the model class will reside, in this case `artisynth.demos.tutorial`. Lines 3-8 import definitions for other classes that will be used.

The model application class is named `ParticleSpring` and declared to extend `RootModel` (line 13), and the `build()` method definition begins at line 15. (A no-args constructor is also needed, but because no other constructors are defined, the compiler creates one automatically.)

To begin, the `build()` method creates a `MechModel` named `"mech"`, and then adds it to the `models` list of the root model

using the `addModel()` method (lines 18-19). Next, two particles, `p1` and `p2`, are created, with masses equal to 2 and initial positions at 0, 0, 0, and 1, 0, 0, respectively (lines 22-23). Then an axial spring is created, with end points set to `p1` and `p2`, and assigned a linear material with a stiffness and damping of 20 and 10 (lines 24-27). Finally, after the particles and the spring are created, they are added to the `particles` and `axialSprings` lists of the `MechModel` using the methods `addParticle()` and `addAxialSpring()` (lines 30-32).

At this point in the code, both particles are defined to be dynamically controlled, so that running the simulation would cause both to fall under the `MechModel`'s default gravity acceleration of $(0, 0, -9.8)$. However, for this example, we want the first particle to remain fixed in place, so we set it to be *non-dynamic* (line 34), meaning that the physical simulation will not update its position in response to forces (Section 3.1.3).

The remaining calls control aspects of how the model is graphically rendered. `setBounds()` (line 37) increases the model's "bounding box" so that by default it will occupy a larger part of the viewer frustum. The convenience method `RenderProps.setSphericalPoints()` is used to set points `p1` and `p2` to render as solid red spheres with a radius of 0.06, while `RenderProps.setCylindricalLines()` is used to set `spring` to render as a solid blue cylinder with a radius of 0.02. More details about setting render properties are given in Section 4.3.

To run this example in ArtiSynth, select All demos > tutorial > ParticleSpring from the Models menu. The model should load and initially appear as in Figure 3.1. Running the model (Section 1.5.3) will cause the second particle to fall and swing about under gravity.

### 3.1.3 Dynamic, parametric, and attached components

By default, a dynamic component is advanced through time in response to the forces applied to it. However, it is also possible to set a dynamic component's `dynamic` property to `false`, so that it does not respond to force inputs. As shown in the example above, this can be done using the method `setDynamic()`:

```
comp.setDynamic (false);
```

The method isDynamic() can be used to query the `dynamic` property.

Dynamic components can also be *attached* to other dynamic components (as mentioned in Section 1.2) so that their positions and velocities are controlled by the *master* components that they are attached to. To attach a dynamic component, one creates an `AttachmentComponent` specifying the attachment connection and adds it to the `MechModel`, as described in Section 3.7. The method isAttached() can be used to determine if a component is attached, and if it is, getAttachment() can be used to find the corresponding `AttachmentComponent`.

Overall, a dynamic component can be in one of three states:

**active**

Component is dynamic and unattached. The method isActive() returns `true`. The component will move in response to forces.

**parametric**

Component is not dynamic, and is unattached. The method isParametric() returns `true`. The component will either remain fixed, or will move around in response to external inputs specifying the component's position and/or velocity. One way to supply such inputs is to use controllers or input probes, as described in Section 5.

**attached**

Component is attached. The method isAttached() returns `true`. The component will move so as to follow the other master component(s) to which it is attached.

### 3.1.4 Custom axial materials

Application authors may create their own axial materials by subclassing AxialMaterial and overriding the functions

```
double computeF (l, ldot, l0, excitation);
double computeDFdl (l, ldot, l0, excitation);
double computeDFdldot (l, ldot, l0, excitation);
boolean isDFdldotZero ();
```

where `excitation` is an additional *excitation* signal $a$, which is used to implement active springs and which in particular is used to implement axial muscles (Section 4.5), for which $a$ is usually in the range $[0,1]$.

The first three methods should return the values of

$$f(l,\dot{l},a), \quad \frac{\partial f(l,\dot{l},a)}{\partial l}, \quad \text{and} \quad \frac{\partial f(l,\dot{l},a)}{\partial \dot{l}}, \tag{3.2}$$

respectively, while the last method should return `true` if $\partial f(l,\dot{l},a)/\partial \dot{l} \equiv 0$; i.e., if it is always equals to 0.

### 3.1.5 Damping parameters

Mechanical models usually contain damping forces in addition to spring-type restorative forces. Damping generates forces that reduce dynamic component velocities, and is usually the major source of energy dissipation in the model. Damping forces can be generated by the spring components themselves, as described above.

A general damping can be set for all particles by setting the `MechModel`'s `pointDamping` property. This causes a force

$$\mathbf{f}_i = -d_p \mathbf{v}_i \tag{3.3}$$

to be applied to all particles, where $d_p$ is the value of the `pointDamping` and $\mathbf{v}_i$ is the particle's velocity.

`pointDamping` can be set and queried using the `MechModel` methods

```
setPointDamping (double d);
double getPointDamping();
```

> In general, whenever a component has a property `propX`, that property can be set and queried in code using methods of the form
>
> ```
> setPropX (T d);
> T getPropX();
> ```
>
> where `T` is the type associated with the property.

`pointDamping` can also be set for particles individually. This property is *inherited* (Section 1.4.3), so that if not set explicitly, it inherits the nearest explicitly set value in an ancestor component.

## 3.2 Rigid bodies

Rigid bodies are implemented in ArtiSynth by the class RigidBody, which is a dynamic component containing a six-dimensional position and orientation state, a corresponding velocity state, an inertia, and an optional surface mesh.

A rigid body is associated with its own 3D spatial coordinate frame, and is a subclass of the more general Frame component. The combined position and orientation of this frame with respect to world coordinates defines the body's *pose*, and the associated 6 degrees of freedom describe its "position" state.

### 3.2.1 Frame markers

ArtiSynth makes extensive use of *markers*, which are (massless) points attached to dynamic components in the model. Markers are used for graphical display, implementing attachments, and transmitting forces back onto the underlying dynamic components.

A *frame marker* is a marker that can be attached to a Frame, and most commonly to a RigidBody (Figure 3.2). They are frequently used to provide the anchor points for attaching springs and, more generally, applying forces to the body.

Frame markers are implemented by the class FrameMarker, which is a subclass of Point. The methods

```
Point3d getLocation();
void setLocation (Point3d r);
```

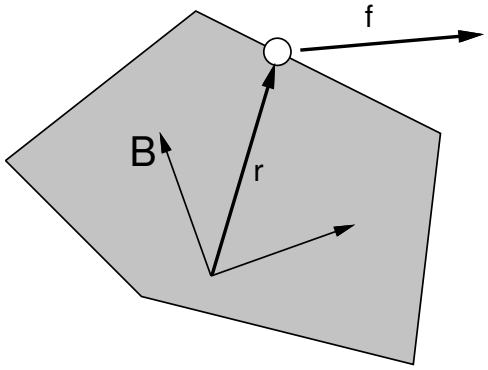



Figure 3.2: A force **f** applied to a frame marker attached to a rigid body. The marker is located at the point **r** with respect to the body coordinate frame B.

get and set the marker's location **r** with respect to the frame's coordinate system. When a 3D force **f** is applied to the marker, it generates a spatial force $\hat{\mathbf{f}}$ (Section A.5) on the frame given by

$$\hat{\mathbf{f}} = \begin{pmatrix} \mathbf{f} \\ \mathbf{r} \times \mathbf{f} \end{pmatrix}. \tag{3.4}$$

Frame markers can be created using a variety of constructors, including

```
FrameMarker ();
FrameMarker (String name);
FrameMarker (Frame frame, Point3d loc);
```

where `FrameMarker()` creates an empty marker, `FrameMarker(name)` creates an empty marker with a name, and `FrameMarker(frame,loc)` creates an unnamed marker attached to `frame` at the location `loc` with respect to the frame's coordinates. Once created, a marker's frame can be set and queried with

```
void setFrame (Frame frame);
Frame getFrame ();
```

A frame marker can be added to a MechModel with the `MechModel` methods

```
void addFrameMarker (FrameMarker mkr);
void addFrameMarker (FrameMarker mkr, Frame frame, Point3d loc);
```

where `addFrameMarker(mkr,frame,loc)` also sets the frame and the marker's location with respect to it.

`MechModel` also supplies convenience methods to create a marker, attach it to a frame, and add it to the model:

```
FrameMarker addFrameMarker (Frame frame, Point3d loc);
FrameMarker addFrameMarkerWorld (Frame frame, Point3d locw);
```

Both methods return the created marker. The first, `addFrameMarker(frame,loc)`, places it at the location `loc` with respect to the frame, while `addFrameMarkerWorld(frame,pos)` places it at `pos` with respect to *world* coordinates.

### 3.2.2 Example: a simple rigid body-spring model

A simple rigid body-spring model is defined in

```
artisynth.demos.tutorial.RigidBodySpring
```

This differs from ParticleSpring only in the `build()` method, which is listed below:

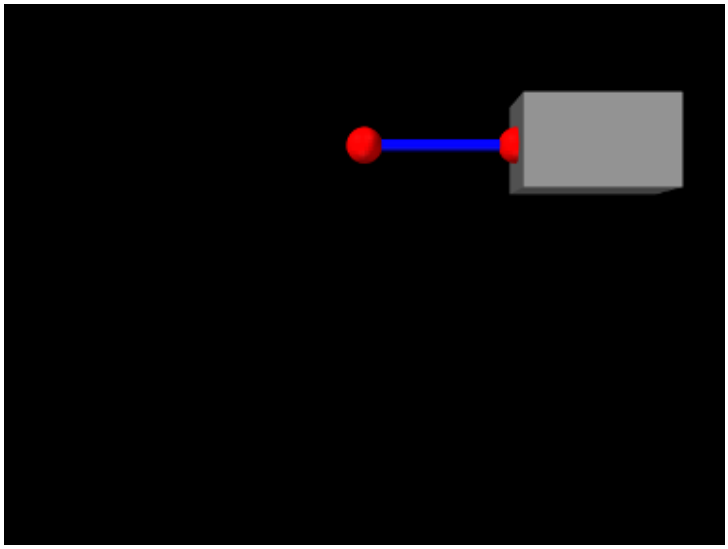


Figure 3.3: RigidBodySpring model loaded into ArtiSynth.

```
1   public void build (String[] args) {
2
3      // create MechModel and add to RootModel
4      MechModel mech = new MechModel ("mech");
5      addModel (mech);
6
7      // create the components
8      Particle p1 = new Particle ("p1", /*mass=*/2, /*x,y,z=*/0, 0, 0);
9      // create box and set its pose (position/orientation):
10     RigidBody box =
11        RigidBody.createBox ("box", /*wx,wy,wz=*/0.5, 0.3, 0.3, /*density=*/20);
12     box.setPose (new RigidTransform3d (/*x,y,z=*/0.75, 0, 0));
13     // create marker point and connect it to the box:
14     FrameMarker mkr = new FrameMarker (/*x,y,z=*/-0.25, 0, 0);
15     mkr.setFrame (box);
16
17     AxialSpring spring = new AxialSpring ("spr", /*restLength=*/0);
18     spring.setPoints (p1, mkr);
19     spring.setMaterial (
20        new LinearAxialMaterial (/*stiffness=*/20, /*damping=*/10));
21
22     // add components to the mech model
23     mech.addParticle (p1);
24     mech.addRigidBody (box);
25     mech.addFrameMarker (mkr);
26     mech.addAxialSpring (spring);
27
28     p1.setDynamic (false);              // first particle set to be fixed
29
30     // increase model bounding box for the viewer
31     mech.setBounds (/*min=*/-1, 0, -1, /*max=*/1, 0, 0);
32     // set render properties for the components
33     RenderProps.setSphericalPoints (p1, 0.06, Color.RED);
34     RenderProps.setSphericalPoints (mkr, 0.06, Color.RED);
35     RenderProps.setCylindricalLines (mkr, 0.02, Color.BLUE);
36  }
```

The differences from `ParticleSpring` begin at line 9. Instead of creating a second particle, a rigid body is created using the factory method RigidBody.createBox(), which takes x, y, z widths and a (uniform) density and creates a box-shaped rigid body complete with surface mesh and appropriate mass and inertia. As the box is initially centered at the

origin, moving it elsewhere requires setting the body's pose, which is done using `setPose()`. The `RigidTransform3d` passed to `setPose()` is created using a three-argument constructor that generates a translation-only transform. Next, starting at line 14, a `FrameMarker` is created for a location $(-0.25, 0, 0)^T$ relative to the rigid body, and attached to the body using its `setFrame()` method.

The remainder of `build()` is the same as for `ParticleSpring`, except that the spring is attached to the frame marker instead of a second particle.

To run this example in ArtiSynth, select All demos > tutorial > RigidBodySpring from the Models menu. The model should load and initially appear as in Figure 3.3. Running the model (Section 1.5.3) will cause the rigid body to fall and swing about under gravity.

### 3.2.3 Creating rigid bodies

As illustrated above, rigid bodies can be created using factory methods supplied by RigidBody. Some of these include:

```
createBox (name, widthx, widthy, widthz, density);
createCylinder (name, radius, height, density, nsides);
createSphere (name, radius, density, nslices);
createEllipsoid (name, radx, rady, radz, density, nslices);
```

The bodies do not need to be named; if no name is desired, then `name` and can be specified as `null`.

In addition, there are also factory methods for creating a rigid body directly from a mesh:

```
createFromMesh (name, mesh, density, scale);
createFromMesh (name, meshFileName, density, scale);
```

These take either a polygonal mesh (Section 2.5), or a file name from which a mesh is read, and use it as the body's surface mesh and then compute the mass and inertia properties from the specified (uniform) density.

> When a body is created directly from a surface mesh, its center of mass will typically *not* be coincident with the origin of its coordinate frame. Section 3.2.6 discusses the implications of this and how to correct it.

Alternatively, one can create a rigid body directly from a constructor, and then set the mesh and inertia properties explicitly:

```
PolygonalMesh femurMesh;
SpatialInertia inertia;

... initialize mesh and inertia appropriately ...

RigidBody body = new RigidBody ("femur");
body.setMesh (femurMesh);
body.setInertia (inertia);
```

### 3.2.4 Pose and velocity

A body's pose can be set and queried using the methods

```
setPose (RigidTransform3d T);    // sets the pose to T
getPose (RigidTransform3d T);    // gets the current pose in T
RigidTransform3d getPose();      // returns the current pose (read-only)
```

These use a RigidTransform3d (Section 2.2) to describe the pose. Body poses are described in world coordinates and specify the transform from body to world coordinates. In particular, the pose for a body A specifies the rigid transform $\mathbf{T}_{AW}$.

Rigid bodies also expose the translational and rotational components of their pose via the properties `position` and `orientation`, which can be queried and set independently using the methods

```
setPosition (Point3d p);        // sets the position to p
getPosition (Point3d p);        // gets the current position in p
Point3d getPosition();          // returns the current position (read-only)

setOrientation (AxisAngle a);   // sets the orientation to a
getOrientation (AxisAngle a);   // gets the current orientation in a
AxisAngle getOrientation();     // returns the current orientation (read-only)
```

The velocity of a rigid body is described using a Twist (Section 2.4), which contains both the translational and rotational velocities. The following methods set and query the spatial velocity as described with respect to world coordinates:

```
setVelocity (Twist v);          // sets the spatial velocity to v
getVelocity (Twist v);          // gets the current spatial velocity in v
Twist getVelocity();            // returns current spatial velocity (read-only)
```

During simulation, unless a rigid body has been set to be *parametric* (Section 3.1.3), its pose and velocity are updated in response to forces, so setting the pose or velocity generally makes sense only for setting initial conditions. On the other hand, if a rigid body is parametric, then it is possible to control its pose during the simulation, but in that case it is better to set its *target pose* and/or *target velocity*, as described in Section 5.3.1.

### 3.2.5 Inertia and the surface mesh

The "mass" of a rigid body is described by its spatial inertia, which is a $6 \times 6$ matrix relating its spatial velocity to its spatial momentum (Section A.6). Within ArtiSynth, spatial inertia is described by a SpatialInertia object, which specifies its mass, center of mass (with respect to body coordinates), and rotational inertia (with respect to the center of mass).

Most rigid bodies are also associated with a polygonal surface mesh, which can be set and queried using the methods

```
setSurfaceMesh (PolygonalMesh mesh);
setSurfaceMesh (PolygonalMesh mesh, String meshFileName);
PolygonalMesh getSurfaceMesh();
```

The second method takes an optional `fileName` argument that can be set to the name of a file from which the mesh was read. Then if the model itself is saved to a file, the model file will specify the mesh using the file name instead of explicit vertex and face information, which can reduce the model file size considerably.

Rigid bodies can also have more than one mesh, as described in Section 3.2.9.

The inertia of a rigid body can be explicitly set using a variety of methods including

```
setInertia (M)                  // set using SpatialInertia M
setInertia (mass, Jxx, Jyy, Jzz); // mass and diagonal rotational inertia
setInertia (mass, J);           // mass and full rotational inertia
setInertia (mass, J, com);      // mass, rotational inertia, center-of-mass
```

and can be queried using

```
getInertia (M);                 // get SpatialInertia in M
getInertia ();                  // return read-only SpatialInertia
```

In practice, it is often more convenient to simply specify a mass or a density, and then use the geometry of the surface mesh (and possibly other meshes, Section 3.2.9) to compute the remaining inertial values. How a rigid body's inertia is computed is determined by its inertiaMethod property, which can be one

**EXPLICIT**

Inertia is set explicitly.

**MASS**

Inertia is determined implicitly from the mesh geometry and the body's mass.

**DENSITY**

Inertia is determined implicitly from the mesh geometry and the body's density (which is multiplied by the mesh volume(s) to determine a mass).

> When using `DENSITY` to determine the inertia, it is generally assumed that the contributing meshes are both polygonal and closed. Meshes which are either open or non-polygonal generally do not have a well-defined volume which can be multiplied by the density to determine the mass.

The inertiaMethod property can be set and queried using

```
setInertiaMethod (InertiaMethod method);
InertiaMethod getInertiaMethod();
```

and its default value is `DENSITY`. Explicitly setting the inertia using one of `setInertia()` methods described above will set `inertiaMethod` to `EXPLICIT`. The method

```
setInertiaFromDensity (density);
```

will (re)compute the inertia using the mesh geometry and a density value and set `inertiaMethod` to `DENSITY`, and the method

```
setInertiaFromMass (mass);
```

will (re)compute the inertia using the mesh geometry and a mass value and set `inertiaMethod` to `MASS`.

Finally, the (assumed uniform) density of the body can be queried using

```
getDensity();
```

> There are some subtleties involved in determining the inertia using either the `DENSITY` or `MASS` methods when the rigid body contains more than one mesh. Details are given in Section 3.2.9.

### 3.2.6 Coordinate frames and the center of mass

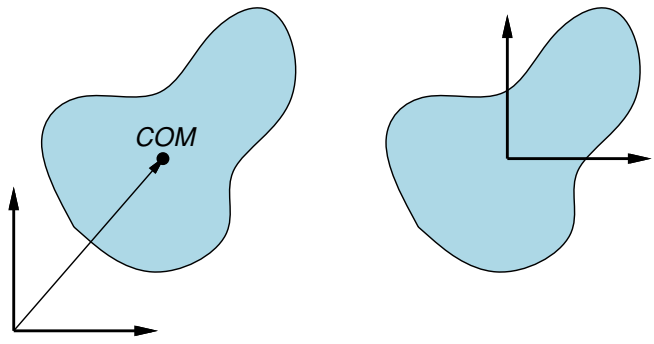



Figure 3.4: Left: rigid body whose coordinate frame B is not coincident with the center of mass (COM). Right: same body, with its coordinate frame translated to be coincident with the COM.

It is important to note that the origin of a body's coordinate frame will not necessarily coincide with its center of mass (COM), and in fact the frame origin does not even have to lie inside the body's surface (Figure 3.4). This typically occurs when a body's inertia is computed directly from its surface mesh (or meshes), as described in Section 3.2.5.

Having the COM differ from the frame origin may lead to some undesired effects. For instance, since the body's spatial velocity is defined with respect to the frame origin and not the COM, if the two are not coincident, then a purely angular body velocity will cause the COM to translate. The body's spatial inertia also becomes more complicated, with non-zero

3 x 3 blocks in the lower left and upper right (Section A.6), which can have a small effect on computational accuracy. Finally, manipulating a body's pose in the ArtiSynth UI (as described in the section "Model Manipulation" in the ArtiSynth User Interface Guide) can also be more cumbersome if the origin is located far from the COM.

There are several ways to ensure that the COM and frame origin are coincident. The most direct is to call the method centerPoseOnCenterOfMass() after the body has been created:

```
String meshFilePath = "/project/geometry/bodyMesh.obj";
double density = 1000;

PolygonalMesh mesh = new PolygonalMesh (meshFilePath); // read in a mesh
RigidBody bodyA = RigidBody.createFromMesh (
   "bodyA", mesh, density, /*scale=*/1); // create body from the mesh
bodyA.centerPoseOnCenterOfMass();        // center body on the COM
```

This will shift the body's frame to be coincident with the COM, while at the same time translating its mesh vertices in the opposite direction so that its mesh (or meshes) don't move with respect to world coordinates. The spatial inertia is updated as well.

Alternatively, if the body is being created from a single mesh, one may transform that mesh to be centered on its COM *before* it is used to define the body. This can be done using the `PolygonalMesh` method translateToCenterOfVolume(), which centers a mesh's vertices on its COM (assuming a uniform density):

```
PolygonalMesh mesh = new PolygonalMesh (meshFilePath); // read in a mesh
mesh.translateToCenterOfVolume();        // center mesh on its COM
RigidBody bodyA = RigidBody.createFromMesh (
   "bodyA", mesh, density, /*scale=*/1); // create body from the mesh
```

### 3.2.7 Damping parameters

As with particles, it is possible to set damping parameters for rigid bodies. Damping can be specified in two different ways:

1. *Translational/rotational* damping which is proportional to a body's translational and rotational velocity;
2. *Inertial* damping, which is proportional to a body's spatial inertia multiplied by its spatial velocity.

Translational/rotational damping is controlled by the `MechModel` properties frameDamping and rotaryDamping, and generates a spatial force centered on each rigid body's coordinate frame given by

$$\hat{\mathbf{f}} = -\begin{pmatrix} d_f \mathbf{v} \\ d_r \boldsymbol{\omega} \end{pmatrix}, \tag{3.5}$$

where $d_f$ and $d_r$ are the frameDamping and rotaryDamping values, and $\mathbf{v}$ and $\boldsymbol{\omega}$ are the translational and angular velocity of the body's coordinate frame. The damping parameters can be set and queried using the `MechModel` methods

```
setFrameDamping (double df)
setRotaryDamping (double dr)
double getFrameDamping()
double getRotaryDamping()
```

These damping parameters can also be set for individual bodies using their own (inherited) frameDamping and rotaryDamping properties.

> For models involving rigid bodies, it is often necessary to set `rotaryDamping` to a non-zero value because `frameDamping` will provide no damping at all when a rigid body is simply rotating about its coordinate frame origin.

Inertial damping is controlled by the `MechModel` property inertialDamping, and generates a spatial force centered on a rigid body's coordinate frame given by

$$\hat{\mathbf{f}} = -d_I \mathbf{M} \hat{\mathbf{v}}, \quad \hat{\mathbf{v}} \equiv \begin{pmatrix} \mathbf{v} \\ \boldsymbol{\omega} \end{pmatrix}, \tag{3.6}$$

where $d_I$ is the inertialDamping, $\mathbf{M}$ is the body's $6 \times 6$ spatial inertia matrix (Section A.6), and $\hat{\mathbf{v}}$ is the body's spatial velocity. The inertial damping property can be set and queried using the `MechModel` methods

```
setInertialDamping (double di)
double getInertialDamping()
```

This parameter can also be set for individual bodies using their own (inherited) inertialDamping property.

> Inertial damping offers two advantages over translational/rotational damping:
>
> 1. It is independent of the location of the body's coordinate frame with respect to its center of mass;
> 2. There is no need to adjust two different translational and rotational parameters or to consider their relative sizes, as these considerations are contained within the spatial inertia itself.

### 3.2.8 Rendering rigid bodies

A rigid body is rendered in ArtiSynth by drawing its mesh (or meshes, Section 3.2.9) and/or coordinate frame.

Meshes are drawn using the face rendering properties described in more detail in Section 4.3. The most commonly used of these are:

- faceColor: A value of type `java.awt.Color` giving the color of mesh faces. The default value is `GRAY`.
- shading: A value of type Renderer.Shading indicating how the mesh should be shaded, with the options being `FLAT`, `SMOOTH`, `METAL`, and `NONE`. The default value is `FLAT`.
- alpha: A double value between 0 and 1 indicating transparency, with transparency increasing as value decreases from 1. The default value is 1.
- faceStyle: A value of type Renderer.FaceStyle indicating which face sides should be drawn, with the options being `FRONT`, `BACK`, `FRONT_AND_BACK`, and `NONE`. The default value is `FRONT`.
- drawEdges: A boolean indicating whether the mesh edges should also be drawn, using either the edgeColor rendering property, or the lineColor property if edgeColor is not set. The default value is `false`.
- edgeWidth: An integer giving the width of the mesh edges in pixels.

These properties, and others, can be set either interactively in the GUI, or in code. To set the render properties in the GUI, select the rigid body or its mesh component, and then right click the mouse and choose Edit render props .... More details are given in the section "Render properties" in the ArtiSynth User Interface Guide.

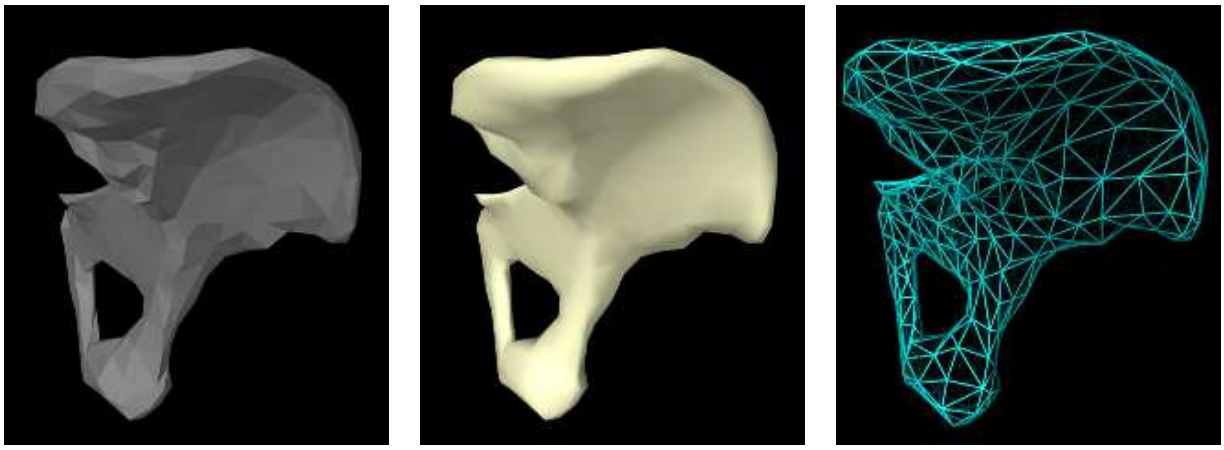


Figure 3.5: Different rendering settings for a rigid body hip mesh showing the default (left), smooth rendering with a lighter color (center), and wireframe (right).

Properties can also be set in code, usually during the `build()` method. Typically this is done using a static method of the RenderProps class that has the form

```
RenderProps.setXXX (comp, value)
```

where XXX is the property name, comp is the component for which the property should be set, and value is the desired value. Some examples are shown in Figure 3.5 for a rigid body hip representation with a fairly coarse mesh. The left image shows the default rendering, using a gray color and flat shading. The center image shows a lighter color and smooth shading, which could be set by the following code fragment:

```
import maspack.render.*;
import maspack.render.Renderer.*;
  ...

  RigidBody hipBody;
  ...

  RenderProps.setFaceColor (hipBody, new Color (255, 255, 204));
  RenderProps.setShading (hipBody, Shading.SMOOTH);
```

Finally, the right image shows the body rendered as a wire frame, which can by done by setting faceStyle to NONE and drawEdges to true:

```
RenderProps.setFaceStyle (hip, FaceStyle.NONE);
RenderProps.setDrawEdges (hip, true);
RenderProps.setEdgeWidth (hip, 2);
RenderProps.setEdgeColor (hip, Color.CYAN);
```

Render properties can also be set in higher level model components, from which their values will be inherited by lower level components that have not explicitly set their own values. For example, setting the faceColor render property in the MechModel will automatically set the face color for all subcomponents which have not explicitly set faceColor. More details on render properties are given in Section 4.3.

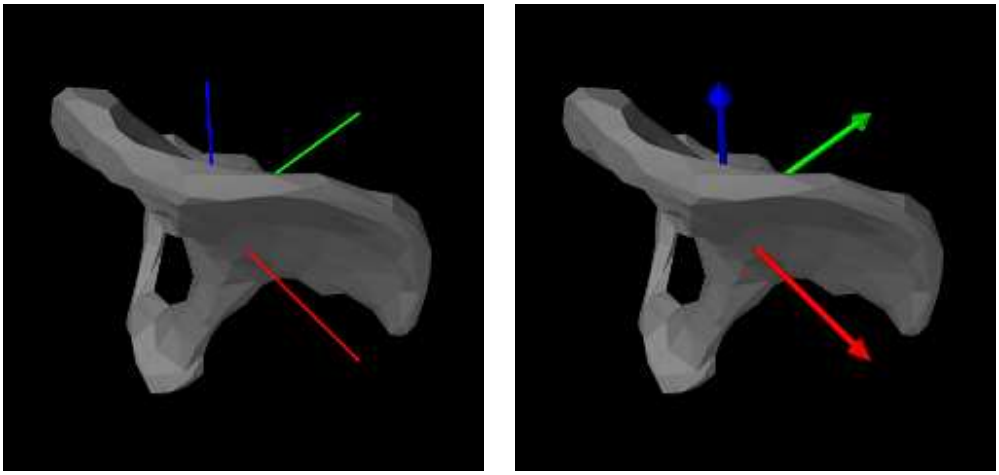


Figure 3.6: Rigid body axes rendered with axisDrawStyle set to LINE (left) and ARROW (right).

In addition to mesh rendering, it is often useful to draw a rigid body's coordinate frame, which can be done using its axisLength and axisDrawStyle properties. Setting axisLength to a positive value will cause the body's three coordinate axes to be drawn, with the indicated length, with the $x$, $y$ and $z$ axes colored red, green, and blue, respectively. The axisDrawStyle property controls how the axes are rendered (Figure 3.6). It has the type Renderer.AxisDrawStyle, and can be set to the following values:

**OFF**

Axes are not rendered.

**LINE**

Axes are rendered as simple red-green-blue lines, with a width given by the joint's lineWidth rendering property.

**ARROW**

Axes are rendered as solid red-green-blue arrows.

As with the rendering proprieties, the axisLength and axisDrawStyle properties can be managed either interactively in the GUI (by selecting the body, right clicking and choosing Edit properties ...), or in code, using the following methods:

```
double getAxisLength()
void setAxisLength (double len)

AxisDrawStyle getAxisDrawStyle()
void setAxisDrawStyle (AxisDrawStyle style)
```

### 3.2.9 Multiple meshes

A `RigidBody` may contain multiple meshes, which can be useful for various reasons:

- It may be desirable to use different meshes for collision detection, inertia computation, and visual presentation;
- Different render properties can be set for different mesh components, allowing the body to be rendered in a more versatile way;
- Different mesh components can be selected individually.

Each rigid body mesh is encapsulated inside a RigidMeshComp component, which is in turn stored in a subcomponent list called `meshes`. Meshes do not need to be instances of PolygonalMesh; instead, they can be any instance of MeshBase, including PointMesh and PolylineMesh.

> The default surface mesh, returned by `getSurfaceMesh()`, is also stored inside a `RigidMeshComp` in the `meshes` list. By default, the surface mesh is the first mesh in the list, but is otherwise defined to be the first mesh in `meshes` which is also an instance of `PolygonalMesh`. The `RigidMeshComp` containing the surface mesh can be obtained using the method `getSurfaceMeshComp()`.

A `RigidMeshComp` contains a number of properties that control how the mesh is displayed and interacts with its rigid body:

**renderProps**

Render properties controlling how the mesh is rendered (see Section 4.3).

**hasMass**

A boolean, which if `true` means that the mesh will contribute to the body's inertia when the inertiaMethod is either `MASS` or `DENSITY`. The default value is `true`.

**massDistribution**

An enumerated type defined by MassDistribution which specifies how the mesh's inertia contribution is determined for a given mass. `VOLUME`, `AREA`, `LENGTH`, and `POINT` indicate, respectively, that the mass is distributed evenly over the mesh's volume, area (faces), length (edges), or points. The default value is determined by the mesh type: `VOLUME` for a closed `PolygonalMesh`, `AREA` for an open `PolygonalMesh`, `LENGTH` for a `PolylineMesh`, and `POINT` for a `PointMesh`. Applications can specify an alternate value providing the mesh has the features to support it. Specifying `DEFAULT` will restore the default value.

**isCollidable**

A boolean, which if `true`, and if the mesh is a `PolygonalMesh`, means that the mesh will take part in collision and wrapping interactions (Chapter 8 and Section 9.3). The default value is `true`, and the get/set accessors have the names `isCollidable()` and `setIsCollidable()`.

**volume**

A double whose value is the volume of the mesh. If the mesh is a `PolygonalMesh`, this is the value returned by its computeVolume() method. Otherwise, the volume is 0, unless setVolume(vol) is used to explicitly set a non-zero volume value.

**mass**

A double whose default value is the product of the density and volume properties. Otherwise, if mass has been explicitly set using setMass(mass), the value is the explicit mass.

**density**

A double whose default value is the rigid body's density. Otherwise, if density has been explicitly set using setDensity(density), the value is the explicit density, or if mass has been explicitly set using setMass(mass), the value is the explicit mass divided by volume.

Note that by default, the density of a `RigidMeshComp` is simply the density setting for the rigid body, and the mass is this times the volume. However, it is possible to set either an explicit mass or a density value that will override this. (Also, explicitly setting a mass will unset any explicit density, and explicitly setting the density will unset any explicit mass.)

When the inertiaMethod of the rigid body is either `MASS` or `DENSITY`, then its inertia is computed from the sum of all the inertias $\mathbf{M}_k$ of the component meshes $k$ for which hasMass is `true`. Each $\mathbf{M}_k$ is computed by the mesh's createInertia(mass,massDistribution) method, using the mass and massDistribution properties of its `RigidMeshComp`.

> When forming the body inertia from the inertia components of individual meshes, no attempt is made to account for mesh overlap. If this is important, the meshes themselves should be modified in advance so that they do not overlap, perhaps by using the CSG primitives described in Section 2.5.7.

Instances of `RigidMeshComp` can be created directly, using constructions such as

```
PolygonalMesh mesh;

... initialize mesh ...

RigidMeshComp mcomp = new RigidMeshComp (mesh);
```

or

```
RigidMeshComp mcomp = new RigidMeshComp ("meshName");
mcomp.setMesh (mesh);
```

after which they can be added or removed from the `meshes` list using the methods

```
void addMeshComp (RigidMeshComp mcomp)
void addMeshComp (RigidMeshComp mcomp, int idx)
int numMeshComps()
boolean removeMeshComp (RigidMeshComp mcomp)
boolean removeMeshComp (String name)
void clearMeshComps()
```

It is also possible to add meshes directly to the `meshes` list, using the methods

```
RigidMeshComp addMesh (MeshBase mesh)
RigidMeshComp addMesh (MeshBase mesh, boolean hasMass, boolean collidable)
```

each of which creates a `RigidMeshComp`, adds it to the mesh list, and returns it. The second method also specifies the values of the hasMass and collidable properties (both of which are `true` by default).

### 3.2.10 Example: a composite rigid body

An example of constructing a rigid body from multiple meshes is defined in

```
artisynth.demos.tutorial.RigidCompositeBody
```

This uses three meshes to construct a rigid body whose shape resembles a dumbbell. The code, with the include files omitted, is listed below:

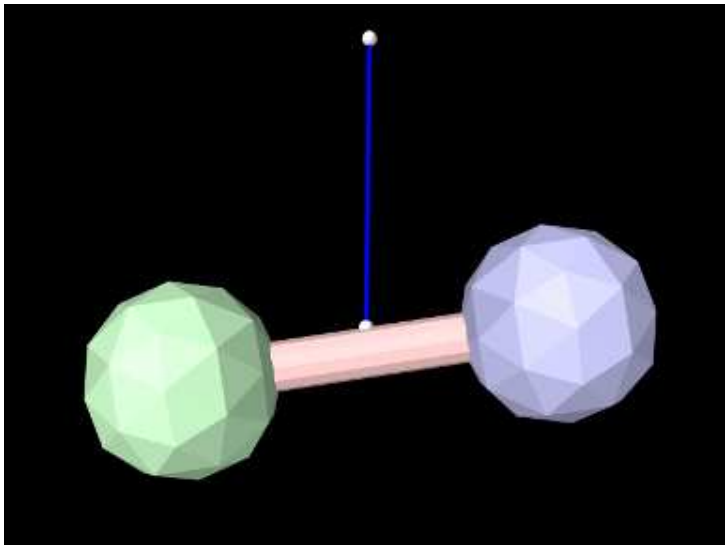


Figure 3.7: `RigidCompositeBody` loaded into ArtiSynth and run for 0.75 seconds. The ball on the right falls less because it has a lower density than the rest of the body.

```
public class RigidCompositeBody extends RootModel {

   public void build (String[] args) {

      // create MechModel and add to RootModel
      MechModel mech = new MechModel ("mech");
      addModel (mech);

      // create the component meshes
      PolygonalMesh ball1 = MeshFactory.createIcosahedralSphere (0.8, 1);
      ball1.transform (new RigidTransform3d (1.5, 0, 0));
      PolygonalMesh ball2 = MeshFactory.createIcosahedralSphere (0.8, 1);
      ball2.transform (new RigidTransform3d (-1.5, 0, 0));
      PolygonalMesh axis = MeshFactory.createCylinder (0.2, 2.0, 12);
      axis.transform (new RigidTransform3d (0, 0, 0, 0, Math.PI/2, 0));

      // create the body and add the component meshes
      RigidBody body = new RigidBody ("body");
      body.setDensity (10);
      body.setFrameDamping (10); // add damping to the body
      body.addMesh (axis);
      RigidMeshComp bcomp1 = body.addMesh (ball1);
      RigidMeshComp bcomp2 = body.addMesh (ball2);
      mech.addRigidBody (body);

      // connect the body to a spring attached to a fixed particle
      Particle p1 = new Particle ("p1", /*mass=*/0, /*x,y,z=*/0, 0, 2);
      p1.setDynamic (false);
      mech.addParticle (p1);
      FrameMarker mkr = mech.addFrameMarkerWorld (body, new Point3d (0, 0, 0.2));
      AxialSpring spring =
         new AxialSpring ("spr", /*k=*/150, /*d=*/0, /*restLength=*/0);
      spring.setPoints (p1, mkr);
      mech.addAxialSpring (spring);

      // set the density for ball1 to be less than the body density
      bcomp1.setDensity (8);

      // set render properties for the component, with the ball
      // meshes having different colors
```

```
41        RenderProps.setFaceColor (body, new Color (250, 200, 200));
42        RenderProps.setFaceColor (bcomp1, new Color (200, 200, 250));
43        RenderProps.setFaceColor (bcomp2, new Color (200, 250, 200));
44        RenderProps.setSphericalPoints (mech, 0.06, Color.WHITE);
45        RenderProps.setCylindricalLines (spring, 0.02, Color.BLUE);
46     }
47  }
```

As in the previous examples, the `build()` method starts by creating a `MechModel` (lines 6-7). Three different meshes (two balls and an axis) are then constructed at lines 10-15, using `MeshFactory` methods (Section 2.5) and transforming each result to an appropriate position/orientation with respect to the body's coordinate frame.

The body itself is constructed at lines 18-24. Its default density is set to 10, and its frame damping (Section 3.2.7) is also set to 10 (the previous rigid body example in Section 3.2.2 relied on spring damping to dissipate energy). The meshes are added using addMesh(), which allocates and returns a RigidMeshComp. For the ball meshes, these are saved in `bcomp1` and `bcomp2` and used later to adjust density and/or render properties.

Lines 27-34 create a simple linear spring, connected to a fixed point `p0` and a marker `mkr`. The marker is created and attached to the body by the `MechModel` method addFrameMarkerWorld(), which places the marker at a known position in world coordinates. The spring is created using an AxialSpring constructor that accepts a name, along with stiffness, damping, and rest length parameters to specify a LinearAxialMaterial.

At line 37, `bcomp1` is used to set the density of `ball1` to 8. Since this is less than the default body density, the inertia component of `ball1` will be lighter than that of `ball2`. Finally, render properties are set at lines 41-45. This includes setting the default face colors for the body and for each ball.

To run this example in ArtiSynth, select All demos > tutorial > RigidCompositeBody from the Models menu. The model should load and initially appear as in Figure 3.7. Running the model (Section 1.5.3) will cause the rigid body to fall and swing about under gravity, with the right ball (`ball1`) not falling as far because it has less density.

## 3.3 Joints and connectors

In a typical mechanical model, many of the rigid bodies are interconnected, either using spring-type components that exert binding forces on the bodies, or through joints and connectors that enforce the connection using hard constraints. This section describes the latter. While the discussion focuses on rigid bodies, joints and connectors can be used more generally with any body that implements the ConnectableBody interface. In particular, this allows joints to also interconnect finite element models, as described in Section 6.6.2.

### 3.3.1 Joints and coordinate frames

Consider two rigid bodies A and B. The pose of body B with respect to body A can be described by the 6 DOF rigid transform $\mathbf{T}_{BA}$. If A and B are unconnected, $\mathbf{T}_{BA}$ may assume any possible value and has a full six degrees of freedom. A *joint* between A and B constrains the set of poses that are possible between the two bodies and reduces the degrees of freedom available to $\mathbf{T}_{BA}$. For ease of use, the constraining action of a joint is described with respect to a pair of local coordinate frames C and D that are connected to frames A and B, respectively, by auxiliary transformations. This allows joints to be placed at locations that do not correspond directly to frames A or B.

The joint frames C and D move with respect to each other as the joint moves. The allowed joint motions therefore correspond to the allowed values of the *joint transform* $\mathbf{T}_{CD}$. Although both frames typically move with their attached bodies, D is considered the *base* frame and C the *motion* frame (this is because when a joint is used to connect a single body to ground, body B is set to `null` and the world frame takes its place). As an example of a joint's constraining effect, consider a hinge joint (Figure 3.8), which allows C to move with respect to D *only* by rotating about the $z$ axis while the origins of C and D remain coincident. Other motions are prohibited. If we let $\theta$ describe the counter-clockwise rotation angle of C about the $z$ axis, then $\mathbf{T}_{CD}$ should always have the form

$$\mathbf{T}_{CD} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{3.7}$$

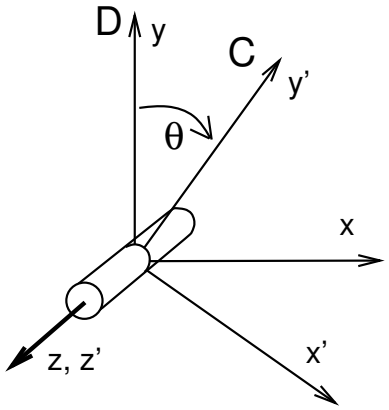



Figure 3.8: Coordinate frames D and C for a hinge joint.

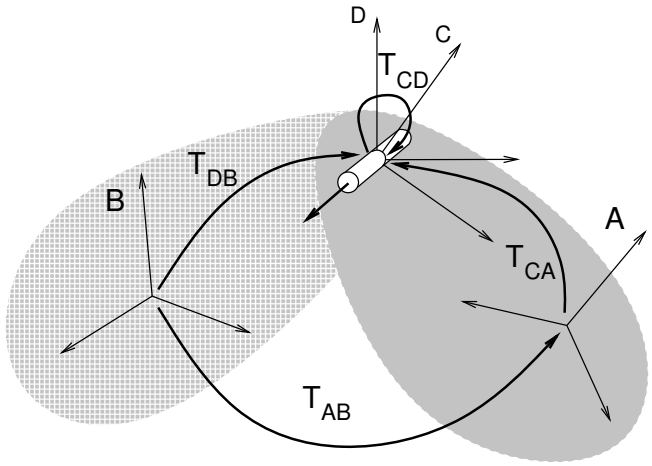



Figure 3.9: Transforms connecting joint coordinate frames C and D with rigid bodies A and B.

When a joint is attached to bodies A and B, frame C is fixed to body A and frame D is fixed to body B. Except in special cases, the joint frames C and D are not coincident with the body frames A and B. Instead, they are located relative to A and B by the transforms $\mathbf{T}_{CA}$ and $\mathbf{T}_{DB}$, respectively (Figure 3.9). Since $\mathbf{T}_{CA}$ and $\mathbf{T}_{DB}$ are both fixed, the joint constraints on $\mathbf{T}_{CD}$ constrain the relative poses of A and B, with $\mathbf{T}_{AB}$ determined from

$$\mathbf{T}_{AB} = \mathbf{T}_{DB}\,\mathbf{T}_{CD}\,\mathbf{T}_{CA}^{-1}. \tag{3.8}$$

(See Section A.2 for a discussion of determining transforms between related coordinate frames).

### 3.3.2 Joint coordinates, constraints, and errors

Each different joint and connector type restricts the motion between two bodies to $M$ degrees of freedom, for some $M < 6$. Sometimes, the joint also defines a set of $M$ coordinates that parameterize these $M$ DOFs. For example, the hinge joint described above is parameterized by $\theta$. Other examples are given in Section 3.4: a 2 DOF cylindrical has coordinates $z$ and $\theta$, a 3 DOF gimbal joint is parameterized by the roll-pitch-yaw angles $\theta$, $\phi$, and $\psi$, etc. When $\mathbf{T}_{CD} = \mathbf{I}$ (where $\mathbf{I}$ is the identity transform), the coordinates are usually all equal to zero, and the joint is said to be in the *zero state*.

As explained in Section 1.2, ArtiSynth uses a full coordinate formulation for dynamic simulation. That means that instead of using joint coordinates to describe system state, it uses the combined full coordinates $\mathbf{q}$ of all dynamic components. For example, a model consisting of a single rigid body connected to ground by a hinge joint will have 6

DOF (corresponding to the 6 DOF of the body), rather than the 1 DOF implied by the hinge joint. The DOF restrictions imposed by the joints are then enforced by a set of linearized constraint relationships

$$\mathbf{G}(\mathbf{q})\mathbf{u} = \mathbf{g}, \qquad \mathbf{N}(\mathbf{q})\mathbf{u} \geq \mathbf{n} \tag{3.9}$$

that restrict the body velocities $\mathbf{u}$ computed at each simulation step, usually by solving an MLCP like (1.6). As explained in Section 1.2, the right side vectors $\mathbf{g}$ and $\mathbf{n}$ in (3.9) contain time derivative terms, which for simplicity much of the following presentation will assume to be 0.

Each joint contributes its own set of constraint equations to (3.9). Typically these take the form of *bilateral*, or equality, constraints

$$\mathbf{G}_\mathrm{J}(\mathbf{q})\mathbf{u} = 0 \tag{3.10}$$

which are added to the system's global bilateral constraint matrix $\mathbf{G}$. $\mathbf{G}_\mathrm{J}$ contains $6-M$ rows providing $6-M$ individual constraints $\mathbf{G}_k$. During simulation, these give rise to $6-M$ constraint forces (corresponding to $\lambda$ in (1.8)) which enforce the constraints.

In some cases, the joint also maintains *unilateral*, or inequality constraints, to keep $\mathbf{T}_{CD}$ out of inadmissible regions. These take the form

$$\mathbf{N}_\mathrm{J}(\mathbf{q})\mathbf{u} \geq 0 \tag{3.11}$$

and are added to the system's global unilateral constraint matrix $\mathbf{N}$. They give rise to constraint forces corresponding to $\theta$ in (1.8). A common use of unilateral constraints is to enforce range limits of the joint coordinates (Section 3.3.5), such as

$$\theta_\mathrm{min} \leq \theta \leq \theta_\mathrm{max}. \tag{3.12}$$

A specific unilateral constraint is added to $\mathbf{N}_\mathrm{J}$ only when $\mathbf{T}_{CD}$ is on or within the boundary of the inadmissible region associated with that constraint. The constraint is then said to be *engaged*. The combined number of bilateral and engaged unilateral constraints for a particular joint should not exceed 6; otherwise, the joint would be overconstrained.

Joint coordinates, when supported for a particular joint, can be both read and set. Setting a coordinate causes the joint transform $\mathbf{T}_{CD}$ to change. To accommodate this, the system adjusts the poses of one or both bodies connected to the joint, along with adjacent bodies connected to them, with preference given to bodies that are not attached to "ground". However, if this is done during simulation, and particularly if one or both of the bodies connected to the joint are moving dynamically, the results will be unpredictable and will likely conflict with the simulation.

Joint coordinates are also often exported as properties. For example, the HingeJoint class (Section 3.4) exports its $\theta$ coordinate as the property theta, which can be accessed in the GUI, or via the accessor methods

```
double getTheta()          // get theta in degrees

void setTheta (double deg) // set theta in degrees
```

Since joint constraints are generally nonlinear, their linearized enforcement at the velocity level by (3.9) will usually produce small errors as the simulation proceeds. These errors are reduced using a position correction step described in Section 4.9.1 and [11]. Errors can also be caused by joint compliance (Section 3.3.8). Both effects mean that the joint transform $\mathbf{T}_{CD}$ may deviate from the allowed values dictated by the joint type. In ArtiSynth, this is accounted for by introducing an additional *constraint* frame G between D and C (Figure 3.10). G is computed to be the nearest frame to C that lies exactly in the joint constraint space. $\mathbf{T}_{GD}$ is therefore a valid joint transform, $\mathbf{T}_{GC}$ accommodates the error, and the whole joint transform is given by the composition

$$\mathbf{T}_{CD} = \mathbf{T}_{GD}\,\mathbf{T}_{CG}. \tag{3.13}$$

If there is no compliance or joint error, then frames G and C are identical, $\mathbf{T}_{CG} = \mathbf{I}$, and $\mathbf{T}_{CD} = \mathbf{T}_{GD}$. Because $\mathbf{T}_{CG}$ describes the joint error, we sometimes refer to it as $\mathbf{T}_{err} = \mathbf{T}_{CG}$.

### 3.3.3 Creating joints

Joint and connector components in ArtiSynth are both derived from the superclass BodyConnector, with joints being further derived from JointBase, which provides support for coordinates. Some of the commonly used joints and connectors are described in Section 3.4.

An application creates a joint by constructing it and adding it to a `MechModel`. Many joints have constructors of the form

```
XXXJoint (bodyA, bodyB, TDW)
```

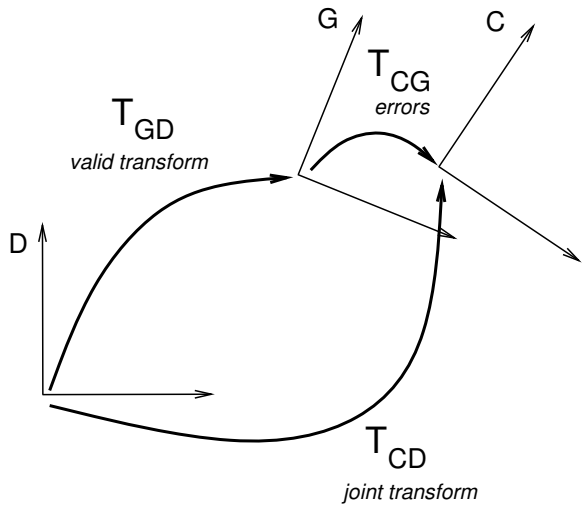



Figure 3.10: 2D schematic showing the joint frames D and C, along with the intermediate frame G that accounts for numeric error and complaint motion.

which specifies the bodies A and B which the joint connects, along with the transform $\mathbf{T}_{DW}$ giving the pose of the joint base frame D in world coordinates. The constructor then assumes that the joint is in the zero state, so that C and D are the same and $\mathbf{T}_{CD} = \mathbf{I}$ and $\mathbf{T}_{CW} = \mathbf{T}_{DW}$, and then computes $\mathbf{T}_{CA}$ and $\mathbf{T}_{DB}$ from

$$\mathbf{T}_{CA} = \mathbf{T}_{AW}^{-1}\, \mathbf{T}_{CW} \tag{3.14}$$

$$\mathbf{T}_{DB} = \mathbf{T}_{BW}^{-1}\, \mathbf{T}_{DW}, \tag{3.15}$$

where $\mathbf{T}_{AW}$ and $\mathbf{T}_{BW}$ are the current poses of A and B.

After the joint is created, it should be added to the system's `MechModel` using addBodyConnector(), as shown in the following code fragment:

```
MechModel mech;
RigidBody bodyA, bodyB;
RigidTransform3d TDW;

... initialize mech, bodyA, bodyB, and TDW ...

HingeJoint joint = new HingeJoint (bodyA, bodyB, TDW);
mech.addBodyConnector (joint);
```

It is also possible to create a joint using its default constructor and attach it to the bodies afterward, using the method setBodies(bodyA,bodyB,TDW), as in the following:

```
HingeJoint joint = new HingeJoint ();
joint.setBodies (bodyA, bodyB, TDW);
mech.addBodyConnector (joint);
```

One reason for doing this is that it allows the joint transform $\mathbf{T}_{CD}$ to be modified (by setting coordinate values) *before* `setBodies()` is called; this is discussed further in Section 3.3.4.

Joints usually offer a number of other constructors that let its world location and body relationships to be specified in different ways. These may include:

```
XXXJoint (bodyA, TCA, bodyB, TDB)

XXXJoint (bodyA, bodyB, TCW, TDW)
```

The first, which is restricted to rigid bodies, allows the application to explicitly specify transforms $\mathbf{T}_{CA}$ and $\mathbf{T}_{DB}$ connecting frames C and D to the body frames A and B, and is useful when $\mathbf{T}_{CA}$ and $\mathbf{T}_{DB}$ are explicitly known, or the

initial value of $\mathbf{T}_{CD}$ is *not* the identity. Likewise, the second constructor allows $\mathbf{T}_{CW}$ and $\mathbf{T}_{DW}$ to be explicitly specified, with $\mathbf{T}_{CD} \neq \mathbf{I}$ if $\mathbf{T}_{CW} \neq \mathbf{T}_{DW}$. For instance, suppose $\mathbf{T}_{CD}$ and $\mathbf{T}_{DW}$ are both known. Then we can use the relationship

$$\mathbf{T}_{CW} = \mathbf{T}_{DW}\,\mathbf{T}_{CD} \tag{3.16}$$

to create the joint as in the following code fragment:

```
MechModel mech;
RigidBody bodyA, bodyB;
RigidTransform3d TDW, TCD;

... initialize mech, bodyA, bodyB, TDW, and TCD ...

// compute TCW:
RigidTransform3d TCW = new RigidTransform3d();
TCW.mul (TDW, TCD);
HingeJoint joint = new HingeJoint (bodyA, bodyB, TCW, TDW);
mech.addBodyConnector (joint);
```

As an alternative to specifying $\mathbf{T}_{DW}$ or its equivalents, some joint types provide constructors that let the application locate specific joint features. These may be easier to use in some cases. For instance, HingeJoint provides a constructor

```
HingeJoint (bodyA, bodyB, originD, zaxis)
```

that specifies origin of D and its $z$ axis (which is the rotation axis), with the remaining orientation of D aligned as closely as possible with the world. SphericalJoint provides a constructor

```
SphericalJoint (bodyA, bodyB, originD)
```

that specifies origin of D and aligns its orientation with the world. Users should consult the source code or API documentation for specific joints to see what special constructors may be available.

Finally, it is possible to use joints to connect a single body to ground (by convention, this is the A body). Most joints provide a constructor of the form

```
XXXJoint (bodyA, TDW)
```

which allows this to be done explicitly. Alternatively, most joint constructors which supply body B will allow this to be specified as `null`, so that body A will be connected to ground by default.

### 3.3.4 Working with coordinates

As mentioned in Section 3.3.2, some joints support coordinates that parameterize the valid motions within the joint transform $\mathbf{T}_{CD}$. All such joints are subclasses of JointBase, which provides some generic methods for querying and setting coordinate values (`JointBase` is in turn a subclass of BodyConnector).

The number of coordinates is returned by the method `numCoordinates()`; if this returns 0, then coordinates are not supported. Each coordinate has an index in the range $0 \ldots M-1$, where $M$ is the number of coordinates. Coordinate values can be queried or set using the following methods:

```
getCoordinate (int idx)               // get coordinate value with index idx
getCoordinates (VectorNd coords)      // get all coordinates values

setCoordinate (int idx, double value) // set coordinate value with index idx
setCoordinates (VectorNd values)      // set all coordinates values
```

Specific joint types usually also provide names for their joint coordinates, along with integer constants describing their indices and methods for accessing their values. For example, CylindricalJoint supports two coordinates, $z$ and $\theta$, along with the following:

```
// coordinate indices
static final int Z_IDX = 0;
static final int THETA_IDX = 1;
```

```
// set/get z value and range
double getZ()
void setZ (double z)

// set/get theta value and range in degrees
double getTheta()
void setTheta (double theta)
```

The coordinate values are also exported as the properties z and theta, allowing them to be set in the GUI. For convenience, particularly in GUI applications, the properties and methods for controlling specific angular coordinates generally use degrees instead of radians.

As discussed in Section 3.3.2, unlike in some multibody simulation systems (such as OpenSim), joint coordinates are *not* fundamental quantities that describe system state. As such, then, coordinates can usually only be set in specific circumstances that avoid simulation conflicts. In general, when joint coordinates are set, the system adjusts the poses of one or both bodies connected to this joint, along with adjacent bodies connected to them, with preference given to bodies that are not attached to "ground". However, if this is done during simulation, and particularly if one or both of the bodies connected to the joint are moving dynamically, the results will be unpredictable and will likely conflict with the simulation.

If a joint has been created with its default constructor and not yet attached to any bodies, then setting joint values will simply set the joint transform $\mathbf{T}_{CD}$. This can be useful in situations where one needs to initialize a joint's $\mathbf{T}_{CD}$ to a non-identity value corresponding to a particular set of joint coordinates:

```
RigidTransform3d TDW; // known location for D frame
double z, theta; // desired initial coordinate values
...
CylindricalJoint joint = new CylindricalJoint();
joint.setZ (z);
joint.setTheta (thetaDeg);
joint.setBodies (bodyA, bodyB, TDW);
```

This can also be done in vector form:

```
RigidTransform3d TDW; // known location for D frame
VectorNd coordValues; // desired initial coordinate values
...
CylindricalJoint joint = new CylindricalJoint();
joint.setCoordinates (coordValues);
joint.setBodies (bodyA, bodyB, TDW);
```

In either of these cases, `setBodies()` will not use $\mathbf{T}_{CD} = \mathbf{I}$ but instead use the value determined by the initial coordinate values.

To determine the $\mathbf{T}_{CD}$ corresponding to a particular set of coordinates, one may use the method

```
void coordinatesToTCD (RigidTransform3d TCD, VectorNd coords)
```

In some cases, within a model's `build()` method, one may wish to set initial coordinates *after* a joint has been attached to its bodies, in order to move those bodies (along with the bodies attached to them) into an initial configuration without having to explicitly calculate the poses from the joint coordinates. As mentioned above, the system will make a decision about which attached bodies are most "free" and adjust their poses accordingly. This is done in the example of the next section.

### 3.3.5 Coordinate limits and locking

It is possible to set limits on a joint coordinate's range, and also to lock a coordinate in place at its current value.

When a joint coordinate hits either an upper or lower range limit, a unilateral constraint is invoked to prevent it from violating the limit, and remains engaged until the joint moves away from the limit. Each range constraint that is engaged reduces the number of joint DOFs by one.

By default, joint range limits are usually disabled (i.e., they are set to $(-\inf, \inf)$). They can be queried and set, for a given joint with index `idx`, using the methods:

```
DoubleInterval getCoordinateRange (int idx)
double getMinCoordinate (int idx)
double getMaxCoordinate (int idx)
void setCoordinateRange (idx, DoubleInterval rng)
```

where range limits for angular coordinates are specified in radians. For convenience, the following methods are also provided which use degrees instead of radians for angular coordinates:

```
DoubleInterval getCoordinateRangeDeg (int idx)
double getMinCoordinateDeg (int idx)
double getMaxCoordinateDeg (int idx)
void setCoordinateRangeDeg (idx, DoubleInterval rng)
```

Range checking can be disabled by setting the range to $(-\inf, \inf)$, or by specifying `rng` as `null`, which implicitly does the same thing.

> Ranges for angular coordinates are not limited to $\pm 180^\circ$ but can instead be set to larger values; the joint will continue to wrap until the limit is reached.

Joint coordinates can also be *locked*, so that they hold their current value and don't move. A joint is locked using a bilateral constraint that prevents motion in either direction and reduces the joint's DOF count by one. The following methods are available for querying or setting a coordinate's locked status:

```
boolean isLocked (int idx)
void setLocked (int idx, boolean locked)
```

As with coordinate values, specific joint types usually provide methods for controlling the ranges and locking status of individual coordinates, with ranges for angular coordinates specified in degrees instead of radians. For example, CylindricalJoint supplies the methods

```
// set/get z range
DoubleInterval getZRange()
void setZRange (double min, double max)

// control z locking
boolean isZLocked()
void setZLocked (boolean locked)

// set/get theta range in degrees
DoubleInterval getThetaRange()
void setThetaRange (double min, double max)
void setThetaRange (DoubleInterval rng)

// control theta locking
boolean isThetaLocked()
void setThetaLocked (boolean locked)
```

The range and locking information is also exported as the properties zRange, thetaRange, zLocked, and thetaLocked, allowing them to be set in the GUI.

### 3.3.6 Example: a simple hinge joint

A simple model showing two rigid bodies connected by a joint is defined in

```
artisynth.demos.tutorial.RigidBodyJoint
```

The build method for this model is given below:

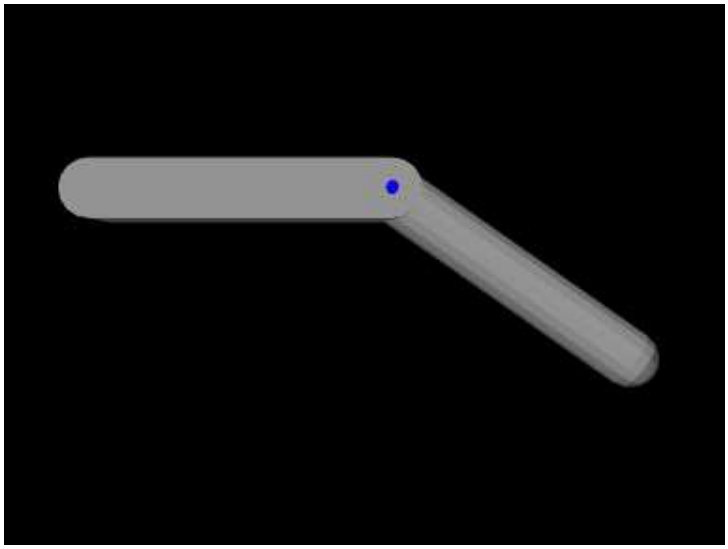


Figure 3.11: RigidBodyJoint model loaded into ArtiSynth.

```
public void build (String[] args) {

   // create MechModel and add to RootModel
   mech = new MechModel ("mech");
   mech.setGravity (0, 0, -98);
   mech.setFrameDamping (1.0);
   mech.setRotaryDamping (4.0);
   addModel (mech);

   PolygonalMesh mesh;  // bodies will be defined using a mesh

   // create first body and set its pose
   mesh = MeshFactory.createRoundedBox (lenx1, leny1, lenz1, /*nslices=*/8);
   RigidTransform3d TMB =
      new RigidTransform3d (0, 0, 0, /*axisAng=*/1, 1, 1, 2*Math.PI/3);
   mesh.transform (TMB);
   bodyB = RigidBody.createFromMesh ("bodyB", mesh, /*density=*/0.2, 1.0);
   bodyB.setPose (new RigidTransform3d (0, 0, 1.5*lenx1, 1, 0, 0, Math.PI/2));
   bodyB.setDynamic (false);

   // create second body and set its pose
   mesh = MeshFactory.createRoundedCylinder (
      leny2/2, lenx2, /*nslices=*/16, /*nsegs=*/1, /*flatBottom=*/false);
   mesh.transform (TMB);
   bodyA = RigidBody.createFromMesh ("bodyA", mesh, 0.2, 1.0);
   bodyA.setPose (new RigidTransform3d (
                     (lenx1+lenx2)/2, 0, 1.5*lenx1, 1, 0, 0, Math.PI/2));

   // create the joint
   RigidTransform3d TDW =
      new RigidTransform3d (lenx1/2, 0, 1.5*lenx1, 1, 0, 0, Math.PI/2);
   HingeJoint joint = new HingeJoint (bodyA, bodyB, TDW);

   // add components to the mech model
   mech.addRigidBody (bodyB);
   mech.addRigidBody (bodyA);
   mech.addBodyConnector (joint);

   joint.setTheta (35);  // set joint position

   // set render properties for components
```

```
42        RenderProps.setFaceColor (joint, Color.BLUE);
43        joint.setShaftLength (4);
44        joint.setShaftRadius (0.2);
45     }
```

A `MechModel` is created as usual at line 4. However, in this example, we also set some parameters for it: setGravity() is used to set the gravity acceleration vector to $(0,0,-98)^T$ instead of the default value of $(0,0,-9.8)^T$, and the `frameDamping` and `rotaryDamping` properties (Section 3.2.7) are set to provide appropriate damping.

Each of the two rigid bodies are created from a mesh and a density. The meshes themselves are created using the factory methods MeshFactory.createRoundedBox() and MeshFactory.createRoundedCylinder() (lines 13 and 22), and then RigidBody.createFromMesh() is used to turn these into rigid bodies with a density of 0.2 (lines 17 and 25). The pose of the two bodies is set using `RigidTransform3d` objects created with x, y, z translation and axis-angle orientation values (lines 18 and 26).

The hinge joint is implemented using HingeJoint, which is constructed at line 32 with the joint coordinate frame D being located in world coordinates by `TDW` as described in Section 3.3.3.

Once the joint is created and added to the `MechModel`, the method setTheta() is used to explicitly set the joint parameter to 35 degrees. The joint transform $\mathbf{T}_{CD}$ is then set appropriately and `bodyA` is moved to accommodate this (`bodyA` being chosen since it is the most free to move).

Finally, joint rendering properties are set starting at line 42. We render the joint as a cylindrical shaft about the rotation axis, using its shaftLength and shaftRadius properties. Joint rendering is discussed in more detail in Section 3.3.10).

To run this example in ArtiSynth, select All demos > tutorial > RigidBodyJoint from the Models menu. The model should load and initially appear as in Figure 3.11. Running the model (Section 1.5.3) will cause `bodyA` to fall and swing under gravity.

### 3.3.7 Constraint forces

During each simulation solve step, the joint velocity constraints described by (3.10) and (3.11) are enforced by bilateral and unilateral constraint forces $\mathbf{f}_g$ and $\mathbf{f}_n$:

$$\mathbf{f}_g = \mathbf{G}_J^T \lambda_J, \quad \mathbf{f}_n = \mathbf{N}_J^T \theta_J. \tag{3.17}$$

Here, $\mathbf{f}_g$ and $\mathbf{f}_n$ are spatial forces (or *wrenches*, Section A.5) acting in the joint coordinate frame C, and $\lambda_J$ and $\theta_J$ are the Lagrange multipliers computed as part of the mechanical system solve (see (1.6) and (1.8)). The sizes of $\lambda_J$ and $\theta_J$ equal the number of bilateral and *engaged* unilateral constraints in the joint; these numbers can be queried for a particular joint using the methods numBilateralConstraints() and numEngagedUnilateralConstraints(). (The number of engaged unilateral constraints may be less than the total number of unilateral constraints; the latter may be queried with numUnilateralConstraints(), while the total number of constraints is returned by numConstraints().

Applications may sometimes need to query the current constraint force values, typically from within a controller or monitor (Section 5.3). The Lagrange multipliers themselves may be obtained with

```
void getBilateralForces (VectorNd lam)

void getUnilateralForces (VectorNd the)
```

which load the multipliers into `lam` or `the` and set their sizes to the number of bilateral or engaged unilateral constraints. Alternatively, one can retrieve the individual multiplier for the constraint indexed by `idx` using

```
double getConstraintForce (int idx);
```

Typically, it is more useful to find the spatial constraint forces $\mathbf{f}_g$ and $\mathbf{f}_n$, which can be obtained with respect to frame C:

```
// place the forces in the wrench f
void getBilateralForcesInC (Wrench f)
void getUnilateralForcesInC (Wrench f)

// convenience methods that allocate the wrench and return it
Wrench getBilateralForcesInC();
Wrench getUnilateralForcesInC();
```

If the attached bodies A and B are rigid bodies, it is also possible to obtain the constraint wrenches experienced by those bodies:

```
// place the forces in the wrench f
void getBilateralForcesInA (Wrench f)
void getUnilateralForcesInA (Wrench f)
void getBilateralForcesInB (Wrench f)
void getUnilateralForcesInB (Wrench f)

// convenience methods that allocate the wrench and return it
Wrench getBilateralForcesInA();
Wrench getUnilateralForcesInA();
Wrench getBilateralForcesInB();
Wrench getUnilateralForcesInB();
```

Constraint wrenches obtained for bodies A or B are given in world coordinates, which is consistent with the forces reported by rigid bodies via their `getForce()` method. To orient the forces into body coordinates, one may use the inverse of the rotation matrix **R** of the body's pose. For example:

```
RigidBody bodyA;

// ... body A initialized, etc. ...

Wrench force = joint.getBilateralForceInA();
force.inverseTransform (bodyA.getPose().R);
```

### 3.3.8 Compliance and regularization

By default, the constraints used to implement joints and couplings are treated as *hard*, so that the system tries to respect the constraint conditions (3.9) as exactly as possible as the simulation proceeds. Sometimes, however, it is desirable to introduce some "softness" into the constraints, whereby constraint forces are determined as a linear function of their distance from the constraint. Adding compliance also allows an application to *regularize* a system of joint constraints that would otherwise be overconstrained, as illustrated in Section 3.3.9.

To describe compliance precisely, consider the bilateral constraint portion of the MLCP in (1.6), which solves for the updated system velocities $\mathbf{u}^{k+1}$ at each time step:

$$\begin{pmatrix} \hat{\mathbf{M}}^k & -\mathbf{G}^T \\ \mathbf{G} & 0 \end{pmatrix} \begin{pmatrix} \mathbf{u}^{k+1} \\ \tilde{\lambda} \end{pmatrix} = \begin{pmatrix} \mathbf{M}\mathbf{u}^k - h\hat{\mathbf{f}}^k \\ 0 \end{pmatrix}. \tag{3.18}$$

Here $\mathbf{G}$ is the system's bilateral constraint matrix, $\tilde{\lambda}$ denotes the constraint impulses (from which the constraint forces $\lambda$ can be determined by $\lambda = \tilde{\lambda}/h$), and for simplicity we have assumed that $\mathbf{G}$ is constant and so the $\mathbf{g}$ term on the lower right side is 0.

Solving (3.18) results in constraint forces that satisfy $\mathbf{G}\mathbf{u}^{k+1} = 0$ precisely, corresponding to hard constraints. To implement soft constraints, start by defining a function $\phi(\mathbf{q})$ that defines the *distances* from each constraint, where $\mathbf{q}$ is the vector of system positions; these distances are the local translational and rotational deviations from each constraint's correct position and are discussed in more detail in Section 4.9.1. Then assume that the constraint forces are a linear function of these distances:

$$\lambda = -\mathbf{C}^{-1}\phi(\mathbf{q}), \tag{3.19}$$

where $\mathbf{C}$ is a diagonal *compliance* matrix that is equivalent to an inverse stiffness matrix. We also note that $\phi$ will be time varying, and that we can approximate its change between time steps as

$$\phi^{k+1} \approx \phi^k + h\dot{\phi}^{k+1}, \quad \text{with} \quad \dot{\phi}^{k+1} = \mathbf{G}\mathbf{u}^{k+1}. \tag{3.20}$$

Next, assume that in using (3.19) to determine $\lambda$ for a particular time step, we use the *average* value of $\phi$ over the step, represented by $\bar{\phi} = (\phi^{k+1} + \phi^k)/2$. Substituting this and (3.20) into (3.19), multiplying by $\mathbf{C}$, and rearranging yields:

$$\mathbf{G}\mathbf{u}^{k+1} + \frac{2\mathbf{C}}{h}\lambda = -\frac{2}{h}\phi^k. \tag{3.21}$$

Then noting that $\tilde{\lambda} = h\lambda$, we obtain a revised form of (3.18),

$$\begin{pmatrix} \hat{\mathbf{M}}^k & -\mathbf{G}^T \\ \mathbf{G} & 2\mathbf{C}/h^2 \end{pmatrix} \begin{pmatrix} \mathbf{u}^{k+1} \\ \tilde{\lambda} \end{pmatrix} = \begin{pmatrix} \mathbf{M}\mathbf{u}^k - h\hat{\mathbf{f}}^k \\ -2\phi^k/h \end{pmatrix}, \tag{3.22}$$

in the which the zeros in the matrix and right hand side have been replaced by compliance terms. The resulting constraint behavior is different from that of (3.18) in two important ways:

1. The joint now allows 6 DOF, with motion along the constrained directions limited by restoring spring constants given by the reciprocals of the diagonal entries of $\mathbf{C}$.
2. If $\mathbf{C}$ has no zero diagonal entries, then the system (3.22) is *regularized* by the $2\mathbf{C}/h^2$ term in the lower right matrix block. This means that the matrix is always non-singular, even if $\mathbf{G}$ is rank deficient, and so compliance offers a way to handle overconstrained models, as discussed further in Section 3.3.9.

Unilateral constraints can be regularized using the same approach, with a distance function defined such that $\phi(\mathbf{q}) \leq 0$.

The reason for specifying soft constraints using compliance instead of stiffness is that by setting $\mathbf{C} = 0$ we can easily handle the case of *infinite* stiffness where the constraints are strictly enforced. The ArtiSynth compliance implementation uses a slightly more complex version of (3.22) that accounts for non-constant $\mathbf{G}$ and also allows for a damping term $-\mathbf{D}\dot{\phi}$, where $\mathbf{D}$ is again a diagonal matrix. For more details, see [9] and [21].

When using compliance, damping is often needed for stability, and, in the case of unilateral constraints, to prevent "bouncing". A good choice for damping is usually *critical damping*, which is discussed further below.

Any joint which is a subclass of BodyConnector allows individual compliance values $C_i$ and damping values $D_i$ to be set for each of the joint's $i$ constraints. These values comprise the diagonal entries in the compliance and damping matrices $\mathbf{C}$ and $\mathbf{D}$, and can be queried and set using the methods

```
VectorNd getCompliance()
void setCompliance (VectorNd compliance)

VectorNd getDamping()
void setCompliance (VectorNd damping)
```

The vectors supplied to the above `set` methods contain the requested compliance or damping values. If their size $n$ is less than `numConstraints()`, then compliance or damping will be set for the first $n$ constraints. Damping for a specific constraint only has an effect if the compliance for that constraint is nonzero.

What compliance and damping values should be specified? Compliance is usually relatively easy to figure out. Each of the joint's individual constraints $i$ corresponds to a row in its bilateral constraint matrix $\mathbf{G}_\text{J}$ or unilateral constraint matrix $\mathbf{N}_\text{J}$, and represents a specific 6 DOF direction along which the spatial velocity $\hat{\mathbf{v}}_{CD}$ (of frame C with respect to D) is restricted (more details on this are given in Section 4.9.1). Each of these constraint directions is usually predominantly linear or rotational; specific descriptions for the constraints of different joints are provided in Section 3.4. To determine compliance for a constraint $i$, estimate the typical force $f$ likely to act along its direction, decide how much displacement $\delta q$ (translational or rotational) along that constraint is desirable, and then set the compliance $C_i$ to the associated inverse stiffness:

$$C_k = \delta q / f. \tag{3.23}$$

Once $C_k$ is determined, the damping $D_k$ can be estimated based on the desired damping ratio $\zeta$, using the formula

$$D_k = 2\zeta\sqrt{M/C_k} \tag{3.24}$$

where $M$ is total mass of the bodies attached to the joint. Typically, the desired damping will be close to critical damping, for which $\zeta = 1$.

Constraints associated with linear motion will typically require different compliance values from those associated with rotation. To make this process easier, joint components allow the setting of collective compliance values for their linear and rotary constraints, using the methods

```
void setLinearCompliance (double c)
double getLinearCompliance()

void setRotaryCompliance (double c)
double getRotaryCompliance()
```

The `set()` methods will set a uniform compliance for all linear or rotary constraints, except for unilateral constraints associated with coordinate limits. At the same time, they will also set an *automatically* computed critical damping value. Likewise, the `get()` methods query these linear or rotary constraints for uniform compliance values (with the corresponding critical damping), and return either that value, or -1 if it does not exist.

Most of the demonstration models for the joints described in Section 3.4 allow these linear and rotary compliance settings to be adjusted interactively using a control panel, enabling users to experimentally gain a feel for their behavior.

To determine programmatically whether a particular constraint is linear or rotary, one can use the joint method

```
VectorNi getConstraintFlags()
```

which returns a vector of information flags for all its constraints. Linear and rotary constraints are indicated by the flags `LINEAR` and `ROTARY`, defined in RigidBodyConstraint.

### 3.3.9 Example: an overconstrained linkage

Situations may occasionally arise in which a model is *overconstrained*, which means that the rows of the bilateral constraint matrix **G** in (3.9) are not all linearly dependent, or in other words, **G** does not have *full row rank*. At present, the ArtiSynth solver has difficultly handling overconstrained models, but these situations can often be handled by adding a small amount of compliance to the constraints. (Overconstraining is not a problem with unilateral constraints **N**, because of the way they are handled by the solver.)

One possible symptom of an overconstrained system is a error message in the application's terminal output, such as

```
Pardiso: num perturbed pivots=12
```

Overconstraining frequently occurs in closed-chain linkages, involving loops in which a jointed sequence of links is connected back on itself. Depending on how the constraints are configured and how redundant they are, the system may still be able to move. A classical example is the *four-bar linkage*, a common version of which consists of four links, or "bars", arranged as a parallelogram and connected by hinge joints at the corners. One link is usually connected to ground, and so the remaining three links together have 18 DOF, while the four hinge joints together remove 20 DOF, overconstraining the system. However, the constraints are redundant in such as way that the linkage still actually has 1 DOF.

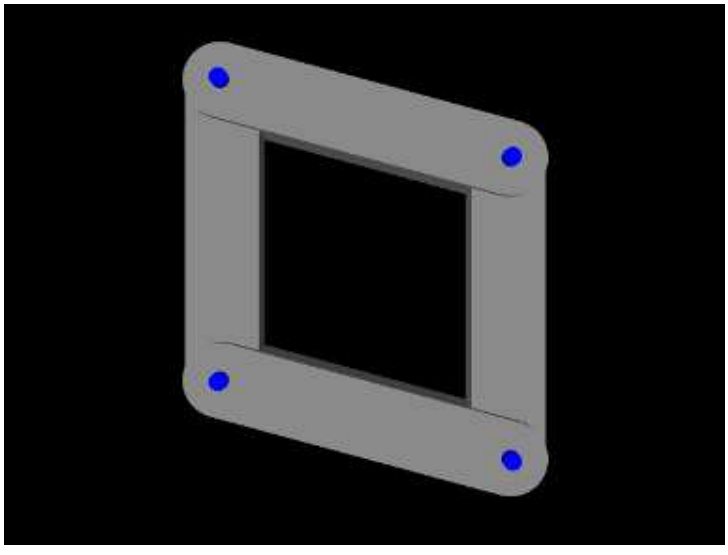


Figure 3.12: FourBarLinkage model, several steps into the simulation.

To model a four-bar in ArtiSynth presently requires adding compliance to the hinge joints. An example of this is defined by the demo program

```
artisynth.demos.tutorial.FourBarLinkage
```

shown in Figure 3.12. The code for the `build()` method and a couple of supporting methods is given below:

```
   /**
    * Create a link with a length of 1.0, width of 0.25, and specified depth
    * and add it to the mech model. The parameters x, z, and deg specify the
    * link's position and orientation (in degrees) in the x-z plane.
    */
   protected RigidBody createLink (
      MechModel mech, String name,
      double depth, double x, double z, double deg) {
      int nslices = 20; // num slices on the rounded mesh ends
      PolygonalMesh mesh =
         MeshFactory.createRoundedBox (1.0, 0.25, depth, nslices);
      RigidBody body = RigidBody.createFromMesh (
         name, mesh, /*density=*/1000.0, /*scale=*/1.0);
      body.setPose (new RigidTransform3d (x, 0, z, 0, Math.toRadians(deg), 0));
      mech.addRigidBody (body);
      return body;
   }

   /**
    * Create a hinge joint connecting one end of link0 with the other end of
    * link1, and add it to the mech model.
    */
   protected HingeJoint createJoint (
      MechModel mech, String name, RigidBody link0, RigidBody link1) {
      // easier to locate the link using TCA and TDB since we know where frames
      // C and D are with respect the link0 and link1
      RigidTransform3d TCA = new RigidTransform3d (0, 0,  0.5, 0, 0, Math.PI/2);
      RigidTransform3d TDB = new RigidTransform3d (0, 0, -0.5, 0, 0, Math.PI/2);
      HingeJoint joint = new HingeJoint (link0, TCA, link1, TDB);
      joint.setName (name);
      mech.addBodyConnector (joint);
      // set joint render properties
      joint.setAxisLength (0.4);
      RenderProps.setLineRadius (joint, 0.03);
      return joint;
   }

   public void build (String[] args) {
      // create a mech model and set rigid body damping parameters
      MechModel mech = new MechModel ("mech");
      addModel (mech);
      mech.setFrameDamping (1.0);
      mech.setRotaryDamping (4.0);

      // create four 'bars' from which to construct the linkage
      RigidBody[] bars = new RigidBody[4];
      bars[0] = createLink (mech, "link0", 0.2, -0.5,  0.0, 0);
      bars[1] = createLink (mech, "link1", 0.3,  0.0,  0.5, 90);
      bars[2] = createLink (mech, "link2", 0.2,  0.5,  0.0, 180);
      bars[3] = createLink (mech, "link3", 0.3,  0.0, -0.5, 270);
      // ground the left bar
      bars[0].setDynamic (false);

      // connect the bars using four hinge joints
      HingeJoint[] joints = new HingeJoint[4];
      joints[0] = createJoint (mech, "joint0", bars[0], bars[1]);
      joints[1] = createJoint (mech, "joint1", bars[1], bars[2]);
      joints[2] = createJoint (mech, "joint2", bars[2], bars[3]);
      joints[3] = createJoint (mech, "joint3", bars[3], bars[0]);

      // Set uniform compliance and damping for all bilateral constraints,
      // which are the first 5 constraints of each joint
      VectorNd compliance = new VectorNd(5);
```

```
64        VectorNd damping = new VectorNd(5);
65        for (int i=0; i<5; i++) {
66           compliance.set (i, 0.000001);
67           damping.set (i, 25000);
68        }
69        for (int i=0; i<joints.length; i++) {
70           joints[i].setCompliance (compliance);
71           joints[i].setDamping (damping);
72        }
73     }
```

Two helper methods are used to construct the model: `createLink()` (lines 6-17), and `createJoint()` (lines 23-36). `createLink()` makes the individual rigid bodies used to build the linkage: a mesh is produced defining the body's shape (a box with rounded ends), and then passed to the RigidBody `createFromMesh()` method which creates the body and sets its inertia according to a specified density. The body's pose is then set so as to center it at $(x,0,z)$ while rotating it about the $y$ axis by the angle `deg` (in degrees). The completed body is then added to the `MechModel mech` and returned.

The second helper method, `createJoint()`, connects two rigid bodies (`link0` and `link1`) together using a `HingeJoint`. Because we know the location of the joint in body-relative coordinates, it is easier to create the joint using the transforms $\mathbf{T}_{CA}$ and $\mathbf{T}_{DB}$ instead of $\mathbf{T}_{DW}$: $\mathbf{T}_{CA}$ locates the joint at the top end of `link0`, at $(0,0,0.5)$, with the $z$ axis parallel to the body's $y$ axis, while $\mathbf{T}_{DB}$ similarly locates the joint at the bottom of `link1`. After the joint is created and added to the `MechModel`, its render properties are set so that its axis drawn as a blue cylinder.

The `build()` method itself begins by creating a `MechModel` and setting damping parameters for the rigid bodies (lines 40-43). Next, `createLink()` is used to create and store the four links (lines 46-50), and the left bar is attached to ground by making it non-dynamic (line 52). The links are then connected together using joints created by `createJoint()` (lines 55-59). Finally, uniform compliance and damping values are set for each of the joint's bilateral constraints, using the `setCompliance()` and `setDamping()` methods (lines 63-72). Values are set for the first five constraints, since for a `HingeJoint` these are the bilateral constraints. The compliance value of $C = 10^{-6}$ was found experimentally to be low enough so as to not cause noticeable deflections in the joints. Given $C$ and an average mass of around $M = 150$ for each link pair, (3.24) suggests the damping factor of $D = 25000$. Note that for this example, very similar settings could be achieved by simply calling

```
   for (int i=0; i<joints.length; i++) {
      joints[i].setLinearCompliance (0.000001);
      joints[i].setRotaryCompliance (0.000001);
   }
```

In principle, we only need to set compliance for the constraints that are redundant, but it can sometimes be difficult to determine exactly which these are. Also, different values are often needed for linear and rotary constraints; that is not necessary here because the links have unit length and so the linear and rotary units have similar scales.

### 3.3.10 Rendering joints

Most joints provide a means to render themselves in order to provide a graphical representation of their position and configuration. Control over this is achieved by setting various properties in the joint component, including both specialized properties and the standard render properties (Section 4.3) used by all renderable components.

All joints which are subclasses of JointBase support rendering of both their C and D coordinate frames, through the properties drawFrameC, drawFrameD, and axisLength. The first two properties are of the type Renderer.AxisDrawStyle (described in detail in Section 3.2.8), and can be set to `LINE` or `ARROW` to enable the coordinate axes to be drawn either as lines or solid arrows. The axisLength property has type `double` and specifies the length with which the axes are drawn. As with all properties, these properties can be set either in the GUI, or in code using accessor methods supplied by the joint:

```
   void setAxisLength (double l)
   double getAxisLength()

   void setDrawFrameC (AxisDrawStyle style)
   (AxisDrawStyle getDrawFrameC()
```

```
void setDrawFrameD (AxisDrawStyle style)
(AxisDrawStyle getDrawFrameD()
```

Another pair of properties used by several joints is shaftLength and shaftRadius, which specify the length and radius used to draw shaft or axis structures associated with the joint. These are rendered as solid cylinders, using the color indicated by the faceColor rendering property. The default value of both properties is 0; if shaftLength is 0, then the structures are not drawn, while if shaftRadius is 0, a default value proportional to shaftLength is used. For example, to enable rendering of a blue shaft along the rotation axis of a hinge joint, one may use the code fragment

```
HingeJoint joint;
...

joint.setShaftLength (0.5); // set shaft dimensions
joint.setShaftRadius (0.05);
RenderProps.setFaceColor (joint, Color.BLUE); // set the color
```

As another example, to enable rendering of a green ball about the center of a spherical joint, one may use the fragment

```
SphericalJoint joint;
...

joint.setJointRadius (0.02); // set the ball size
RenderProps.setFaceColor (joint, Color.GREEN); // set the color
```

Specific joints may define additional properties to control how they are rendered.

## 3.4 Joint components

ArtiSynth supplies a number of basic joints and connectors in the package `artisynth.core.mechmodels`, the most common of which are described here.

Many of the descriptions are associated with a demonstration model, named `XXXJointDemo`, where `XXX` is the joint type. These demos are located in the package `artisynth.demos.mech`, and can be loaded by selecting All demos > mech > XXXJointDemo from the Models menu. When run, they can be interactively controlled, using either the pull tool (see the section "Pull Manipulation" in the ArtiSynth User Interface Guide), or the interactive control panel. The control panel allows the adjustment of coordinate values and ranges (if supported), some of the render properties, and the different compliance and damping properties (Section 3.3.8). One can inspect the source code for each demo in its `.java` file located in the folder `<ARTISYNTH_HOME>/src/artisynth/demos/mech`.

### 3.4.1 Hinge joint

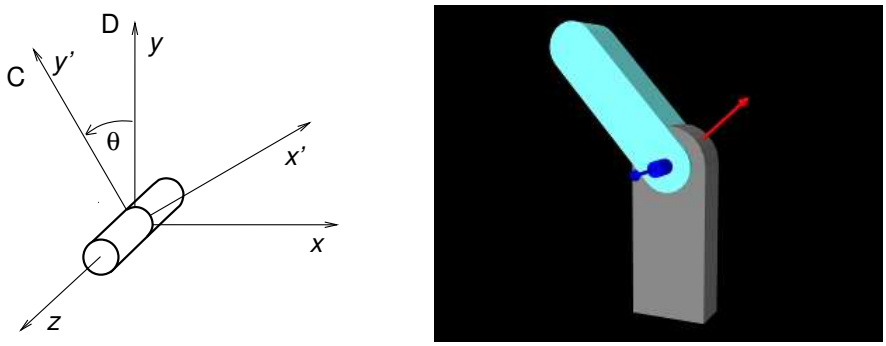



Figure 3.13: Coordinate frames (left) and demo model (right) for the hinge joint.

The HingeJoint (Figure 3.13) is a 1 DOF joint that constrains motion between frames C and D to a simple rotation about the $z$ axis of D. It implements six constraints and one coordinate $\theta$ (Table 3.1), to which the joint transform $\mathbf{T}_{CD}$ is

related by

$$\mathbf{T}_{CD} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

The value and ranges for $\theta$ are exported by the properties theta and thetaRange, and the $\theta$ coordinate index is defined by the constant `THETA_IDX`. For rendering, the properties shaftLength and shaftRadius control the size of a shaft drawn about the rotation axis, using the faceColor rendering property. A demo is provided by `artisynth.demos.mech.HingeJointDemo`.

In addition to the standard constructors described in Section 3.3.3,

```
HingeJoint (bodyA, bodyB, originD, zaxis)
```

creates a hinge joint with a specified origin and $z$ axis direction for frame D (in world coordinates), and frames C and D coincident.

| Index | type/name | description |
|---|---|---|
| 0 | bilateral | restricts translation along $x$ |
| 1 | bilateral | restricts translation along $y$ |
| 2 | bilateral | restricts translation along $z$ |
| 3 | bilateral | restricts rotation about $x$ |
| 4 | bilateral | restricts rotation about $y$ |
| 5 | unilataral | enforces limits on $\theta$ |
| 0 | $\theta$ | counter-clockwise rotation of $C$ about the $z$ axis |

Table 3.1: Constraints (top) and coordinates (bottom) for the hinge joint.

### 3.4.2 Slider joint

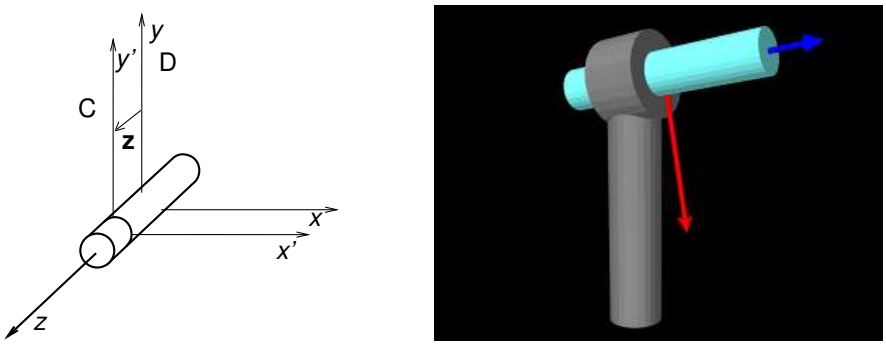



Figure 3.14: Coordinate frames (left) and demo model (right) for the slider joint.

The SliderJoint (Figure 3.14) is a 1 DOF joint that constrains motion between frames C and D to a simple translation along the $z$ axis of D. It implements six constraints and one coordinate $z$ (Table 3.2), to which the joint transform $\mathbf{T}_{CD}$ is related by

$$\mathbf{T}_{CD} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

The value and ranges for $z$ are exported by the properties z and zRange, and the $z$ coordinate index is defined by the constant `Z_IDX`. For rendering, the properties shaftLength and shaftRadius control the size of the shaft drawn about the sliding axis, using the faceColor rendering property. A demo is provided by `artisynth.demos.mech.SliderJointDemo`.

In addition to the standard constructors described in Section 3.3.3,

```
SliderJoint (bodyA, bodyB, originD, zaxis)
```

creates a slider joint with a specified origin and $z$ axis direction for frame D (in world coordinates), and frames C and D coincident.

| Index | type/name | description |
|---|---|---|
| 0 | bilateral | restricts translation along $x$ |
| 1 | bilateral | restricts translation along $y$ |
| 2 | bilateral | restricts rotation about $x$ |
| 3 | bilateral | restricts rotation about $y$ |
| 4 | bilateral | restricts rotation about $z$ |
| 5 | unilataral | enforces limits on the $z$ coordinate |
| 0 | $z$ | translation of $C$ along the $z$ axis |

Table 3.2: Constraints (top) and coordinates (bottom) for the slider joint.

### 3.4.3 Cylindrical joint

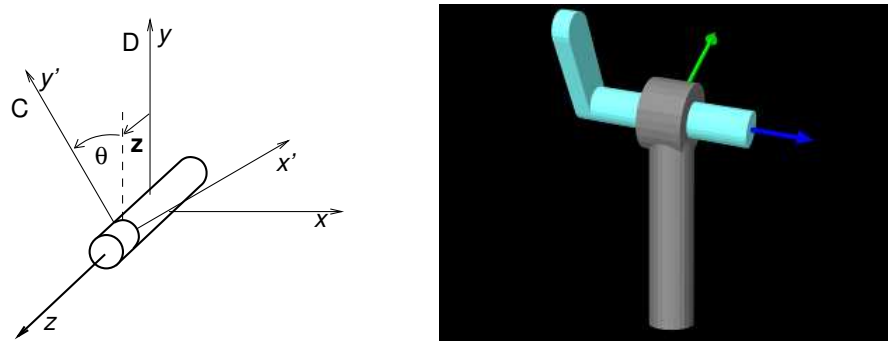



Figure 3.15: Coordinate frames (left) and demo model (right) for the cylindrical joint.

The CylindricalJoint (Figure 3.15) is a 2 DOF joint that constrains motion between frames C and D to translation and rotation along and about the $z$ axis of D. It implements six constraints and two coordinates $z$ and $\theta$ (Table 3.3), to which the joint transform $\mathbf{T}_{CD}$ is related by

$$\mathbf{T}_{CD} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

The value and ranges for $z$ and $\theta$ are exported by the properties z, theta, zRange and thetaRange, and the coordinate indices are defined by the constants `Z_IDX` and `THETA_IDX`. For rendering, the properties shaftLength and shaftRadius control the size of a shaft drawn about the sliding/rotation axis, using the faceColor rendering property. A demo is provided by `artisynth.demos.mech.CylindricalJointDemo`.

In addition to the standard constructors described in Section 3.3.3,

```
CylindricalJoint (bodyA, bodyB, originD, zaxis)
```

creates a cylindrical joint with a specified origin and $z$ axis direction for frame D (in world coordinates), and frames C and D coincident.

### 3.4.4 Slotted hinge joint

The SlottedHingeJoint (Figure 3.16) is a 2 DOF joint that constrains motion between frames C and D to translation along the $x$ axis and rotation about the $z$ axis of D. It implements six constraints and two coordinates $x$ and $\theta$ (Table 3.4), to which the joint transform $\mathbf{T}_{CD}$ is related by

$$\mathbf{T}_{CD} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 & x \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{3.25}$$

| Index | type/name | description |
|---|---|---|
| 0 | bilateral | restricts translation along $x$ |
| 1 | bilateral | restricts translation along $y$ |
| 2 | bilateral | restricts rotation about $x$ |
| 3 | bilateral | restricts rotation about $y$ |
| 4 | unilataral | enforces limits on the $z$ coordinate |
| 5 | unilataral | enforces limits on the $\theta$ coordinate |
| 0 | $z$ | translation of $C$ along the $z$ axis |
| 1 | $\theta$ | rotation of $C$ about the $z$ axis |

Table 3.3: Constraints (top) and coordinates (bottom) for the cylindrical joint.

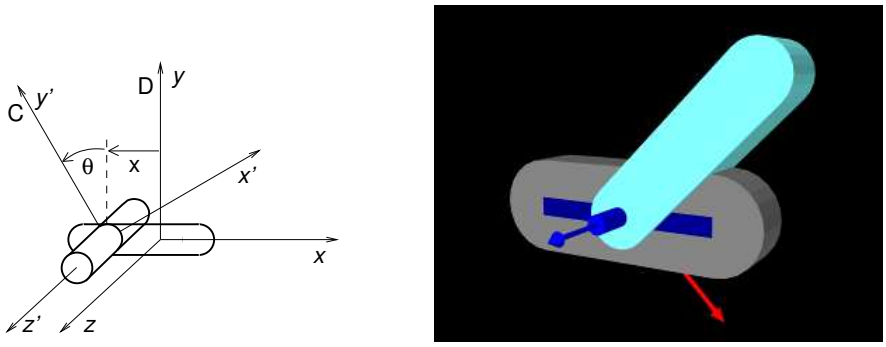



Figure 3.16: Coordinate frames (left) and demo model (right) for the slotted hinge joint.

The value and ranges for $x$ and $\theta$ are exported by the properties x, theta, xRange and thetaRange, and the coordinate indices are defined by the constants `X_IDX` and `THETA_IDX`. For rendering, the properties shaftLength and shaftRadius control the size of a shaft drawn about the rotation axis, while slotWidth and slotDepth control the width and depth of a slot drawn along the sliding ($x$) axis; both are drawn using the faceColor rendering property. When rendering the slot, its bounds along the $x$ axis are set to xRange by default. However, this may be too large, particularly if xRange is unbounded. As an alternate, the property slotRange will be used instead if its range (i.e., the upper bound minus the lower bound) exceeds 0. A demo of `SlottedHingeJoint` is provided by `artisynth.demos.mech.SlottedHingeJointDemo`.

In addition to the standard constructors described in Section 3.3.3,

```
SlottedHingeJoint (bodyA, bodyB, originD, zaxis)
```

creates a slotted hinge joint with a specified origin and $z$ axis direction for frame D (in world coordinates), and frames C and D coincident.

| Index | type/name | description |
|---|---|---|
| 0 | bilateral | restricts translation along $y$ |
| 1 | bilateral | restricts translation along $z$ |
| 2 | bilateral | restricts rotation about $x$ |
| 3 | bilateral | restricts rotation about $y$ |
| 4 | unilataral | enforces limits on the $x$ coordinate |
| 5 | unilataral | enforces limits on the $\theta$ coordinate |
| 0 | $x$ | translation of $C$ along the $x$ axis |
| 1 | $\theta$ | rotation of $C$ about the $z$ axis |

Table 3.4: Constraints (top) and coordinates (bottom) for the slotted hinge joint.

### 3.4.5 Universal joint

The UniversalJoint (Figure 3.17) is a 2 DOF joint that allows C two rotational degrees of freedom with respect to D: a *roll* rotation $\theta$ about D's $z$ axis, followed by a *pitch* rotation $\phi$ about the rotated $y'$ axis. It implements six constraints and

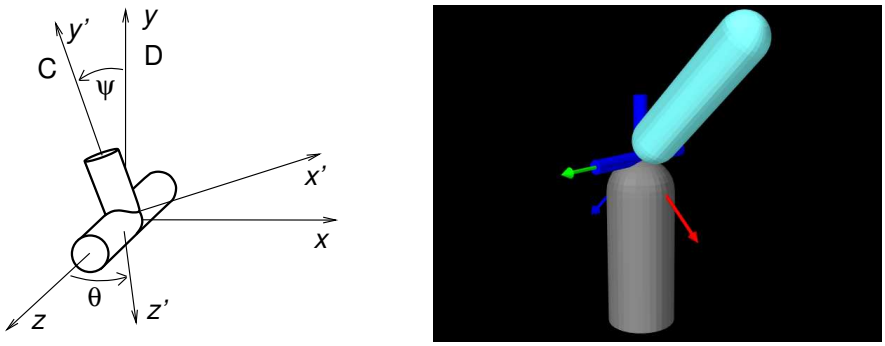



Figure 3.17: Coordinate frames (left) and demo model (right) for the universal joint.

| Index | type/name | description |
|---|---|---|
| 0 | bilateral | restricts translation along $x$ |
| 1 | bilateral | restricts translation along $y$ |
| 2 | bilateral | restricts translation along $z$ |
| 3 | bilateral | restricts rotation about the final $x$ axis of C |
| 4 | unilataral | enforces limits on the roll coordinate |
| 5 | unilataral | enforces limits on the pitch coordinate |
| 0 | $\theta$ (roll) | first rotation of $C$ about the $z$ axis of D |
| 1 | $\phi$ (pitch) | second rotation of $C$ about the rotated $y'$ axis |

Table 3.5: Constraints (top) and coordinates (bottom) for the universal joint.

the two coordinates $\theta$ and $\phi$ (Table 3.5), to which the joint transform $\mathbf{T}_{CD}$ is related by

$$\mathbf{T}_{CD} = \begin{pmatrix} c_\theta c_\phi & -s_\theta & c_\theta s_\phi & 0 \\ s_\theta c_\phi & c_\theta & s_\theta s_\phi & 0 \\ -s_\phi & 0 & c_\phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

where

$$c_\theta \equiv \cos(\theta),\ s_\theta \equiv \sin(\theta),\ c_\phi \equiv \cos(\phi),\ s_\phi \equiv \sin(\phi).$$

The value and ranges for $\theta$ and $\phi$ are exported by the properties roll, pitch, rollRange and pitchRange, and the coordinate indices are defined by the constants `ROLL_IDX` and `PITCH_IDX`. For rendering, the properties shaftLength and shaftRadius control the size of shafts drawn about the roll and pitch axes, while jointRadius specifies the radius of a ball drawn around the origin of D; both are drawn using the faceColor rendering property. A demo is provided by `artisynth.demos.mech.UniversalJointDemo`.

### 3.4.6 Skewed universal joint

The SkewedUniversalJoint (Figure 3.18) is a version of the universal joint in which the pitch axis is skewed relative to its nominal direction by an angle $\alpha$. More precisely, let $x'$ and $y'$ be the $x$ and $y$ axes of C after the initial roll rotation. For a regular universal joint, the pitch axis is $y'$, whereas for a skewed universal joint it is $y'$ rotated by $\alpha$ clockwise about $x'$. The joint still has 2 DOF, but the space of allowed rotations is reduced.

The constraints and the coordinates are the same as for the universal joint, although the relationship between $\mathbf{T}_{CD}$ is now more complicated. With $c_\theta$, $s_\theta$, $c_\phi$, and $s_\phi$ defined as for the universal joint, $\mathbf{T}_{CD}$ is given by

$$\mathbf{T}_{CD} = \begin{pmatrix} c_\theta c_\phi - s_\theta s_\alpha s_\phi & -s_\theta \beta - s_\alpha c_\theta s_\phi & c_\alpha (c_\theta s_\phi - s_\alpha s_\theta v_\phi) & 0 \\ s_\theta c_\phi + c_\theta s_\alpha s_\phi & c_\theta \beta - s_\alpha s_\theta s_\phi & c_\alpha (s_\theta s_\phi + s_\alpha c_\theta v_\phi) & 0 \\ -c_\alpha s_\phi & c_\alpha s_\alpha v_\phi & s_\alpha^2 + c_\alpha^2 c_\phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

where

$$c_\alpha \equiv \cos(\alpha), \quad s_\alpha \equiv \sin(\alpha), \quad v_\phi \equiv 1 - c_\phi, \quad \beta \equiv c_\alpha^2 + s_\alpha^2 c_\phi.$$

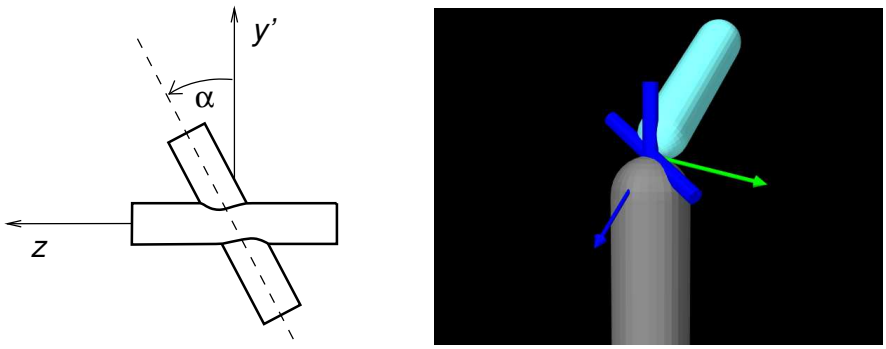



Figure 3.18: Left: diagram for a skewed universal joint, showing the pitch axis (dotted line) skewed by an angle $\alpha$ relative to its nominal direction along the $y'$ axis. Right: demo model with skew angle of 30°.

Rendering is controlled using the properties shaftLength, shaftRadius and jointRadius in the same way as for the `UniversalJoint`. A demo is provided by calling `artisynth.demos.mech.UniversalJointDemo` with the model arguments `-skew <angDeg>`, where `<angDeg>` is the desired skew angle in degrees.

Constructors for skewed universal joints take the standard forms described in Section 3.3.3, with an additional argument at the end indicating the skew angle:

```
SkewedUniveralJoint (bodyA, TCA, bodyB, TCB, skewAngle)

SkewedUniveralJoint (bodyA, bodyB, TDW, skewAngle)

SkewedUniveralJoint (bodyA, bodyB, TCW, TDW, skewAngle)
```

In addition, the constructor

```
SkewedUniveralJoint (bodyA, bodyB, originD, rollAxis, pitchAxis)
```

creates a skewed universal joint specifying the origin of frame D together with the directions of the roll and pitch axes (in world coordinates). Frames C and D are coincident and the skew angle is inferred from the angle between the axes.

### 3.4.7 Gimbal joint

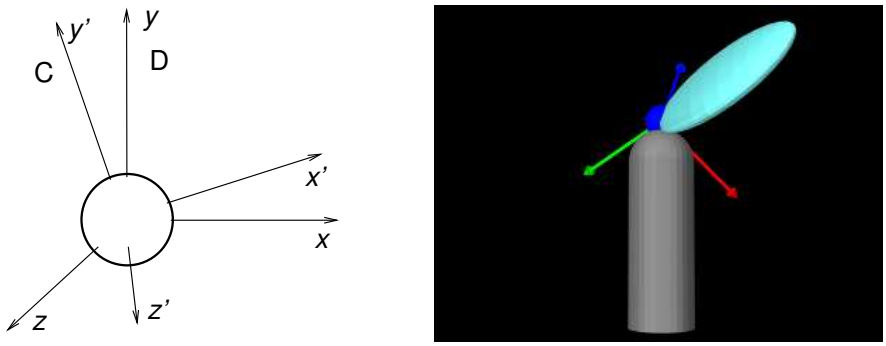



Figure 3.19: Coordinate frames (left; rotation angles not shown) and demo model (right) for the gimbal joint.

The GimbalJoint (Figure 3.19) is a 3 DOF spherical joint that anchors the origins of C and D together but otherwise allows C complete rotational freedom. The rotational degrees of freedom are parameterized by three roll-pitch-yaw angles, denoted by $\theta, \phi, \psi$, which define a rotation $\theta$ about D's $z$ axis, followed by a second rotation $\phi$ about the rotated $y'$ axis, followed by a third rotation $\psi$ about the final $x''$ axis. It implements six constraints and the three coordinates

$\theta, \phi, \psi$ (Table 3.6), to which the joint transform $\mathbf{T}_{CD}$ is related by

$$\mathbf{T}_{CD} = \begin{pmatrix} c_\theta c_\phi & c_\theta s_\phi s_\psi - s_\theta c_\psi & c_\theta s_\phi c_\psi + s_\theta s_\psi & 0 \\ s_\theta c_\phi & s_\theta s_\phi s_\psi + c_\theta c_\psi & s_\theta s_\phi c_\psi - c_\theta s_\psi & 0 \\ -s_\phi & c_\phi s_\psi & c_\phi c_\psi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

where

$$c_\theta \equiv \cos(\theta),\ s_\theta \equiv \sin(\theta),\ c_\phi \equiv \cos(\phi),\ s_\phi \equiv \sin(\phi),\ c_\psi \equiv \cos(\psi),\ s_\psi \equiv \sin(\psi).$$

The value and ranges for $\theta, \phi, \psi$ are exported by the properties roll, pitch, yaw, rollRange, pitchRange, and yawRange, and the coordinate indices are defined by the constants `ROLL_IDX`, `PITCH_IDX`, and `YAW_IDX`. For rendering, the property jointRadius specifies the radius of a ball drawn around the origin of D, using the faceColor rendering property. A demo is provided by `artisynth.demos.mech.GimbalJointDemo`.

In addition to the standard constructors described in Section 3.3.3,

```
GimbalJoint (bodyA, bodyB, originD)
```

creates a gimbal joint with a specified origin for frame D (in world coordinates), and frames C and D coincident and world aligned.

> The constraints implementing `GimbalJoint` are designed so that it is immune to *gimbal lock*, in which a degree of freedom is lost when $\phi = \pm\pi/2$. However, the coordinate values themselves are not immune to this singularity, and neither are the unilateral constraints which enforce limits on their values. Therefore, if coordinate limits are implemented, the joint should be deployed so as try and avoid pitch values near $\pm\pi/2$.

| Index | type/name | description |
|---|---|---|
| 0 | bilateral | restricts translation along $x$ |
| 1 | bilateral | restricts translation along $y$ |
| 2 | bilateral | restricts translation along $z$ |
| 3 | unilataral | enforces limits on the roll coordinate |
| 4 | unilataral | enforces limits on the pitch coordinate |
| 5 | unilataral | enforces limits on the yaw coordinate |
| 0 | $\theta$ (roll) | first rotation of $C$ about the $z$ axis of D |
| 1 | $\phi$ (pitch) | second rotation of $C$ about the rotated $y'$ axis |
| 2 | $\psi$ (yaw) | third rotation of $C$ about the final $x''$ axis |

Table 3.6: Constraints (top) and coordinates (bottom) for the gimbal joint.

### 3.4.8 Spherical joint

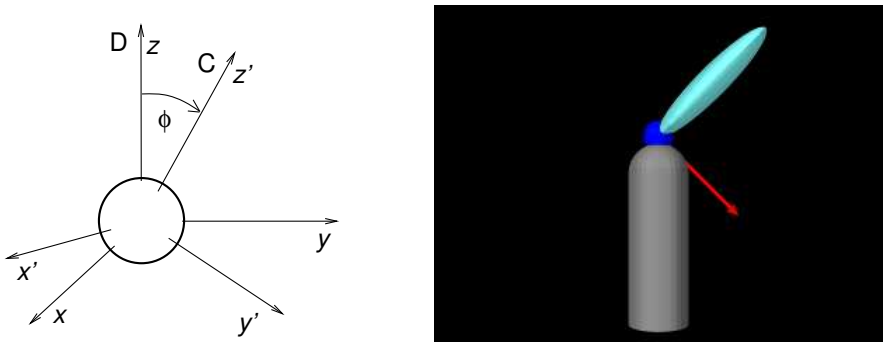



Figure 3.20: Left: coordinate frames of the spherical joint, showing the tilt angle $\phi$ between the $z$ axes of C and D. Right: demo model for the spherical joint.

The SphericalJoint (Figure 3.20) is a 3 DOF spherical joint that, like GimbalJoint, anchors the origins of C and D together but otherwise allows C complete rotational freedom. `SphericalJoint` does not implement any coordinates, and so is conceptually more like a *ball* joint. However, it does provide two choices for limiting its rotation:

- A limit on the *tilt* angle $\phi$ between the $z$ axes of D and C, such that

$$\phi \leq \phi_{\max}. \tag{3.26}$$

  This is intended to emulate the limit imposed by a ball joint socket.

- A limit on the total rotation, defined as follows: Let $(\mathbf{u}, \theta)$ be the axis-angle representation of the rotation matrix of $\mathbf{T}_{CD}$, normalized such that $\theta \geq 0$ and $\|u\| = 1$, and let $\mathbf{r}_{\max}$ be a three-vector giving maximum rotation angles with $x$, $y$, and $z$ components. Then $\theta$ is constrained by

$$\theta \leq \|\mathbf{r}_{\max} \circ \mathbf{u}\|, \tag{3.27}$$

  where $\circ$ denotes the element-wise product. If the components of $\mathbf{r}_{\max}$ are set to a uniform value $\theta_{\max}$, this simplifies to $\theta \leq \theta_{\max}$.

These limits can be enabled by setting the joint's properties isTiltLimited and isRotationLimited, respectively, where enabling one disables the other. The limit values $\phi_{\max}$ and $\mathbf{r}_{\max}$ are managed using the properties maxTilt and maxRotation, and setting either automatically enables tilt or rotation limiting, as appropriate. Finally, the tilt angle $\phi$ can be queried using the (read-only) tilt property. For rendering, the property jointRadius specifies the radius of a ball drawn around the origin of D, using the faceColor rendering property. A demo of the `SphericalJoint` is provided by `artisynth.demos.mech.SphericalJointDemo`.

In addition to the standard constructors described in Section 3.3.3,

```
SphericalJoint (bodyA, bodyB, originD)
```

creates a spherical joint with a specified origin for frame D (in world coordinates), and frames C and D coincident and world aligned.

> One should use the rotation limit with some caution, as the orientations which it prohibits can be somewhat hard to predict, particularly when $\mathbf{r}_{\max}$ has non-uniform values.

| Index | type/name | description |
|---|---|---|
| 0 | bilateral | restricts translation along $x$ |
| 1 | bilateral | restricts translation along $y$ |
| 2 | bilateral | restricts translation along $z$ |
| 3 | unilataral | enforces either the "tilt" or "rotation" limits |

Table 3.7: Constraints for the spherical joint.

### 3.4.9 Planar joint

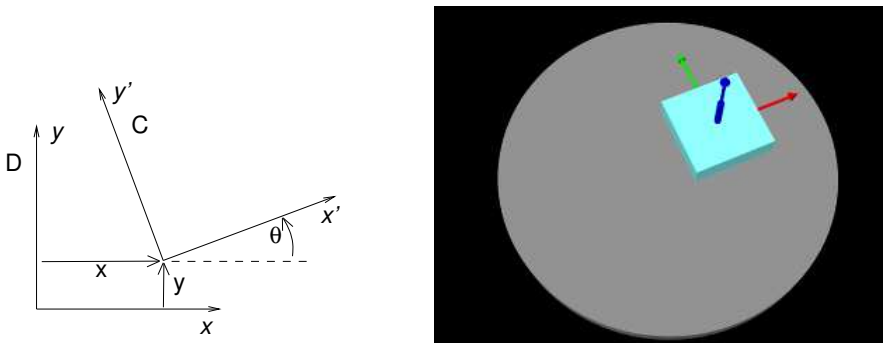



Figure 3.21: Coordinate frames (left) and demo model (right) for the planar joint.

The PlanarJoint (Figure 3.21) is a 3 DOF joint that constrains C to translation in the $x$-$y$ plane and rotation about the $z$ axis of D. It implements six constraints and three coordinates $x$, $y$ and $\theta$ (Table 3.8), to which the joint transform $\mathbf{T}_{CD}$ is

related by

$$\mathbf{T}_{CD} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 & x \\ \sin(\theta) & \cos(\theta) & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

The value and ranges for $x$, $y$ and $\theta$ are exported by the properties x, y, theta, xRange, yRange and thetaRange, and the coordinate indices are defined by the constants `X_IDX`, `Y_IDX` and `THETA_IDX`. A planar joint can be rendered as a square centered on the origin of D, using face rendering properties and with a size given by the planeSize property. For example,

```
PlanarJoint joint;
...
joint.setPlaneSize (5.0);
RenderProps.setFaceColor (joint, Color.LIGHT_GRAY);
```

will cause `joint` to be drawn as a light gray square with size 5.0. The default value of planeSize is 0, so drawing the plane is disabled by default. Also, the default faceStyle rendering property for `PlanarConnector` is set to `FRONT_AND_BACK`, so that the plane (when drawn) can be seen from both sides. A shaft about the rotation axis can also be drawn, as controlled by the properties shaftLength and shaftRadius and using the faceColor rendering property. A demo is provided by `artisynth.demos.mech.PlanarJointDemo`.

In addition to the standard constructors described in Section 3.3.3,

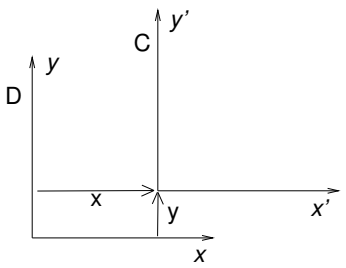
```
PlanarJoint (bodyA, bodyB, originD, zaxis)
```

creates a planar joint with a specified origin and $z$ axis direction for frame D (in world coordinates), and frames C and D coincident.

| Index | type/name | description |
|---|---|---|
| 0 | bilateral | restricts translation along $z$ |
| 1 | bilateral | restricts rotation about $x$ |
| 2 | bilateral | restricts rotation about $y$ |
| 3 | unilataral | enforces limits on the $x$ coordinate |
| 3 | unilataral | enforces limits on the $y$ coordinate |
| 5 | unilataral | enforces limits on the $\theta$ coordinate |
| 0 | $x$ | translation of $C$ along the $x$ axis of D |
| 1 | $y$ | translation of $C$ along the $y$ axis of D |
| 2 | $\theta$ | rotation of $C$ about the $z$ axis of D |

Table 3.8: Constraints (top) and coordinates (bottom) for the planar joint.

### 3.4.10 Planar translation joint



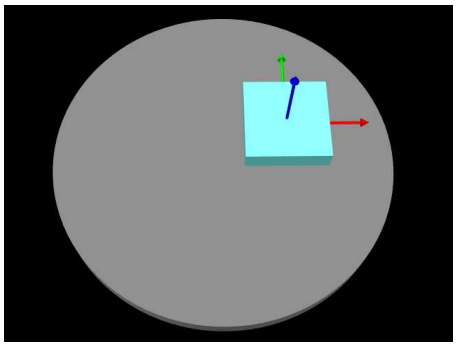



Figure 3.22: Coordinate frames (left) and demo model (right) for the planar translation joint.

| Index | type/name | description |
|---|---|---|
| 0 | bilateral | restricts translation along $z$ |
| 1 | bilateral | restricts rotation about $x$ |
| 2 | bilateral | restricts rotation about $y$ |
| 3 | bilateral | restricts rotation about $z$ |
| 4 | unilataral | enforces limits on the $x$ coordinate |
| 5 | unilataral | enforces limits on the $y$ coordinate |
| 0 | $x$ | translation of $C$ along the $x$ axis of D |
| 1 | $y$ | translation of $C$ along the $y$ axis of D |

Table 3.9: Constraints (top) and coordinates (bottom) for the planar translation joint.

The PlanarTranslationJoint (Figure 3.22) is a 2 DOF joint that is the same as the planar joint without rotation: C is restricted to translation in the $x$-$y$ plane of D. It implements six constraints and two coordinates $x$ and $y$ (Table 3.9), to which the joint transform $\mathbf{T}_{CD}$ is related by

$$\mathbf{T}_{CD} = \begin{pmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

The value and ranges for $x$ and $y$ are exported by the properties x, y, xRange and yRange, and the coordinate indices are defined by the constants `X_IDX` and `Y_IDX`. A planar translation joint can be rendered as a square centered on the origin of D, using face rendering properties and with a size given by the planeSize property, in the same way as described for `PlanarJoint`. A demo is provided by `artisynth.demos.mech.PlanarJointDemo`.

In addition to the standard constructors described in Section 3.3.3,

```
PlanarTranslationJoint (bodyA, bodyB, originD, zaxis)
```

creates a planar translation joint with a specified origin and $z$ axis direction for frame D (in world coordinates), and frames C and D coincident.

### 3.4.11 Ellipsoid joint

The EllipsoidJoint is a 4 DOF joint that provides similar functionality to the ellipsoidal and scapulothoracic joints available in OpenSim. It allows the origin of C to slide around on the surface of an ellipsoid centered on the origin of D, together with two additional rotational degrees of freedom.

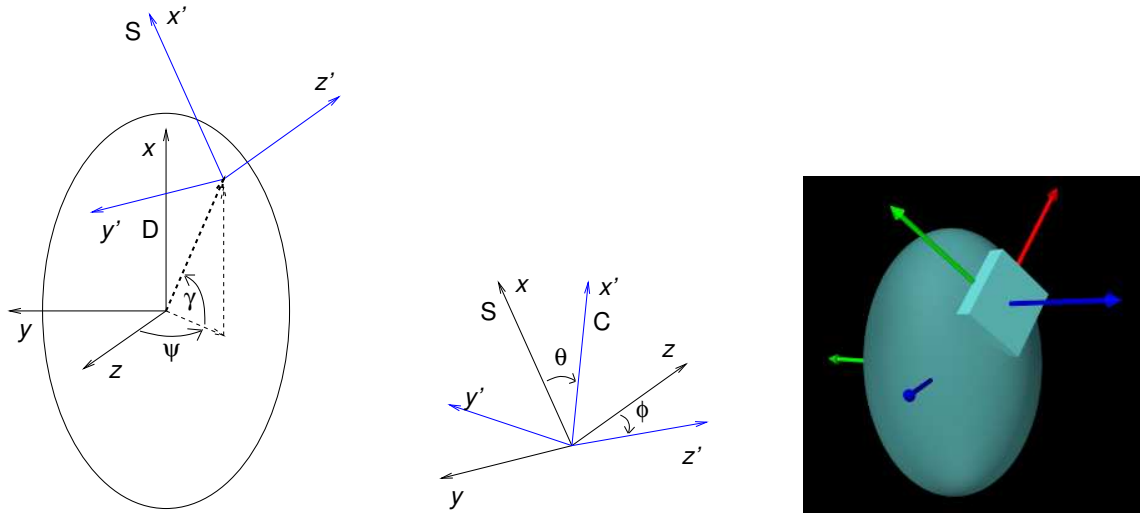



Figure 3.23: Ellipsoidal joint. Left: frame relationships between D and S (blue). Middle: frame relationships between S and C (blue). Right: the demo model `EllipsoidJointDemo`, with $\psi$, $\gamma$, $\theta$ and $\phi$ set to 45°, 30°, −40°, and 20°.

The joint kinematics is easiest to describe in terms of an intermediate frame S whose origin lies on the ellipsoid surface, with its position controlled by two coordinates: a *longitude* angle $\psi$, and a *latitude* angle $\gamma$ (Figure 3.23, left). Frame C has the same origin as S, with two additional coordinates, $\theta$ and $\phi$ which allow it to rotate about S (Figure 3.23, middle). If the transform $\mathbf{T}_{SD}$ from S to D and the (rotational-only) transform $\mathbf{T}_{CS}$ from C to S are given by

$$\mathbf{T}_{SD} = \begin{pmatrix} \mathbf{R}_{SD} & \mathbf{p}_{SD} \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{T}_{CS} = \begin{pmatrix} \mathbf{R}_{CS} & 0 \\ 0 & 1 \end{pmatrix},$$

then $\mathbf{T}_{CD}$ is given by

$$\mathbf{T}_{CD} = \mathbf{T}_{SD}\mathbf{T}_{CS} = \begin{pmatrix} \mathbf{R}_{SD}\mathbf{R}_{CS} & \mathbf{p}_{SD} \\ 0 & 1 \end{pmatrix}.$$

The six constraints and four coordinates of the ellipsoidal joint are described in table 3.10.

| Index | type/name | description |
|---|---|---|
| 0 | bilateral | restricts C to the ellipsoid surface and limits rotation |
| 1 | bilateral | restricts C to the ellipsoid surface and limits rotation |
| 2 | unilateral | enforces limits on the $\psi$ coordinate |
| 3 | unilateral | enforces limits on the $\gamma$ coordinate |
| 4 | unilataral | enforces limits on the $\theta$ coordinate |
| 5 | unilataral | enforces limits on the $\phi$ coordinate |
| 0 | $\psi$ | longitude angle for origin of S (and C) on the ellipsoid |
| 1 | $\gamma$ | latitude angle for origin of S (and C) on the ellipsoid |
| 2 | $\theta$ | first rotation of C about the $z$ axis of S |
| 3 | $\phi$ | second rotation of C about rotated $x$ (or $x'$ if $\alpha \neq 0$) |

Table 3.10: Constraints (top) and coordinates (bottom) for the ellipsoidal joint.

For frame S, if $A$, $B$ and $C$ are the ellipsoid semi-axis lengths for the $x$, $y$, and $z$ axes, and $c_\psi$, $s_\psi$, $c_\gamma$, and $s_\gamma$ are the cosines and sines of $\psi$ and $\gamma$, we can show that

$$\mathbf{p}_{SD} = \begin{pmatrix} A\,s_\gamma \\ -B\,s_\psi c_\gamma \\ C\,c_\psi c_\gamma \end{pmatrix}.$$

For the orientation of S, the $z$ axis of S is parallel to the surface normal and the $x$ axis is parallel to the tangent direction imparted by the latitudinal velocity $\dot{\gamma}$. That means $x$ and $z$ axes are parallel to the direction vectors $\mathbf{d}_x$ and $\mathbf{d}_z$ given by

$$\mathbf{d}_x = \begin{pmatrix} A\,c_\gamma \\ B\,s_\psi s_\gamma \\ -C\,c_\psi s_\gamma \end{pmatrix} \quad \text{and} \quad \mathbf{d}_z = \begin{pmatrix} p_x/A^2 \\ p_y/B^2 \\ p_z/C^2 \end{pmatrix}. \tag{3.28}$$

The columns of $\mathbf{R}_{SD}$ are then given by the normalized values of $\mathbf{d}_x$, $\mathbf{d}_z \times \mathbf{d}_x$, and $\mathbf{d}_z$, respectively.

The rotation $\mathbf{R}_{CS}$ is formed by a rotation $\theta$ about the $z$ axis, followed by a rotation of $\phi$ about the new $x$ axis. Letting $c_\theta$, $s_\theta$, $c_\phi$, and $s_\phi$ be the cosines and sines of $\theta$ and $\phi$, we then have

$$\mathbf{R}_{CS} = \begin{pmatrix} c_\theta & -s_\theta c_\phi & s_\theta s_\phi \\ s_\theta & c_\theta c_\phi & -c_\theta s_\phi \\ 0 & s_\phi & c_\phi \end{pmatrix}.$$

If desired, the $\phi$ rotation can instead be performed about a modified axis $x'$ that makes an angle $\alpha$ with respect to $x$ in the $x$-$y$ plane. $\alpha$ is controlled by the joint's alpha property (default value 0) and corresponds to the "winging" angle of the OpenSim scapulothoracic joint. If $\alpha \neq 0$, $\mathbf{R}_{CS}$ takes the more complex form

$$\mathbf{R}_{CS} = \begin{pmatrix} c_0 c_\alpha + s_0 c_\phi s_\alpha & c_0 s_\alpha - s_0 c_\phi c_\alpha & s_0 s_\phi \\ s_0 c_\alpha - c_0 c_\phi s_\alpha & s_0 s_\alpha + c_0 c_\phi c_\alpha & -c_0 s_\phi \\ -s_\phi s_\alpha & s_\phi c_\alpha & c_\phi \end{pmatrix}$$

where $c_0$, $s_0$, $c_\alpha$, and $s_\alpha$ are the cosines and sines of $\theta + \alpha$ and $\alpha$, respectively.

Within an `EllipsoidJoint`, the values and ranges for $\psi$, $\gamma$, $\theta$ and $\phi$ are exported by the properties longitude, latitude, theta, phi, longitudeRange, latitudeRange, thetaRange, and phiRange, and the coordinate indices are defined by the

constants `LONGITUDE_IDX`, `LATITUDE_IDX`, `THETA_IDX`, and `PHI_IDX`. For rendering, the property drawEllipsoid specifies whether the ellipsoid surface should be drawn; if `true`, it will be drawn using the joint's face rendering properties. A demo is provided by `artisynth.demos.mech.EllipsoidJointDemo`.

Ellipsoid joints can be created with the following constructors:

```
EllipsoidJoint (A, B, C, alpha, openSimCompatible)

EllipsoidJoint (rbodyA, TCA, rbodyB, TDB, A, B, C, alpha, openSimCompatible)

EllipsoidJoint (cbodyA, cbodyB, TCW, TDW, A, B, C)
```

The first of these creates a joint that is *not* attached to any bodies; attachment can be done later using one of the `setBodies()` methods. Its semi-axis lengths are given by `A`, `B`, and `C`, its $\alpha$ angle is given by `alpha`, and the argument `openSimCompatible`, if `true`, makes the joint kinematics compatible with OpenSim (Section 3.4.11.1). The second constructor creates a joint and then attaches it to rigid bodies `rbodyA` and `rbodyB`, with the specified $\mathbf{T}_{CA}$ and $\mathbf{T}_{DB}$ transformations. The third constructor creates a joint and attaches it to connectable bodies `cbodyA` and `cbodyB`, with the locations of the C and D frames specified in world coordinates by `TCW` and `TDW`.

> Unlike in many joints, $\mathbf{T}_{CD}$ is *not* the identity when the joint coordinates are all 0. That is because the origin of C must lie on the ellipsoid surface, and since D is at the center of the ellipsoid, $\mathbf{T}_{CD}$ can never be the identity. In particular, when all coordinate values are 0, $\mathbf{R}_{CD} = \mathbf{I}$ but $\mathbf{p}_{CD} = \mathbf{p}_{SD} = (0,0,C)^T$.

#### 3.4.11.1 OpenSim compatibility

The `openSimCompatible` argument in some of the joint's constructors makes the kinematics compatible with the ellipsoidal joint used by OpenSim. This means that $\mathbf{R}_{SD}$ is computed differently: in OpenSim, instead of using (3.28), the $z$ and $x$ axis directions of $\mathbf{R}_{SD}$ are computed using

$$\mathbf{d}_x = \begin{pmatrix} c_\gamma \\ s_\psi s_\gamma \\ -c_\psi s_\gamma \end{pmatrix} \quad \text{and} \quad \mathbf{d}_z = \begin{pmatrix} p_x/A \\ p_y/B \\ p_z/C \end{pmatrix}. \tag{3.29}$$

In particular, this means that the $z$ axis is only approximately parallel to the ellipsoid surface normal.

> In OpenSim, the axes of the C frame of both the ellipsoid and scapulothoracic joints are oriented differently that those of the ArtiSynth joint: they are rotated by $-\pi/2$ about $z$, so that the $x$ and $y$ axes correspond to the $-y$ and $x$ axes of the ArtiSynth joint.

### 3.4.12 Solid joint

The SolidJoint is a 0 DOF joint that rigidly constrains C to D. It implements six constraints and no coordinates (Table 3.11) and the resulting $\mathbf{T}_{CD}$ is the identity.

> There aren't normally many uses for solid joints. If one wishes to create a complex rigid body by joining together a variety of shapes, this can be done more efficiently by making these shapes mesh components of a single rigid body (Section 3.2.9).

### 3.4.13 Planar Connector

The PlanarConnector (Figure 3.24) is a 5 DOF connector that attaches the origin of C to the $x$-$y$ plane of D. C is completely free to rotate, and to translate within the $x$-$y$ plane. Only motion in the $z$ direction is restricted. `PlanarConnector` implements one constraint and has no coordinates (Table 3.12).

A `PlanarConnector` constrains a point on body A (located at the origin of C) to move within a plane on body B. Several planar connectors can be employed to constrain body motions in more complicated ways, although one must

| Index | type/name | description |
|---|---|---|
| 0 | bilateral | restricts translation along $x$ |
| 1 | bilateral | restricts translation along $y$ |
| 2 | bilateral | restricts translation along $z$ |
| 3 | bilateral | restricts rotation about $x$ |
| 4 | bilateral | restricts rotation about $y$ |
| 5 | bilateral | restricts rotation about $z$ |

Table 3.11: Constraints for the solid joint.

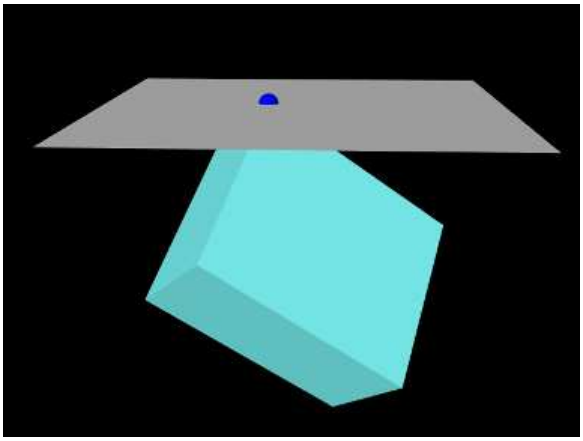


Figure 3.24: Demo model for the planar connector, in which a corner point of a box is constrained to the $x$-$y$ plane of D.

be careful to avoid overconstraining the system. The connector can also be configured to function *unilaterally*, via its unilateral property, in which case the point is constrained to lie in the half-space defined by $z \geq 0$ with respect to D. Several unilateral `PlanarConnector`s can therefore be used to implement a cheap and approximate collision mechanism with fixed collision points.

> When set to function unilaterally, overconstraining the system is not an issue because of the way in which ArtiSynth solves unilateral constraints.

A planar connector can be rendered as a square centered on the origin of D, using face rendering properties and with a size given by the planeSize property. The point attached to A can also be rendered using point rendering properties. For example,

```
PlanarConnector connector;
...
connector.setPlaneSize (5.0);
RenderProps.setFaceColor (connector, Color.LIGHT_GRAY);
RenderProps.setSphericalPoints (connector, 0.1, Color.BLUE);
```

will cause `connector` to be drawn as a light gray square with size 5, and for the point on body A to be drawn as a blue sphere with radius 0.1. The default value of planeSize is 0, so drawing the plane is disabled by default. Also, the default faceStyle rendering property for `PlanarConnector` is set to `FRONT_AND_BACK`, so that the plane (when drawn) can be seen from both sides.

Constructors for the `PlanarConnector` include

```
PlanarConnector (bodyA, pCA, bodyB, TDB)

PlanarConnector (bodyA, pCA, TDW)

PlanarConnector (bodyA, bodyA, TDW)
```

where `pCA` gives the connection point of body A *with respect to frame A*, `TDB` gives the transform from frame D to frame B, and `TDW` gives the transform from frame D to world.

| Index | type/name | description |
|---|---|---|
| 0 | bilateral *or* unilateral | restricts translation along $z$ |

Table 3.12: Constraints for the planar connector.

### 3.4.14 Segmented Planar Connector

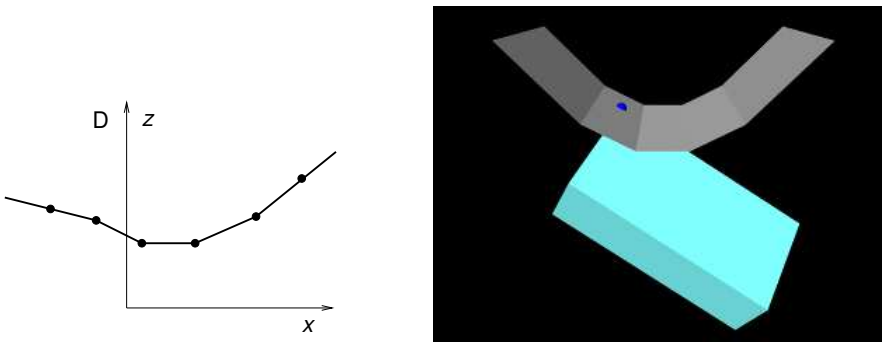



Figure 3.25: Left: cross-section in the $x$-$z$ plane of frame $D$ showing the segments of a segmented planar connector, with the points defining the segments shown as black dots. Right: demo model for the segmented planar connector.

| Index | type/name | description |
|---|---|---|
| 0 | bilateral *or* unilateral | restricts translation normal to the surface |

Table 3.13: Constraints for the segmented planar connector.

The SegmentedPlanarConnector (Figure 3.25) is a 5 DOF connector that generalizes `PlanarConnector` to a piecewise linear surface, to which the origin of $C$ is constrained while $C$ is otherwise completely free to rotate. The surface is specified by a sequence of 2D points defining a piecewise linear curve in the $x$-$z$ plane of D (Figure 3.25, left). This curve does not need to be a function; the segment nearest to C is the one used to enforce the constraint at any given time. The surface has infinite extent and is extrapolated beyond the first and last segments. It implements one constraint and has no coordinates (Table 3.13).

By appropriate choice of segments, a `SegmentedPlanarConnector` can approximate any surface defined by a curve in the $x$-$z$ plane. As with `PlanarConnector`, it can also be configured as unilateral, constraining the origin of $C$ to lie on the side of the surface defined by the normal vectors $\mathbf{n}_k$ of each segment $k$. If $\mathbf{p}_{k-1}$ and $\mathbf{p}_k$ are the points in the $x$-$z$ plane defining the $k$-th segment, and $\hat{\mathbf{y}}$ is the $y$ axis unit vector, then $\mathbf{n}_k$ is given by

$$\mathbf{n}_k = \frac{\mathbf{u} \times \hat{\mathbf{y}}}{\|\mathbf{u} \times \hat{\mathbf{y}}\|}, \quad \mathbf{u} \equiv \mathbf{p}_k - \mathbf{p}_{k-1}. \tag{3.30}$$

The properties controlling the rendering of a segmented planar connector are the same as for a planar connector, with each of the individual plane segments drawn as a rectangle whose length along the $y$ axis is controlled by planeSize.

Constructors for a `SegmentedPlanarConnector` are analogous to those used for `PlanarConnector`,

```
SegmentedPlanarConnector (bodyA, pCA, bodyB, TDB, segs)

SegmentedPlanarConnector (bodyA, pCA, TDW, segs)

SegmentedPlanarConnector (bodyA, bodyA, TDW, segs)
```

where `segs` is an additional argument of type `double[]` giving the 2D coordinates defining the segments in the $x$-$z$ plane.

### 3.4.15 Legacy Joints

ArtiSynth maintains three legacy joint for compatibility with earlier software:

- RevoluteJoint is identical to the HingeJoint, except that its coordinate $\theta$ is oriented *clockwise* about the $z$ axis instead of *counter-clockwise*. Rendering is also done differently, with shafts about the rotation axis drawn using line rendering properties.
- RollPitchJoint is identical to the UniversalJoint, except that its roll-pitch coordinates $\theta, \phi$ are computed with respect to the rotation $\mathbf{R}_{DC}$ from frame D to C, instead of the rotation $\mathbf{R}_{CD}$ from frame C to D. Rendering is also done differently, with shafts along the roll and pitch axes drawn using line rendering properties, and the ball around the origin of D drawn using point rendering properties.
- SphericalRpyJoint is identical to the GimbalJoint, except that its roll-pitch-yaw coordinates $\theta, \phi, \psi$ are computed with respect to the rotation $\mathbf{R}_{DC}$ from frame D to C, instead of the rotation $\mathbf{R}_{CD}$ from frame C to D. Rendering is also done differently, with the ball around the origin of D drawn using point rendering properties.

## 3.5 Frame springs

Another way to connect two rigid bodies together is to use a *frame spring*, which is a six dimensional spring that generates restoring forces and moments between coordinate frames.

### 3.5.1 Frame spring coordinate frames

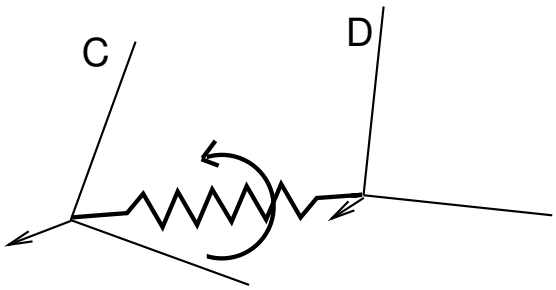



Figure 3.26: A frame spring connecting two coordinate frames D and C.

The basic idea of a frame spring is shown in Figure 3.26. It generates restoring forces and moments on two frames C and D which are a function of $\mathbf{T}_{DC}$ and $\hat{\mathbf{v}}_{DC}$ (the spatial velocity of frame D with respect to frame C).

Decomposing forces into stiffness and damping terms, the force $\mathbf{f}_C$ and moment $\tau_C$ acting on C can be expressed as

$$\begin{aligned} \mathbf{f}_C &= \mathbf{f}_k(\mathbf{T}_{DC}) + \mathbf{f}_d(\hat{\mathbf{v}}_{DC}) \\ \tau_C &= \tau_k(\mathbf{T}_{DC}) + \tau_d(\hat{\mathbf{v}}_{DC}). \end{aligned} \tag{3.31}$$

where the translational and rotational forces $\mathbf{f}_k$, $\mathbf{f}_d$, $\tau_k$, and $\tau_d$ are general functions of $\mathbf{T}_{DC}$ and $\hat{\mathbf{v}}_{DC}$.

The forces acting on D are equal and opposite, so that

$$\begin{aligned} \mathbf{f}_D &= -\mathbf{f}_C, \\ \tau_D &= -\tau_C. \end{aligned} \tag{3.32}$$

If frames C and D are attached to a pair of rigid bodies A and B, then a frame spring can be used to connect them in a manner analogous to a joint. As with joints, C and D generally do not coincide with the body frames, and are instead offset from them by fixed transforms $\mathbf{T}_{CA}$ and $\mathbf{T}_{DB}$ (Figure 3.27).

### 3.5.2 Frame materials

The restoring forces (3.31) generated in a frame spring depend on the *frame material* associated with the spring. Frame materials are defined in the package `artisynth.core.materials`, and are subclassed from FrameMaterial. The most

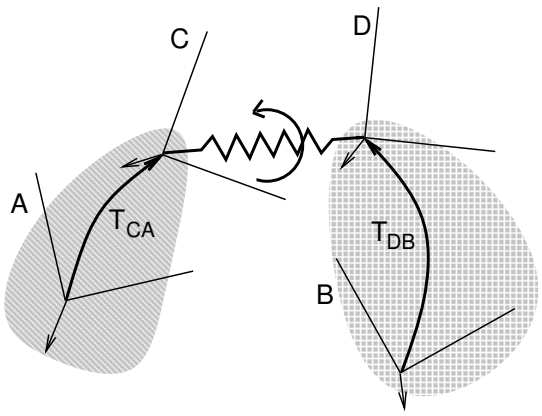



Figure 3.27: A frame spring connecting two rigid bodies A and B.

basic type of material is a LinearFrameMaterial, in which the restoring forces are determined from

$$\mathbf{f}_C = \mathbf{K}_t\,\mathbf{x}_{DC} + \mathbf{D}_t\,\mathbf{v}_{DC}$$
$$\tau_C = \mathbf{K}_r\,\hat{\theta}_{DC} + \mathbf{D}_r\,\omega_{DC}$$

where $\hat{\theta}_{DC}$ gives the small angle approximation of the rotational components of $\mathbf{X}_{DC}$ with respect to the $x$, $y$, and $z$ axes, and

$$\mathbf{K}_t \equiv \begin{pmatrix} k_{tx} & 0 & 0 \\ 0 & k_{ty} & 0 \\ 0 & 0 & k_{tz} \end{pmatrix}, \mathbf{D}_t \equiv \begin{pmatrix} d_{tx} & 0 & 0 \\ 0 & d_{ty} & 0 \\ 0 & 0 & d_{tz} \end{pmatrix},$$
$$\mathbf{K}_r \equiv \begin{pmatrix} k_{rx} & 0 & 0 \\ 0 & k_{ry} & 0 \\ 0 & 0 & k_{rz} \end{pmatrix}, \mathbf{D}_r \equiv \begin{pmatrix} d_{rx} & 0 & 0 \\ 0 & d_{ry} & 0 \\ 0 & 0 & d_{rz} \end{pmatrix}.$$

are the stiffness and damping matrices. The diagonal values defining each matrix are stored in the 3-dimensional vectors $\mathbf{k}_t$, $\mathbf{k}_r$, $\mathbf{d}_t$, and $\mathbf{d}_r$ which are exposed as the `stiffness`, `rotaryStiffness`, `damping`, and `rotaryDamping` properties of the material. Each of these specifies stiffness or damping values along or about a particular axis. Specifying different values for different axes will result in anisotropic behavior.

Other frame materials offering nonlinear behavior may be defined in `artisynth.core.materials`.

### 3.5.3 Creating frame springs

Frame springs are implemented by the class FrameSpring. Creating a frame spring generally involves instantiating this class, and then setting the material, the bodies A and B, and the transforms $\mathbf{T}_{CA}$ and $\mathbf{T}_{DB}$.

A typical construction sequence might look like this:

```
FrameSpring spring = new FrameSpring ("springA");
spring.setMaterial (new LinearFrameMaterial (kt, kr, dt, dr));
spring.setFrames (bodyA, bodyB, TDW);
```

The material is set using setMaterial(). The example above uses a `LinearFrameMaterial`, created with a constructor that sets $\mathbf{k}_t$, $\mathbf{k}_r$, $\mathbf{d}_t$, and $\mathbf{d}_r$ to uniform Isotropic values specified by `kt`, `kr`, `dt`, and `dr`.

The bodies and transforms can be set in the same manner as for joints (Section 3.3.3), with the methods setFrames(bodyA,bodyB,TDW) and setFrames(bodyA,TCA,bodyB,TDB) assuming the role of the `setBodies()` methods used for joints. The former takes D specified in world coordinates and computes $\mathbf{T}_{CA}$ and $\mathbf{T}_{DB}$ assuming that there is no initial spring displacement (i.e., that $\mathbf{T}_{DC} = \mathbf{I}$), while the latter allows $\mathbf{T}_{CA}$ and $\mathbf{T}_{DB}$ to be specified explicitly with $\mathbf{T}_{DC}$ assuming whatever value is implied.

Frame springs and joints are often placed together, using the same transforms $\mathbf{T}_{CA}$ and $\mathbf{T}_{DB}$, with the spring providing restoring forces to help keep the joint within prescribed bounds.

As with joints, a frame spring can be connected to only a single body, by specifying `frameB` as `null`. Frame B is then taken to be the world coordinate frame W.

### 3.5.4 Example: two bodies connected by a frame spring

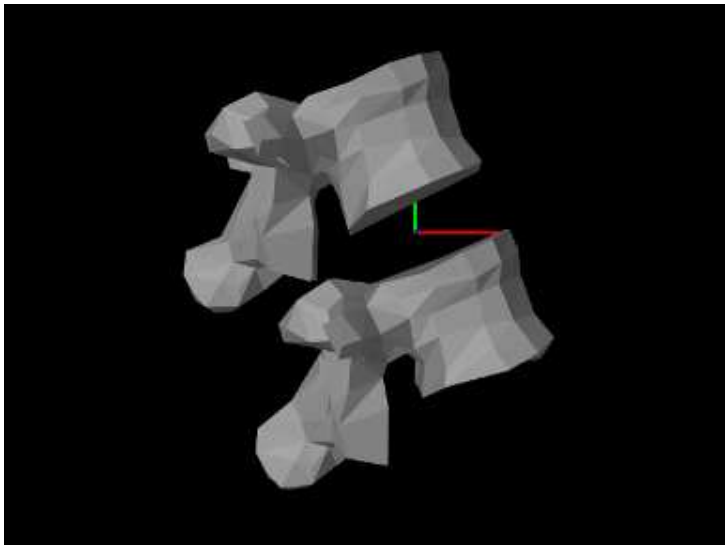


Figure 3.28: LumbarFrameSpring model loaded into ArtiSynth.

A simple model showing two simplified lumbar vertebrae, modeled as rigid bodies and connected by a frame spring, is defined in

```
artisynth.demos.tutorial.LumbarFrameSpring
```

The definition for the entire model class is shown here:

```
1  package artisynth.demos.tutorial;
2
3  import java.io.IOException;
4  import java.io.File;
5  import java.awt.Color;
6  import artisynth.core.modelbase.*;
7  import artisynth.core.mechmodels.*;
8  import artisynth.core.materials.*;
9  import artisynth.core.workspace.RootModel;
10 import maspack.matrix.*;
11 import maspack.geometry.*;
12 import maspack.render.*;
13 import maspack.util.PathFinder;
14
15 /**
16  * Demo of two rigid bodies connected by a 6 DOF frame spring
17  */
18 public class LumbarFrameSpring extends RootModel {
19
20    double density = 1500;
21
22    // path from which meshes will be read
23    private String geometryDir = PathFinder.getSourceRelativePath (
24       LumbarFrameSpring.class, "../mech/geometry/");
25
26    // create and add a rigid body from a mesh
27    public RigidBody addBone (MechModel mech, String name) throws IOException {
28       PolygonalMesh mesh = new PolygonalMesh (new File (geometryDir+name+".obj"));
29       RigidBody rb = RigidBody.createFromMesh (name, mesh, density, /*scale=*/1);
30       mech.addRigidBody (rb);
31       return rb;
32    }
```

```
33
34    public void build (String[] args) throws IOException {
35
36       // create mech model and set it's properties
37       MechModel mech = new MechModel ("mech");
38       mech.setGravity (0, 0, -1.0);
39       mech.setFrameDamping (0.10);
40       mech.setRotaryDamping (0.001);
41       addModel (mech);
42
43       // create two rigid bodies and second one to be fixed
44       RigidBody lumbar1 = addBone (mech, "lumbar1");
45       RigidBody lumbar2 = addBone (mech, "lumbar2");
46       lumbar1.setPose (new RigidTransform3d (-0.016, 0.039, 0));
47       lumbar2.setDynamic (false);
48
49       // flip entire mech model around
50       mech.transformGeometry (
51          new RigidTransform3d (0, 0, 0, 0, 0, Math.toRadians (90)));
52
53       //create and add the frame spring
54       FrameSpring spring = new FrameSpring (null);
55       spring.setMaterial (
56          new LinearFrameMaterial (
57             /*ktrans=*/100, /*krot=*/0.01, /*dtrans=*/0, /*drot=*/0));
58       spring.setFrames (lumbar1, lumbar2, lumbar1.getPose());
59       mech.addFrameSpring (spring);
60
61       // set render properties for components
62       RenderProps.setLineColor (spring, Color.RED);
63       RenderProps.setLineWidth (spring, 3);
64       spring.setAxisLength (0.02);
65       RenderProps.setFaceColor (mech, new Color (238, 232, 170)); // bone color
66    }
67 }
```

For convenience, the code to create and add each vertebrae is wrapped into the method `addBone()` defined at lines 27-32. This method takes two arguments: the `MechModel` to which the bone should be added, and the name of the bone. Surface meshes for the bones are located in `.obj` files located in the directory `../mech/geometry` relative to the source directory for the model itself. PathFinder.getSourceRelativePath() is used to find a proper path to this directory (see Section 2.6) given the model class type (`LumbarFrameSpring.class`), and this is stored in the static string `geometryDir`. Within `addBone()`, the directory path and the bone name are used to create a path to the bone mesh itself, which is in turn used to create a `PolygonalMesh` (line 28). The mesh is then used in conjunction with a `density` to create a rigid body which is added to the `MechModel` (lines 29-30) and returned.

The `build()` method begins by creating and adding a `MechModel`, specifying a low value for gravity, and setting the rigid body damping properties `frameDamping` and `rotaryDamping` (lines 37-41). (The damping parameters are needed here because the frame spring itself is created with no damping.) Rigid bodies representing the vertebrae `lumbar1` and `lumbar2` are then created by calling `addBone()` (lines 44-45), `lumbar1` is translated by setting the origin of its pose to $(-0.016, 0.039, 0)^T$, and `lumbar2` is set to be fixed by making it non-dynamic (line 47).

At this point in the construction, if the model were to be loaded, it would appear as in Figure 3.29. To change the viewpoint to that seen in Figure 3.28, we rotate the entire model about the $x$ axis (line 50). This is done using transformGeometry(X), which transforms the geometry of an entire model using a rigid or affine transform. This method is described in more detail in Section 4.7.

The frame spring is created and added at lines 54-59, using the methods described in Section 3.5.3, with frame D set to the (initial) pose of `lumbar1`.

Render properties are set starting at line 62. By default, a frame spring renders as a pair of red, green, blue coordinate axes showing frames C and D, along with a line connecting them. The line width and the color of the connecting line are controlled by the line render properties `lineWidth` and `lineColor`, while the length of the coordinate axes is controlled by the special frame spring property `axisLength`.

To run this example in ArtiSynth, select All demos > tutorial > LumbarFrameSpring from the Models menu. The model should load and initially appear as in Figure 3.28. Running the model (Section 1.5.3) will cause `lumbar1` to fall slightly

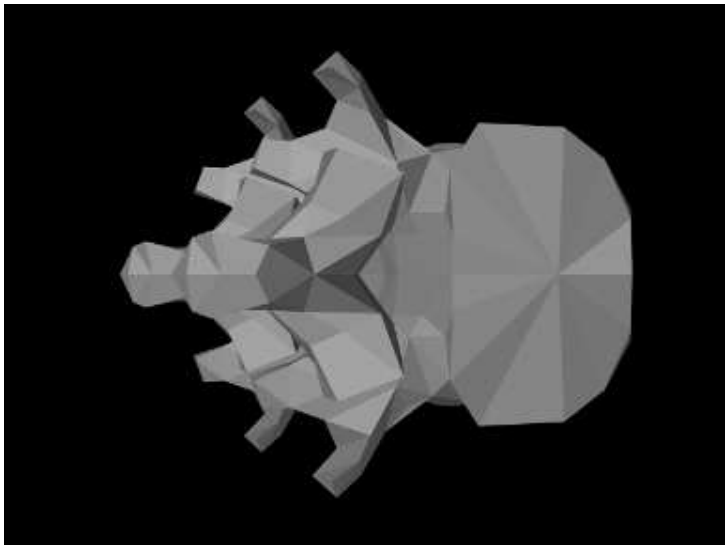


Figure 3.29: LumbarFrameSpring model as it would appear if not rotated about the $x$ axis.

under gravity until the frame spring arrests the motion. To get a sense of the spring's behavior, one can interactively apply forces to `lumbar1` using the pull tool (see the section "Pull Manipulation" in the ArtiSynth User Interface Guide).

## 3.6 Other point-based forces

All ArtiSynth components which are capable of exerting forces, including the axial springs of Section 3.1 and the frame springs of Section 3.5, are subclasses of ForceComponent. Other force effector types include PointPlaneForce and PointMeshForce, described in this section.

### 3.6.1 Forces between points and planes or meshes

A PointPlaneForce component produces forces between a point and a plane, based on the point's signed distance $d$ from the plane. Similarly, a PointMeshForce component produces forces between one or more points and a closed polygonal mesh, based on the signed distance $d$ to the mesh. These components thus implement "soft" constraint forces that bind points to either a plane or a mesh. If the component's unilateral property is set to `true`, as described below, forces are applied only when $d < 0$. This allows the simulation of force-based contact between points and planes or meshes.

> `PointPlaneForce` and `PointMeshForce` can be used for particles, FEM nodes and point-based markers, all of which are subclasses of `Point`.

These force components result in a force $\mathbf{f}$ acting on the point given by

$$\mathbf{f} = f(d,\dot{d})\,\mathbf{n}, \qquad f(d,\dot{d}) = -\text{sgn}(d) f_K(|d|) - D\dot{d},$$

where $d$ is the signed distance to the plane (or mesh), $\mathbf{n}$ is the plane normal (or surface normal at the nearest mesh point), $f_K(d)$ is a stiffness force and $D$ is a damping constant. The stiffness force may be either linear or quadratic, according to

$$f_K(d) = \begin{cases} Kd & \text{(linear)} \\ Kd^2 & \text{(quadratic)} \end{cases}$$

where $K$ is a stiffness constant and the choice of linear or quadratic is controlled by the component's forceType property.

If the component's unilateral property is set to `true`, then the computed force will be 0 whenever $d > 0$:

$$\mathbf{f} = \begin{cases} f(d,\dot{d})d & d \le 0 \\ 0 & d > 0 \end{cases}$$

This allows plane and mesh force objects to implement "soft" collisions.

> In the unilateral case, a quadratic force function provides smoother force transitions at $d = 0$.

`PointPlaneForce` may be created with the following constructors:

| | |
|---|---|
| PointPlaneForce (Point p, Plane plane) | Plane force component for point `p` |
| PointPlaneForce (Point p, Vector3d nrml, Point3d center) | Plane is specified by its normal and center. |

Each of these creates a force component for a single point `p`, with the plane specified by either a `Plane` object or by its normal and center.

`PointMeshForce` may be created with the following constructors:

| | |
|---|---|
| PointMeshForce (MeshComponent mcomp) | Mesh force for the mesh component `mcomp`. |
| PointMeshForce (String name, MeshComponent mcomp) | Named mesh force for mesh `mcomp`. |
| PointMeshForce (String name, MeshComponent mcomp, double K, double D) | Named mesh force with specified stiffness and damping. |

These create `PointMeshForce` for a mesh contained within a MeshComponent (Section 3.8). The mesh must a polygonal mesh composed of triangles.

The points associated with a `PointMeshForce` are added to after creation. The following methods are used to manage the point set:

| | |
|---|---|
| void addPoint (Point p) | Adds point `p`. |
| boolean removePoint (Point p) | Removes point `p`. |
| int numPoints() | Returns the number of points. |
| Point getPoint (int idx) | Returns the idx-th point. |
| void clearAllPoints() | Removes all points. |

`PointPlaneForce` and `PointMeshForce` export a variety of properties that control their force behavior and appearance:

- stiffness: A double value giving the stiffness constant $K$. Default value 1.0.
- damping: A double value giving the damping constant $D$. Default value 0.0.
- forceType: A value of type PointPlaneForce.ForceType or PointMeshForce.ForceType which describes the stiffness force, with the options being `LINEAR` and `QUADRATIC`. Default value `LINEAR`.
- unilateral: A boolean value, which if `true` means that no force will be generated for $d > 0$. Default value `false` for `PointPlaneForce` and `true` for `PointMeshForce`.
- enabled: A boolean value, which if `false` disables the component so that it will generate no force. Default value `true`.
- planeSize: For `PointPlaneForce` only, a double value that gives the size of the plane for rendering purposes only. Default value 1.0.

These properties can be set either interactively in the GUI, or in code using their accessor methods.

### 3.6.2 Example: point plane forces

An example using `PointPlaneForce` is given by

```
artisynth.demos.tutorial.PointPlaneForces
```

which implements soft collision between a particle and two planes. The `build()` method is shown below:

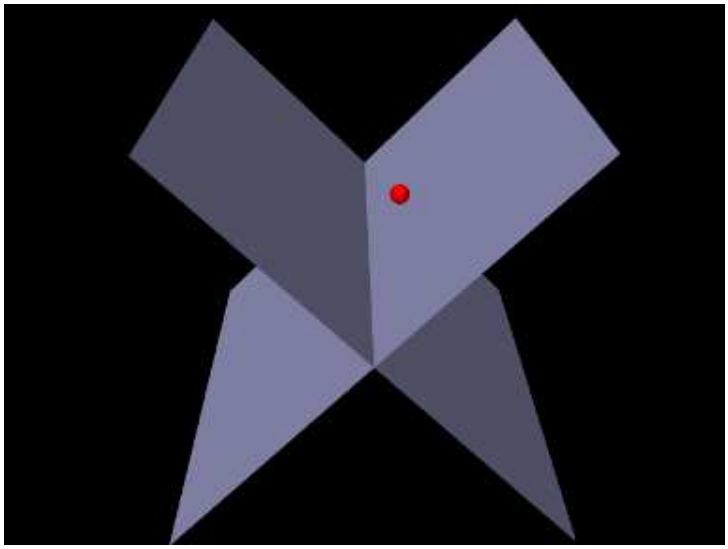


Figure 3.30: PointPlaneForces model running in ArtiSynth.

```
1     public void build (String[] args) {
2        MechModel mech = new MechModel ("mech");
3        addModel (mech);
4
5        // create the particle that will collide with the planes
6        Particle p = new Particle ("part", 1, 1.0, 0, 2.0);
7        mech.addParticle (p);
8
9        // create the PointPlaneForce for the left plane
10       double stiffness = 1000;
11       PointPlaneForce ppfL =
12          new PointPlaneForce (p, new Vector3d (1, 0, 1), Point3d.ZERO);
13       ppfL.setStiffness (stiffness);
14       ppfL.setPlaneSize (5.0);
15       ppfL.setUnilateral (true);
16       mech.addForceEffector (ppfL);
17
18       // create the PointPlaneForce for the right plane
19       PointPlaneForce ppfR =
20          new PointPlaneForce (p, new Vector3d (-1, 0, 1), Point3d.ZERO);
21       ppfR.setStiffness (stiffness);
22       ppfR.setPlaneSize (5.0);
23       ppfR.setUnilateral (true);
24       mech.addForceEffector (ppfR);
25
26       // render properties: make the particle red, and the planes blue-gray
27       RenderProps.setSphericalPoints (mech, 0.1, Color.RED);
28       RenderProps.setFaceColor (mech, new Color (0.7f, 0.7f, 0.9f));
29    }
30 }
```

A `MechModel` is created in the usual way (lines 2-3), followed by a particle (lines 6-7). Then two `PointPlaneForces` are created to act on this particle (lines 10-24), each centered on the origin, with plane normals of $(1,0,1)$ and $(-1,0,1)$, respectively. The forces are unilateral and linear (the default), with a stiffness of 1000. Each plane's size is set to 5; this is for rendering purposes only, as the planes have infinite extent for purposes of force calculation. Lastly, rendering properties are set (lines 27-28) to make the particle appear as a red sphere and the planes blue-gray.

To run this example in ArtiSynth, select All demos > tutorial > PointPlaneForces from the Models menu. When run, the particle will drop and bounce off the planes (Figure 3.30), finally coming to rest along the line where they intersect.

This model does not include damping, and so damping effects are due entirely to the default implicit integrator

`ConstrainedBackwardEuler`. If the integrator is changed to an explicit one, using a directive such as

```
    mech.setIntegrator (Integrator.RungeKutta4);
```

then the ball will continue to bounce during the simulation.

### 3.6.3 Example: point mesh forces

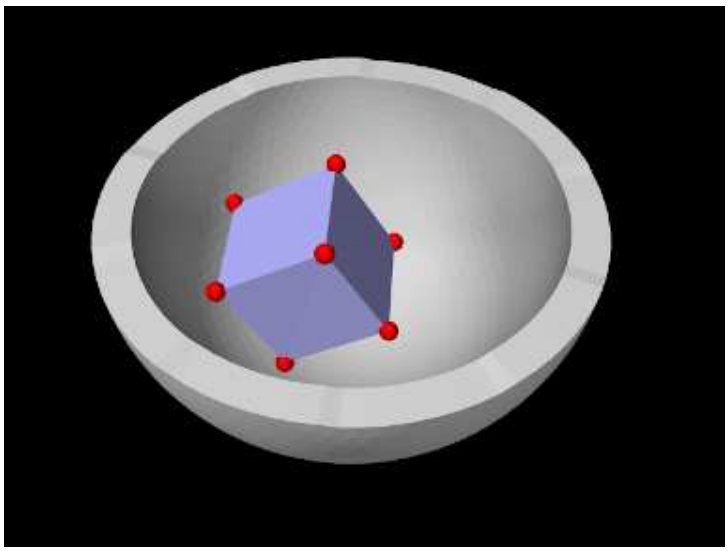


Figure 3.31: PointMeshForces model running in ArtiSynth.

An example using `PointMeshForce` is given by

```
artisynth.demos.tutorial.PointMeshForces
```

which implements soft collision between a rigid cube and a bowl-shaped mesh, using a `PointMeshForce` to create force between the mesh and markers at the cube corners. The `build()` method is shown below:

```
1     public void build (String[] args) throws IOException {
2        // create a MechModel and add it to the root model
3        MechModel mech = new MechModel ("mech");
4        mech.setInertialDamping (1.0);
5        addModel (mech);
6
7        // create the cube as a rigid body
8        RigidBody cube = RigidBody.createBox (
9           "cube", /*wx*/0.5, /*wy*/0.5, /*wz*/0.5, /*density*/1000);
10       mech.addRigidBody (cube);
11       // position the cube to drop into the bowl
12       cube.setPose (new RigidTransform3d (0.3, 0, 1));
13
14       // add a marker to each vertex of the cube
15       for (Vertex3d vtx : cube.getSurfaceMesh().getVertices()) {
16          // vertex positions will be in cube local coordinates
17          mech.addFrameMarker (cube, vtx.getPosition());
18       }
19       // create the bowl for the cube to drop into
20       String geoDir = PathFinder.getSourceRelativePath (this, "data/");
21       PolygonalMesh mesh = new PolygonalMesh (geoDir + "bowl.obj");
22       mesh.triangulate(); // mesh must be triangulated
23       FixedMeshBody bowl = new FixedMeshBody (mesh);
24       mech.addMeshBody (bowl);
25
```

```
26        // Create a PointMeshForce to produce collision interaction between the
27        // bowl and the cube. Use the markers as the mesh-colliding points.
28        PointMeshForce pmf =
29           new PointMeshForce ("pmf", bowl, /*stiffness*/2000000, /*damping*/1000);
30        for (FrameMarker m : mech.frameMarkers()) {
31           pmf.addPoint (m);
32        }
33        pmf.setForceType (ForceType.QUADRATIC);
34        mech.addForceEffector (pmf);
35
36        // render properties: make cube blue-gray, bowl light gray, and
37        // the markers red spheres
38        RenderProps.setFaceColor (cube, new Color (0.7f, 0.7f, 1f));
39        RenderProps.setFaceColor (bowl, new Color (0.8f, 0.8f, 0.8f));
40        RenderProps.setSphericalPoints (mech, 0.04, Color.RED);
41     }
42  }
```

A `MechModel` is created (lines 3-5), with inertialDamping set to 1.0 to help reduce oscillations as the cube bounces off the bowl. The cube itself is created as a `RigidBody` and positioned so as to fall into the bowl under gravity (lines 8-12). A frame marker is placed at each of the cube's corners, using the (local) positions of its surface mesh vertices to determine the marker locations (lines 13-18); these markers will act as the collision points.

The bowl is constructed as a FixedMeshBody containing a mesh read from the folder `"data/"` located beneath the source folder of the model (lines 8-12). A `PointMeshForce` is then allocated for the bowl (lines 28-34), with unilateral behavior (the default) and a quadratic stiffness force with $K = 200000$ and $D = 1000$. Each marker point is added to it to enforce the collision. Lastly, rendering properties are set (lines 38-40) to set the colors for the cube and bowl and make the markers appear as a red spheres.

To run this example in ArtiSynth, select All demos > tutorial > PointMeshForces from the Models menu. When run, the cube will drop into the bowl and bounce around (Figure 3.31).

> Because of the discrete nature of the simulation, force-based collision handling is subject to the same time step limitations as constraint-based collisions (Section 8.9). In particular, insufficiently small time steps, or too small a stiffness, may cause objects to pass through each other. Insufficient damping may also result in excessive bouncing.

## 3.7 Attachments

ArtiSynth provides the ability to rigidly attach dynamic components to other dynamic components, allowing different parts of a model to be connected together. Attachments are made by adding to a `MechModel` special *attachment* components that manage the attachment physics as described briefly in Section 1.2.

### 3.7.1 Point attachments

Point attachments allow particles and other point-based components to be attached to other, more complex components, such as frames, rigid bodies, or finite element models (Section 6.4). Point attachments are implemented by creating attachment components that are instances of PointAttachment. Modeling applications do not generally handle the attachment components directly, but instead create them implicitly using the following `MechModel` method:

```
attachPoint (Point p1, PointAttachable comp);
```

This attaches a point `p1` to any component which implements the interface PointAttachable, indicating that it is capable creating an attachment to a point. Components that implement `PointAttachable` currently include rigid bodies, particles, and finite element models. The attachment is created based on the the current position of the point and component in question. For attaching a point to a rigid body, another method may be used:

```
attachPoint (Point p1, RigidBody body, Point3d loc);
```

This attaches `p1` to `body` at the point `loc` specified in body coordinates. Finite element attachments are discussed in Section 6.4.

Once a point is attached, it will be in the *attached* state, as described in Section 3.1.3. Attachments can be removed by calling

```
detachPoint (Point p1);
```

### 3.7.2 Example: model with particle attachments

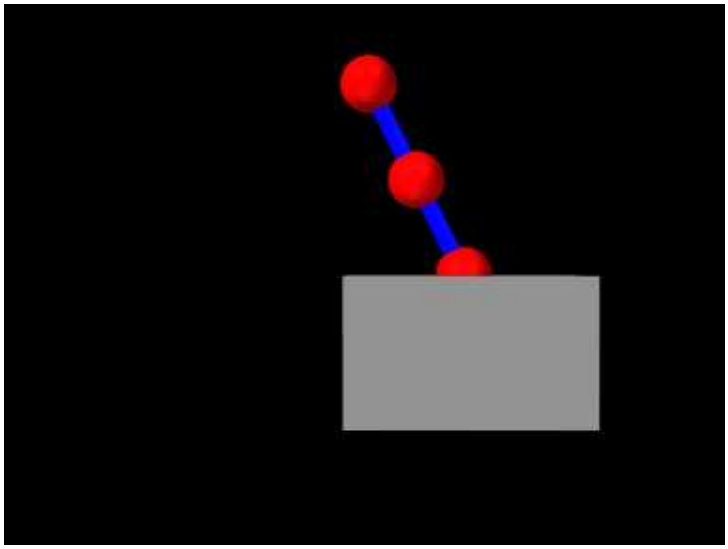


Figure 3.32: ParticleAttachment model loaded into ArtiSynth.

A model illustrating particle-particle and particle-rigid body attachments is defined in

```
artisynth.demos.tutorial.ParticleAttachment
```

and most of the code is shown here:

```
1   public Particle addParticle (MechModel mech, double x, double y, double z) {
2      // create a particle at x, y, z and add it to mech
3      Particle p = new Particle (/*name=*/null, /*mass=*/.1, x, y, z);
4      mech.addParticle (p);
5      return p;
6   }
7
8   public AxialSpring addSpring (MechModel mech, Particle p1, Particle p2){
9      // create a spring connecting p1 and p2 and add it to mech
10     AxialSpring spr = new AxialSpring (/*name=*/null, /*restLength=*/0);
11     spr.setMaterial (new LinearAxialMaterial (/*k=*/20, /*d=*/10));
12     spr.setPoints (p1, p2);
13     mech.addAxialSpring (spr);
14     return spr;
15  }
16
17  public void build (String[] args) {
18
19     // create MechModel and add to RootModel
20     MechModel mech = new MechModel ("mech");
21     addModel (mech);
22
23     // create the components
24     Particle p1 = addParticle (mech, 0, 0, 0.55);
```

```
25        Particle p2 = addParticle (mech, 0.1, 0, 0.35);
26        Particle p3 = addParticle (mech, 0.1, 0, 0.35);
27        Particle p4 = addParticle (mech, 0, 0, 0.15);
28        addSpring (mech, p1, p2);
29        addSpring (mech, p3, p4);
30        // create box and set its pose (position/orientation):
31        RigidBody box =
32           RigidBody.createBox ("box", /*wx,wy,wz=*/0.5, 0.3, 0.3, /*density=*/20);
33        box.setPose (new RigidTransform3d (/*x,y,z=*/0.2, 0, 0));
34        mech.addRigidBody (box);
35
36        p1.setDynamic (false);               // first particle set to be fixed
37
38        // set up the attachments
39        mech.attachPoint (p2, p3);
40        mech.attachPoint (p4, box, new Point3d (0, 0, 0.15));
41
42        // increase model bounding box for the viewer
43        mech.setBounds (/*min=*/-0.5, 0, -0.5, /*max=*/0.5, 0, 0);
44        // set render properties for the components
45        RenderProps.setSphericalPoints (mech, 0.06, Color.RED);
46        RenderProps.setCylindricalLines (mech, 0.02, Color.BLUE);
47     }
```

The code is very similar to `ParticleSpring` and `RigidBodySpring` described in Sections 3.1.2 and 3.2.2, except that two convenience methods, `addParticle()` and `addSpring()`, are defined at lines 1-15 to create particles and spring and add them to a `MechModel`. These are used in the `build()` method to create four particles and two springs (lines 24-29), along with a rigid body box (lines 31-34). As with the other examples, particle `p1` is set to be non-dynamic (line 36) in order to fix it in place and provide a ground.

The attachments are added at lines 39-40, with `p2` attached to `p3` and `p4` connected to the box at the location $(0, 0, 0.15)$ in box coordinates.

Finally, render properties are set starting at line 43. In this example, point and line render properties are set for the entire `MechModel` instead of individual components. Since render properties are inherited, this will implicitly set the specified render properties in all subcomponents for which these properties are not explicitly set (either locally or in an intermediate ancestor).

To run this example in ArtiSynth, select All demos > tutorial > ParticleAttachment from the Models menu. The model should load and initially appear as in Figure 3.32. Running the model (Section 1.5.3) will cause the box to fall and swing under gravity.

### 3.7.3 Frame attachments

Frame attachments allow rigid bodies and other frame-based components to be attached to other components, including frames, rigid bodies, or finite element models (Section 6.6). Frame attachments are implemented by creating attachment components that are instances of FrameAttachment.

As with point attachments, modeling applications do not generally handle frame attachment components directly, but instead create and add them implicitly using the following `MechModel` methods:

```
attachFrame (Frame frame, FrameAttachable comp);

attachFrame (Frame frame, FrameAttachable comp, RigidTransform3d TFW);
```

These attach `frame` to any component which implements the interface FrameAttachable, indicating that it is capable of creating an attachment to a frame. Components that implement `FrameAttachable` currently include frames, rigid bodies, and finite element models. For the first method, the attachment is created based on the the current position of the frame and component in question. For the second method, the attachment is created so that the initial pose of the frame (in world coordinates) is described by `TFW`.

Once a frame is attached, it will be in the *attached* state, as described in Section 3.1.3. Frame attachments can be removed by calling

```
   detachFrame (Frame frame);
```

> While it is possible to create composite rigid bodies using `FrameAttachments`, this is much less computationally efficient (and less accurate) than creating a single rigid body through mesh merging or similar techniques.

### 3.7.4 Example: model with frame attachments

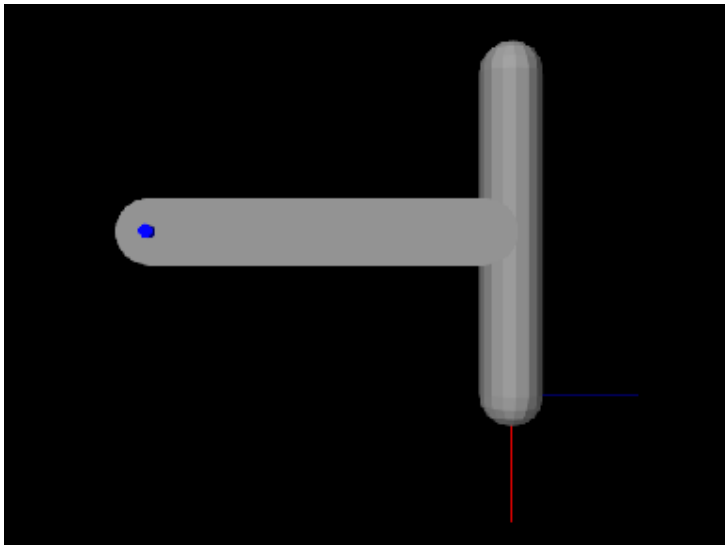


Figure 3.33: FrameBodyAttachment model loaded into ArtiSynth.

A model illustrating rigidBody-rigidBody and frame-rigidBody attachments is defined in

```
artisynth.demos.tutorial.FrameBodyAttachment
```

Most of the code is identical to that for `RigidBodyJoint` as described in Section 3.3.6, except that the joint is further to the left and connects `bodyB` to ground, rather than to `bodyA`, and the initial pose of `bodyA` is changed so that it is aligned vertically. `bodyA` is then connected to `bodyB`, and an auxiliary frame is created and attached to `bodyA`, using code at the end of the `build()` method as shown here:

```
1   public void build (String[] args) {
2
3      ... create model mostly similar to RigidBodyJoint ...
4
5      // now connect bodyA to bodyB using a FrameAttachment
6      mech.attachFrame (bodyA, bodyB);
7
8      // create an auxiliary frame and add it to the mech model
9      Frame frame = new Frame();
10     mech.addFrame (frame);
11
12     // set the frames axis length > 0 so we can see it
13     frame.setAxisLength (4.0);
14     // set the attached frame's pose to that of bodyA ...
15     RigidTransform3d TFW = new RigidTransform3d (bodyA.getPose());
16     // ... plus a translation of lenx2/2 along the x axis:
17     TFW.mulXyz (lenx2/2, 0, 0);
18     // finally, attach the frame to bodyA
19     mech.attachFrame (frame, bodyA, TFW);
20  }
```

To run this example in ArtiSynth, select All demos > tutorial > FrameBodyAttachment from the Models menu. The model should load and initially appear as in Figure 3.32. The frame attached to `bodyA` is visible in the lower right corner. Running the model (Section 1.5.3) will cause both bodies to fall and swing about the joint under gravity.

## 3.8 Mesh components

ArtiSynth models frequently incorporate 3D mesh geometry, as defined by the geometry classes PolygonalMesh, PolylineMesh, and PointMesh described in Section 2.5. Within a model, these basic classes are typically enclosed within container components that are subclasses of MeshComponent. Commonly used instances of these include

**RigidMeshComp**

Introduced in Section 3.2.9, these contain the mesh geometry for rigid bodies, and are stored in a rigid body subcomponent list named `meshes`. Their mesh vertex positions are fixed with respect to their body's coordinate frame.

**FemMeshComp**

Contain mesh geometry associated with finite element models (Chapter 6), including surfaces meshes and embedded mesh geometry, and are stored in a subcomponent list of the FEM model named `meshes`. Their mesh vertex positions change as the FEM model deforms.

**SkinMeshBody**

Described in detail in Chapter 11, these describe "skin" geometry that encloses both rigid bodies and/or FEM models. Their mesh vertex positions change as the underlying bodies move and deform. Skinning meshes may be placed anywhere, but are typically stored in the `meshBodies` component list of a `MechModel`.

**FixedMeshBody**

Described further below, these store arbitrary mesh geometry (polygonal, polyline, and point) and provide (like rigid bodies) a rigid coordinate frame that allows the mesh to be positioned and oriented arbitrarily. As their name suggests, their mesh vertex positions are fixed with respect to this coordinate frame. Fixed body meshes may be placed anywhere, but are typically stored in the `meshBodies` component list of a `MechModel`.

### 3.8.1 Fixed mesh bodies

As mentioned above, FixedMeshBody can be used for placing arbitrary mesh geometry within an ArtiSynth model. These mesh bodies are non-dynamic: they do not interact or collide with other model components, and they function primarily as 3D graphical objects. They can be created from primary mesh components using constructors such as:

| | |
|---|---|
| FixedMeshBody (MeshBase mesh) | Create an unnamed body containing a specified mesh. |
| FixedMeshBody (String name, MeshBase mesh) | Create a named body containing a specified mesh. |

It should be noted that the primary meshes are not copied and are instead stored by reference, and so any subsequent changes to them will be reflected in the mesh body. As with rigid bodies, fixed mesh bodies contain a coordinate frame, or *pose*, that describes the position and orientation of the body with respect to world coordinates. Methods to control the pose include:

| | |
|---|---|
| RigidTransform3d getPose()<br>void setPose (RigidTransform3d XFrameToWorld) | Returns the pose of the body (with respect to world).<br>Sets the pose of the body. |
| Point3d getPosition()<br>void setPosition (Point3d pos) | Returns the body position (pose translation component).<br>Sets the body position. |
| AxisAngle getOrientation()<br>void setOrientation (AxisAngle axisAng) | Returns the body orientation (pose rotation component).<br>Sets the body orientation. |

Once created, a canonical place for storing mesh bodies is the `MechModel` component list `meshBodies`. Methods for maintaining this list include:

| | |
|---|---|
| ComponentListView<MeshComponent> meshBodies() | Returns the `meshBodies` list. |
| void addMeshBody (MeshComponent mcomp) | Adds `mcomp` to `meshBodies`. |
| boolean removeMeshBody (MeshComponent mcomp) | Removes `mcomp` from `meshBodies`. |
| void clearMeshBodies() | Clears the `meshBodies` list. |

Meshes used for instantiating fixed mesh bodies are typically read from files (Section 2.5.5), but can also be created using factory methods (Section 2.5.1). As an example, the following code fragment creates a torus mesh using a factory method, set its pose, and then adds it to a `MechModel`:

```
MechModel mech;
...
PolygonalMesh mesh = MeshFactory.createTorus (
   /*rmajor=*/1.0, /*rminor=*/0.2, /*nmajor=*/32, /*nminor=*/10);
FixedMeshBody mbody = new FixedMeshBody ("torus", mesh);
mbody.setPose (
   new RigidTransform3d (/*xyz=*/0,0,0, /*rpy=*/0,0,Math.toRadians(90)));
mech.addMeshBody (mbody);
```

Rendering of mesh bodies is controlled using the same properties described in Section 3.2.8 for rigid bodies, including the renderProps subproperties faceColor, shading, alpha, faceStyle, and drawEdges, and the properties axisLength and axisDrawStyle for displaying a body's coordinate frame.

### 3.8.2 Example: adding mesh bodies to MechModel

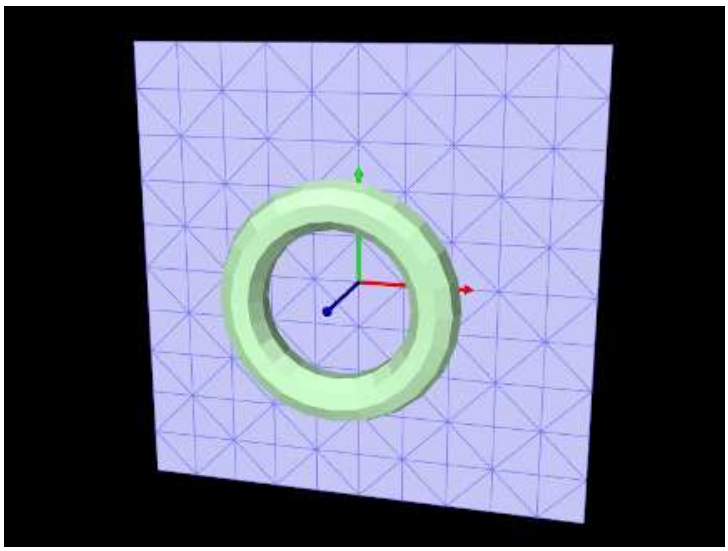


Figure 3.34: FixedMeshes model loaded into ArtiSynth.

An simple application that creates a pair of fixed meshes and adds them to a `MechModel` is defined in

```
artisynth.demos.tutorial.FixedMeshes
```

The `build()` method for this is shown below:

```
1     public void build (String[] args) {
2        // create a MechModel to add the mesh bodies to
3        MechModel mech = new MechModel ("mech");
4        addModel (mech);
5
6        // read torus mesh from a file stored in the folder "data" located under
7        // the source folder of this model:
8        String filepath = getSourceRelativePath ("data/torus_9_24.obj");
9        PolygonalMesh mesh = null;
```

```
10        try {
11           mesh = new PolygonalMesh (filepath);
12        }
13        catch (Exception e) {
14           System.out.println ("Error reading file " + filepath + ": " + e);
15           return;
16        }
17        // create a FixedMeshBody containing the mesh and add it to the MechModel
18        FixedMeshBody torus = new FixedMeshBody ("torus", mesh);
19        mech.addMeshBody (torus);
20
21        // create a square mesh with a factory method and add it to the MechModel
22        mesh = MeshFactory.createRectangle (
23           /*width=*/3.0, /*width=*/3.0, /*ndivsx=*/10, /*ndivsy=*/10,
24           /*addTextureCoords=*/false);
25        FixedMeshBody square = new FixedMeshBody ("square", mesh);
26        mech.addMeshBody (square);
27        // reposition the square: translate it along y and rotate it about x
28        square.setPose (
29           new RigidTransform3d (/*xyz=*/0,0.5,0,/*rpy=*/0,0,Math.toRadians(90)));
30
31        // set rendering properties:
32        // make torus pale green
33        RenderProps.setFaceColor (torus, new Color (0.8f, 1f, 0.8f));
34        // show square coordinate frame using solid arrows
35        square.setAxisLength (0.75);
36        square.setAxisDrawStyle (AxisDrawStyle.ARROW);
37        // make square blue gray with mesh edges visible
38        RenderProps.setFaceColor (square, new Color (0.8f, 0.8f, 1f));
39        RenderProps.setDrawEdges (square, true);
40        RenderProps.setEdgeColor (square, new Color (0.5f, 0.5f, 1f));
41        RenderProps.setFaceStyle (square, FaceStyle.FRONT_AND_BACK);
42     }
```

After creating a `MechModel` (lines 3-4), a torus shaped mesh is imported from the file `data/torus_9_24.obj` (lines 8-16). As described in Section 2.6, the `RootModel` method `getSourceRelativePath()` is used to locate this file relative to the model's source folder. The file path is used in the `PolygonalMesh` constructor, which is enclosed within a `try/catch` block to handle possible I/O exceptions. Once imported, the mesh is used to instantiate a `FixedMeshBody` named `"torus"` which is added to the `MechModel` (lines 18-19).

Another mesh, representing a square, is created with a factory method and used to instantiate a mesh body named `"square"` (lines 22-26). The factory method specifies both the size and triangle density of the mesh in the $x$-$y$ plane. Once created, the square's pose is set to a 90 degree rotation about the $x$ axis and a translation of 0.5 along $y$ (lines 28-29).

Rendering properties are set at lines 33-41. The torus is made pale green by setting it face color; the coordinate frame for the square is made visible as solid arrows using the `axisLength` and `axisDrawStyle` properties; and the square is made blue gray, with its edges made visible and drawn using a darker color, and its face style set to `FRONT_AND_BACK` so that it's visible from either side.

To run this example in ArtiSynth, select All demos > tutorial > FixedMeshes from the Models menu. The model should load and initially appear as in Figure 3.34.

# Chapter 4

# Mechanical Models II

This section provides additional material on building basic multibody-type mechanical models.

## 4.1 Simulation control properties

Both RootModel and MechModel contain properties that control the simulation behavior.

### 4.1.1 Simulation step size

One of the most important properties is `maxStepSize`. By default, simulation proceeds using the `maxStepSize` value defined for the root model. A `MechModel` (or any other type of `Model`) contained in the root model's `models` list may also request a smaller step size by specifying a smaller value for its own `maxStepSize` property. For all models, the `maxStepSize` may be set and queried using

```
void setMaxStepSize (double maxh);
double getMaxStepSize();
```

### 4.1.2 Integrator

Another important simulation property is `integrator` in `MechModel`, which determines the type of integrator used for the physics simulation. The value type of this property is the enumerated type `MechSystemSolver.Integrator`, for which the following values are currently defined:

**ForwardEuler**

First order forward Euler integrator. Unstable for stiff systems.

**SymplecticEuler**

First order symplectic Euler integrator, more energy conserving that forward Euler. Unstable for stiff systems.

**RungeKutta4**

Fourth order Runge-Kutta integrator, quite accurate but also unstable for stiff systems.

**ConstrainedBackwardEuler**

First order backward order integrator. Generally stable for stiff systems.

**Trapezoidal**

Second order trapezoidal integrator. Generally stable for stiff systems, but slightly less so than `ConstrainedBackwardEuler`.

The term "Unstable for stiff systems" means that the integrator is likely to go unstable in the presence of "stiff" systems, which typically include systems containing finite element models, unless the simulation step size is set to an extremely small value. The default value for `integrator` is `ConstrainedBackwardEuler`.

> Stiff systems tend to arise in models containing interconnected deformable elements, for which the step size should not exceed the propagation time across the smallest element, an effect known as the Courant-Friedrichs-Lewy (CFL) condition. Larger stiffness and damping values decrease the propagation time and hence the allowable step size.

### 4.1.3 Position stabilization

Another `MechModel` simulation property is `stabilization`, which controls the stabilization method used to correct drift from position constraints and correct interpenetrations due to collisions (Chapter 8). The value type of this property value is the enumerated type `MechSystemSolver.PosStabilization`, which presently has two values:

**GlobalMass**

Uses only a diagonal mass matrix for the MLCP that is solved to determine the position corrections. This is the default method.

**GlobalStiffness**

Uses a stiffness-corrected mass matrix for the MLCP that is solved to determine the position corrections. Slower than `GlobalMass`, but more likely to produce stable results, particularly for problems involving FEM collisions.

## 4.2 Units

ArtiSynth is primarily "unitless", in the sense that it does not define default units for the fundamental physical quantities of time, length, and mass. Although time is generally understood to be in seconds, and often declared as such in method arguments and return values, there is no hard requirement that it be interpreted as seconds. There are no assumptions at all regarding length and mass. Some components may have default parameter values that reflect a particular choice of units, such as `MechModel`'s default gravity value of $(0, 0, -9.8)^T$, which is associated with the MKS system, but these values can always be overridden by the application.

Nevertheless, it is important, and up to the application developer to ensure, that units be *consistent*. For example, if one decides to switch length units from meters to centimeters (a common choice), then all units involving length will have to be scaled appropriately. For example, density, whose fundamental units are $m/d^3$, where $m$ is mass and $d$ is distance, needs to be scaled by $1/100^3$, or 0.000001, when converting from meters to centimeters.

Table 4.1 lists a number of common physical quantities used in ArtiSynth, along with their associated fundamental units.

### 4.2.1 Scaling units

For convenience, many ArtiSynth components, including `MechModel`, implement the interface ScalableUnits, which provides the following methods for scaling mass and distance units:

```
scaleDistance (s);     // scale distance units by s
scaleMass (s);         // scale mass units by s
```

A call to one of these methods should cause all physical quantities within the component (and its descendants) to be scaled as required by the fundamental unit relationships as shown in Table 4.1.

Converting a `MechModel` from meters to centimeters can therefore be easily done by calling

```
mech.scaleDistance (100);
```

As an example, adding the following code to the end of the `build()` method in `RigidBodySpring` (Section 3.2.2)

| unit | fundamental units | |
|---|---|---|
| time | $t$ | |
| distance | $d$ | |
| mass | $m$ | |
| velocity | $d/t$ | |
| acceleration | $d/t^2$ | |
| force | $md/t^2$ | |
| work/energy | $md^2/t^2$ | |
| torque | $md^2/t^2$ | same as energy (somewhat counter intuitive) |
| angular velocity | $1/t$ | |
| angular acceleration | $1/t^2$ | |
| rotational inertia | $md^2$ | |
| pressure | $m/(dt^2)$ | |
| Young's modulus | $m/(dt^2)$ | |
| Poisson's ratio | 1 | no units; it is a ratio |
| density | $m/d^3$ | |
| linear stiffness | $m/t^2$ | |
| linear damping | $m/t$ | |
| rotary stiffness | $md^2/t^2$ | same as torque |
| rotary damping | $md^2/t$ | |
| mass damping | $1/t$ | used in FemModel |
| stiffness damping | $t$ | used in FemModel |

Table 4.1: Physical quantities and their representation in terms of the fundamental units of mass ($m$), distance ($d$), and time ($t$).

```
System.out.println ("length=" + spring.getLength());
System.out.println ("density=" + box.getDensity());
System.out.println ("gravity=" + mech.getGravity());
mech.scaleDistance (100);
System.out.println ("");
System.out.println ("scaled length=" + spring.getLength());
System.out.println ("scaled density=" + box.getDensity());
System.out.println ("scaled gravity=" + mech.getGravity());
```

will scale the distance units by 100 and print the values of various quantities before and after scaling. The resulting output is:

```
length=0.5
density=20.0
gravity=0.0 0.0 -9.8

scaled length=50.0
scaled density=2.0E-5
scaled gravity=0.0 0.0 -980.0
```

> It is important not to confuse scaling units with scaling the actual geometry or mass. Scaling units should change all physical quantities so that the simulated behavior of the model remains unchanged. If the distance-scaled version of `RigidBodySpring` shown above is run, it should behave exactly the same as the non-scaled version.

## 4.3 Render properties

All ArtiSynth components that are renderable maintain a property `renderProps`, which stores a RenderProps object that contains a number of subproperties used to control an object's rendered appearance.

In code, the `renderProps` property for an object can be set or queried using the methods

| property | purpose | usual default value |
|---|---|---|
| visible | whether or not the component is visible | `true` |
| alpha | transparency for diffuse colors (range 0 to 1) | 1 (opaque) |
| shading | shading style: (`FLAT`, `SMOOTH`, `METAL`, `NONE`) | `FLAT` |
| shininess | shininess parameter (range 0 to 128) | 32 |
| specular | specular color components | `null` |
| faceStyle | which polygonal faces are drawn (`FRONT`, `BACK`, `FRONT_AND_BACK`, `NONE`) | `FRONT` |
| faceColor | diffuse color for drawing faces | `GRAY` |
| backColor | diffuse color used for the backs of faces. If `null`, `faceColor` is used. | `null` |
| drawEdges | hint that polygon edges should be drawn explicitly | `false` |
| colorMap | color mapping properties (see Section 4.3.3) | `null` |
| normalMap | normal mapping properties (see Section 4.3.3) | `null` |
| bumpMap | bump mapping properties (see Section 4.3.3) | `null` |
| edgeColor | diffuse color for edges | `null` |
| edgeWidth | edge width in pixels | 1 |
| lineStyle: | how lines are drawn (`CYLINDER`, `LINE`, or `SPINDLE`) | `LINE` |
| lineColor | diffuse color for lines | `GRAY` |
| lineWidth | width in pixels when `LINE` style is selected | 1 |
| lineRadius | radius when `CYLINDER` or `SPINDLE` style is selected | 1 |
| pointStyle | how points are drawn (`SPHERE` or `POINT`) | `POINT` |
| pointColor | diffuse color for points | `GRAY` |
| pointSize | point size in pixels when `POINT` style is selected | 1 |
| pointRadius | sphere radius when `SPHERE` style is selected | 1 |

Table 4.2: Render properties and their default values.

```
setRenderProps (RenderProps props); // set render properties
RenderProps getRenderProps();       // get render properties (read-only)
```

Render properties can also be set in the GUI by selecting one or more components and the choosing Set render props ... in the right-click context menu. More details on setting render properties through the GUI can be found in the section "Render properties" in the ArtiSynth User Interface Guide.

For many components, the default value of `renderProps` is `null`; i.e., no `RenderProps` object is assigned by default and render properties are instead inherited from ancestor components further up the hierarchy. The reason for this is because `RenderProps` objects are fairly large (many kilobytes), and so assigning a unique one to every component could consume too much memory. Even when a `RenderProps` object is assigned, most of its properties are inherited by default, and so only those properties which are explicitly set will differ from those specified in ancestor components.

### 4.3.1 Render property taxonomy

In general, the properties in `RenderProps` are used to control the color, size, and style of the three primary rendering primitives: faces, lines, and points. Table 4.2 contains a complete list. Values for the `shading`, `faceStyle`, `lineStyle` and `pointStyle` properties are defined using the following enumerated types: Renderer.Shading, Renderer.FaceStyle, Renderer.PointStyle, and Renderer.LineStyle. Colors are specified using `java.awt.Color`.

To increase and improve their visibility, both the line and point primitives are associated with styles (`CYLINDER`, `SPINDLE`, and `SPHERE`) that allow them to be rendered using 3D surface geometry.

Exactly how a component interprets its render properties is up to the component (and more specifically, up to the rendering method for that component). Not all render properties are relevant to all components, particularly if the rendering does not use all of the rendering primitives. For example, Particle components use only the point primitives and AxialSpring components use only the line primitives. For this reason, some components use subclasses of `RenderProps`, such as PointRenderProps and LineRenderProps, that expose only a subset of the available render properties. All renderable components provide the method createRenderProps() that will create and return a `RenderProps` object suitable for that component.

### 4.3.2 Setting render properties

When setting render properties, it is important to note that the value returned by getRenderProps() should be treated as *read-only* and should *not* be used to set property values. For example, applications should *not* do the following:

```
particle.getRenderProps().setPointColor (Color.BLUE);
```

This can cause problems for two reasons. First, `getRenderProps()` will return `null` if the object does not currently have a `RenderProps` object. Second, because `RenderProps` objects are large, ArtiSynth may try to share them between components, and so by setting them for one component, the application my inadvertently set them for other components as well.

Instead, `RenderProps` provides a static method for each property that can be used to set that property's value for a specific component. For example, the correct way to set `pointColor` is

```
RenderProps.setPointColor (particle, Color.BLUE);
```

One can also set render properties by calling setRenderProps() with a predefined `RenderProps` object as an argument. This is useful for setting a large number of properties at once:

```
RenderProps props = new RenderProps();
props.setPointColor (Color.BLUE);
props.setPointRadius (2);
props.setPointStyle (RenderProps.PointStyle.SPHERE);

...

particle.setRenderProps (props);
```

For setting each of the color properties within `RenderProps`, one can use either `Color` objects or `float[]` arrays of length 3 giving the RGB values. Specifically, there are methods of the form

```
props.setXXXColor (Color color)
props.setXXXColor (float[] rgb)
```

as well as the static methods

```
RenderProps.setXXXColor (Renderable r, Color color)
RenderProps.setXXXColor (Renderable r, float[] rgb)
```

where `XXX` corresponds to `Point`, `Line`, `Face`, `Edge`, and `Back`. For `Edge` and `Back`, both `color` and `rgb` can be given as `null` to clear the indicated color. For the specular color, the associated methods are

```
props.setSpecular (Color color)
props.setSpecular (float[] rgb)
RenderProps.setSpecular (Renderable r, Color color)
RenderProps.setSpecular (Renderable r, float[] rgb)
```

> Note that even though components may use a subclass of `RenderProps` internally, one can always use the base `RenderProps` class to set values; properties which are not relevant to the component will simply be ignored.

Finally, as mentioned above, render properties are inherited. Values set high in the component hierarchy will be inherited by descendant components, unless those descendants (or intermediate components) explicitly set overriding values. For example, a `MechModel` maintains its own `RenderProps` (and which is never null). Setting its `pointColor` property to `RED` will cause *all* point-related components within that `MechModel` to be rendered as red *except* for components that set their `pointColor` to a different property.

There are typically three levels in a `MechModel` component hierarchy at which render properties can be set:

- The `MechModel` itself;
- Lists containing components;

- Individual components.

For example, consider the following code:

```
MechModel mech = new MechModel ("mech");

Particle p1 = new Particle (/*name=*/null, 2, 0, 0, 0);
Particle p2 = new Particle (/*name=*/null, 2, 1, 0, 0);
Particle p3 = new Particle (/*name=*/null, 2, 1, 1, 0);

mech.addParticle (p1);
mech.addParticle (p2);
mech.addParticle (p3);

RenderProps.setPointColor (mech, Color.BLUE);
RenderProps.setPointColor (mech.particles(), Color.GREEN);
RenderProps.setPointColor (p3, Color.RED);
```

Setting the `MechModel` render property `pointColor` to `BLUE` will cause all point-related items to be rendered blue by default. Setting the `pointColor` render property for the particle list (returned by `mech.particles()`) will override this and cause all particles in the list to be rendered green by default. Lastly, setting `pointColor` for `p3` will cause it to be rendered as red.

### 4.3.3 Texture mapping

Render properties can also be set to apply texture mapping to objects containing polygonal meshes in which texture coordinates have been set. Supported is provided for color, normal and bump mapping, although normal and bump mapping are only available under the OpenGL 3 version of the ArtiSynth renderer.

Texture mapping is controlled through the colorMap, normalMap, and bumpMap properties of `RenderProps`. These are composite properties with a default value of `null`, but applications can set them to instances of ColorMapProps, NormalMapProps, and BumpMapProps, respectively, to provide the source images and parameters for the associated mapping. The two most important properties exported by all of these `MapProps` objects are:

**enabled**

A boolean indicating whether or not the mapping is enabled.

**fileName**

A string giving the file name of the supporting source image.

`NormalMapProps` and `BumpMapProps` also export scaling, which scales the x-y components of the normal map or the depth of the bump map. Other exported properties control mixing with underlying colors, and how texture coordinates are both filtered and managed when they fall outside the canonical range $[0, 1]$. Full details on texture mapping and its support by the ArtiSynth renderer are given in the "Rendering" section of the Maspack Reference Manual.

To set up a texture map, one creates an instance of the appropriate `MapProps` object and uses this to set either the colorMap, normalMap, or bumpMap property of `RenderProps`. For a specific renderable, the map properties can be set using the static methods

```
void RenderProps.setColorMap (Renderable r, ColorMapProps tprops);
void RenderProps.setNormalMap (Renderable r, NormalMapProps tprops);
void RenderProps.setBumpMap (Renderable r, BumpMapProps tprops);
```

When initializing the `PropMaps` object, it is often sufficient to just set enabled to `true` and `fileName` to the full path name of the source image. Normal and bump maps also often require adjustment of their scaling properties. The following static methods are available for setting the enabled and fileName subproperties within a renderable:

```
void RenderProps.setColorMapEnabled (Renderable r, boolean enabled);
void RenderProps.setColorMapFileName (Renderable r, String fileName);

void RenderProps.setNormalMapEnabled (Renderable r, boolean enabled);
```

```
void RenderProps.setNormalMapFileName (Renderable r, String fileName);

void RenderProps.setBumpMapEnabled (Renderable r, boolean enabled);
void RenderProps.setBumpMapFileName (Renderable r, String fileName);
```

Normal and bump mapping only work under the OpenGL 3 version of the ArtiSynth viewer, and also do not work if the shading property of `RenderProps` is set to `NONE` or `FLAT`.

Texture mapping properties can be set within ancestor nodes of the component hierarchy, to allow file names and other parameters to be propagated throughout the hierarchy. However, when this is done, it is still necessary to ensure that the corresponding mapping properties for the relevant descendants are non-`null`. That's because mapping properties themselves are not inherited; only their subproperties are. If a mapping property for any given object is `null`, the associated mapping will be disabled. A non-`null` mapping property for an object will be created automatically by calling one of the `setXXXEnabled()` methods listed above. So when setting up ancestor-controlled mapping, one may use a construction like this:

```
RenderProps.setColorMap (ancestor, tprops);
RenderProps.setColorMapEnabled (descendant0, true);
RenderProps.setColorMapEnabled (descendant1, true);
```

Then colorMap subproperties set within `ancestor` will be inherited by `descendant0` and `descendant1`.

As indicated above, texture mapping will only be applied to components containing rendered polygonal meshes for which appropriate texture coordinates have been set. Determining such texture coordinates that produce appropriate results for a given source image is often non-trivial; this so-called "u-v mapping problem" is difficult in general and is highly dependent on the mesh geometry. ArtiSynth users can handle the problem of assigning texture coordinates in several ways:

- Use meshes which already have appropriate texture coordinates defined for a given source image. This generally means that mesh is specified by a file that contains the required texture coordinates. The mesh should then be read from this file (Section 2.5.5) and then used in the construction of the relevant components. For example, the application can read in a mesh containing texture coordinates and then use it to create a `RigidBody` via the method RigidBody.createFromMesh().
- Use a simple mesh object with predefined texture coordinates. The class MeshFactory provides the methods

  ```
  PolygonalMesh createRectangle (width, height, xdivs, ydivs, addTextureCoords);

  PolygonalMesh createSphere (radius, nslices, nlevels, addTextureCoords)
  ```

  which create rectangular and spherical meshes, along with canonical canonical texture coordinates if `addTextureCoords` is `true`. Coordinates generated by `createSphere()` are defined so that $(0,0)$ and $(1,1)$ map to the spherical coordinates $(-\pi,\pi)$ (at the south pole) and $(\pi,0)$ (at the north pole). Source images can be relatively easy to find for objects with canonical coordinates.
- Compute custom texture coordinates and set them within the mesh using setTextureCoords().

An example where texture mapping is applied to spherical meshes to make them appear like tennis balls is defined in

`artisynth.demos.tutorial.SphericalTextureMapping`

and listing for this is given below:

```
1 package artisynth.demos.tutorial;
2
3 import java.awt.Color;
4
5 import maspack.geometry.*;
```

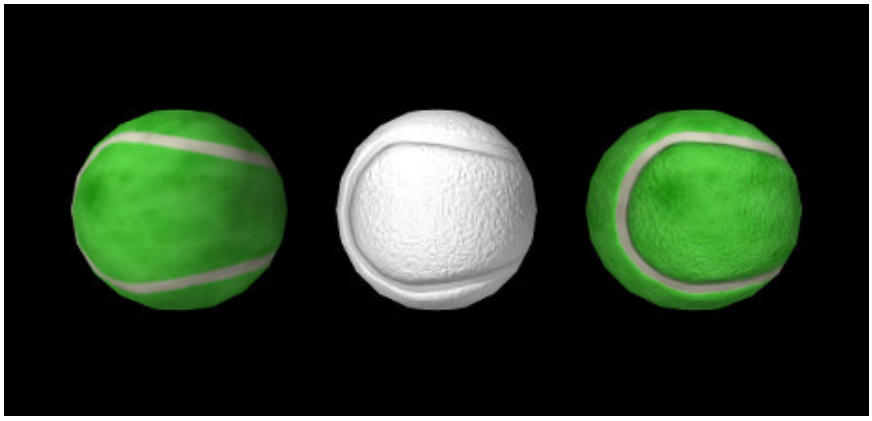


Figure 4.1: Color and bump mapping applied to spherical meshes. Left: color mapping only. Middle: bump mapping only. Right: combined color and bump mapping.

```
6  import maspack.matrix.RigidTransform3d;
7  import maspack.render.*;
8  import maspack.render.Renderer.ColorMixing;
9  import maspack.render.Renderer.Shading;
10 import maspack.util.PathFinder;
11 import maspack.spatialmotion.SpatialInertia;
12 import artisynth.core.mechmodels.*;
13 import artisynth.core.workspace.RootModel;
14
15 /**
16  * Simple demo showing color and bump mapping applied to spheres to make them
17  * look like tennis balls.
18  */
19 public class SphericalTextureMapping extends RootModel {
20
21    RigidBody createBall (
22       MechModel mech, String name, PolygonalMesh mesh, double xpos) {
23       double density = 500;
24       RigidBody ball =
25          RigidBody.createFromMesh (name, mesh.clone(), density, /*scale=*/1);
26       ball.setPose (new RigidTransform3d (/*x,y,z=*/xpos, 0, 0));
27       mech.addRigidBody (ball);
28       return ball;
29    }
30
31    public void build (String[] args) {
32
33       // create MechModel and add to RootModel
34       MechModel mech = new MechModel ("mech");
35       addModel (mech);
36
37       double radius = 0.0686;
38       // create the balls
39       PolygonalMesh mesh = MeshFactory.createSphere (
40          radius, 20, 10, /*texture=*/true);
41
42       RigidBody ball0 = createBall (mech, "ball0", mesh, -2.5*radius);
43       RigidBody ball1 = createBall (mech, "ball1", mesh, 0);
44       RigidBody ball2 = createBall (mech, "ball2", mesh, 2.5*radius);
45
46       // set up the basic render props: no shininess, smooth shading to enable
47       // bump mapping, and an underlying diffuse color of white to combine with
48       // the color map
49       RenderProps.setSpecular (mech, Color.BLACK);
50       RenderProps.setShading (mech, Shading.SMOOTH);
```

```
51        RenderProps.setFaceColor (mech, Color.WHITE);
52        // create and add the texture maps (provided free courtesy of
53        // www.robinwood.com).
54        String dataFolder = PathFinder.expand (
55           "${srcdir SphericalTextureMapping}/data");
56
57        ColorMapProps cprops = new ColorMapProps();
58        cprops.setEnabled (true);
59        // no specular coloring since ball should be matt
60        cprops.setSpecularColoring (false);
61        cprops.setFileName (dataFolder + "/TennisBallColorMap.jpg");
62
63        BumpMapProps bprops = new BumpMapProps();
64        bprops.setEnabled (true);
65        bprops.setScaling ((float)radius/10);
66        bprops.setFileName (dataFolder + "/TennisBallBumpMap.jpg");
67
68        // apply color map to balls 0 and 2. Can do this by setting color map
69        // properties in the MechModel, so that properties are controlled in one
70        // place - but we must then also explicitly enable color mapping in
71        // the surface mesh components for balls 0 and 2.
72        RenderProps.setColorMap (mech, cprops);
73        RenderProps.setColorMapEnabled (ball0.getSurfaceMeshComp(), true);
74        RenderProps.setColorMapEnabled (ball2.getSurfaceMeshComp(), true);
75
76        // apply bump map to balls 1 and 2. Again, we do this by setting
77        // the render properties for their surface mesh components
78        RenderProps.setBumpMap (ball1.getSurfaceMeshComp(), bprops);
79        RenderProps.setBumpMap (ball2.getSurfaceMeshComp(), bprops);
80     }
81  }
```

The `build()` method uses the internal method `createBall()` to generate three rigid bodies, each defined using a spherical mesh that has been created with MeshFactory.createSphere() with `addTextureCoords` set to `true`. The remainder of the `build()` method sets up the render properties and the texture mappings. Two texture mappings are defined: a color mapping and bump mapping, based on the images `TennisBallColorMap.jpg` and `TennisBallBumpMap.jpg` (Figure 4.2), both located in the subdirectory `data` relative to the demo source file. PathFinder.expand() is used to determine the full data folder name relative to the source directory. For the bump map, it is important to set the scaling property to adjust the depth amplitude to relative to the sphere radius.

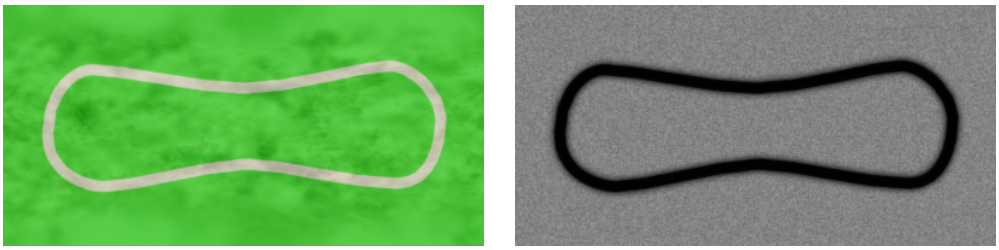


Figure 4.2: Color and bump map images used in the texture mapping example. These map to spherical coordinates on the mesh.

Color mapping is applied to balls 0 and 2, and bump mapping to balls 1 and 2. This is done by setting color map and/or bump map render properties in the components holding the actual meshes, which in this case is the mesh components for the balls' surfaces meshes, obtained using `getSurfaceMeshComp()`. As mentioned above, it is also possible to set these render properties in an ancestor component, and that is done here by setting the colorMap render property of the `MechModel`, but then it is also necessary to enable color mapping within the individual mesh components, using `RenderProps.setColorMapEnabled()`.

To run this example in ArtiSynth, select All demos > tutorial > SphericalTextureMapping from the Models menu. The model should load and initially appear as in Figure 4.1. Note that if ArtiSynth is run with the legacy OpenGL 2 viewer (command line option `-GLVersion 2`), bump mapping will not be supported and will not appear.

## 4.4 Custom rendering

It is often useful to add custom rendering to an application. For example, one may wish to render forces acting on bodies, or marker positions that are not otherwise assigned to a component. Custom rendering is relatively easy to do, as described in this section.

### 4.4.1 Component `render()` methods

All renderable ArtiSynth components implement the IsRenderable interface, which contains the methods prerender() and render(). As its name implies, `prerender()` is called prior to rendering and is discussed in Section 4.4.4. The `render()` method, discussed here, performs the actual rendering. It has the signature

```
void render (Renderer renderer, int flags)
```

where the Renderer supplies a large set of methods for performing the rendering, and `flags` is described in the API documentation. The methods supplied by the renderer include ones for drawing simple primitives such as points, lines, and triangles; simple 3D shapes such as cubes, cones, cylinders and spheres; and text components.

> A full description of the `Renderer` can be obtained by checking its API documentation and also by consulting the "Rendering" section of the Maspack Reference Manual.

A small sampling of the methods supplied by a `Renderer` include:

| Control render settings: | |
|---|---|
| void setColor(Color c) | set the current color. |
| void setPointSize(float size) | Set the point size, in pixels. |
| void setLineWidth(float width) | Set the line width, in pixels. |
| void setShading(Shading shading) | Set the shading model. |
| **Draw simple pixel-based primitives:** | |
| void drawPoint(Vector3d p) | Draw a point. |
| void drawLine(Vector3d p0, Vector3d p1) | Draw a line between points. |
| void drawLine(Vector3d p0, Vector3d p1, Vector3d p2) | Draw a triangle from three points. |
| **Draw solid 3D shapes:** | |
| void drawSphere(Vector3d p, double radius) | Draw a sphere centered at `p`. |
| void drawCylinder(Vector3d p0, Vector3d p1, double radius, boolean capped) | Draw a cylinder between `p0` and `p1`. |
| void drawArrow(Vector3d p, Vector3d dir, double scale, double radius, boolean capped) | Draw an arrow starting at point `p` and extending to point `p + scale*dir`. |
| void drawBox(Vector3d p, Vector3d widths) | Draw a box centered on `p`. |
| **Draw points and lines based on specified render properties:** | |
| void drawPoint(RenderProps props, Vector3d p, boolean highlight) | Draw a point. |
| void drawLine(RenderProps props, Vector3d p0, Vector3d p1, boolean highlight) | Draw a line between points. |

For example, the `render()` implementation below renders three spheres connected by pixel-based lines, as show in Figure 4.3 (left):

```
public void render (Renderer renderer, int flags) {
   // sphere centers
   Vector3d p0 = new Vector3d (-0.5, 0, 0.5);
   Vector3d p1 = new Vector3d (0.5, 0, 0.5);
   Vector3d p2 = new Vector3d (0, 0, -0.5);

   // draw the spheres using a golden-yellow color
   renderer.setColor (new Color (1f, 0.8f, 0f));
   double radius = 0.1;
```

```
      renderer.drawSphere (p0, radius);
      renderer.drawSphere (p1, radius);
      renderer.drawSphere (p2, radius);

      // connect spheres by pixel-based lines
      renderer.setColor (Color.RED);
      renderer.setLineWidth (3);
      renderer.drawLine (p0, p1);
      renderer.drawLine (p1, p2);
      renderer.drawLine (p2, p0);
   }
```

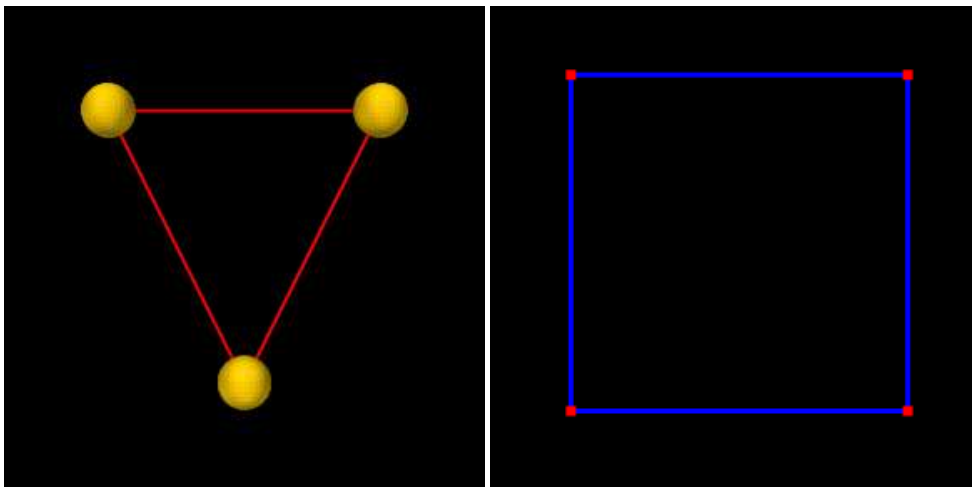


Figure 4.3: Rendered images produced with sample implementations of `render()`.

A `Renderer` also contains *draw mode* methods to implement the drawing of points, lines and triangles in a manner similar to the *immediate mode* of legacy OpenGL,

```
      renderer.beginDraw (drawModeType);

         ... define vertices and normals ...

      renderer.endDraw();
```

where `drawMode` is an instance of Renderer.DrawMode and includes points, lines, line strips and loops, triangles, and triangle strips and fans. The draw mode methods include:

| | |
|---|---|
| void beginDraw(DrawMode mode) | Begin a draw mode sequence. |
| void addVertex(Vector3d vtx) | Add a vertex to the object being drawn. |
| void setNormal(Vector3d nrm) | Sets the current vertex normal for the object being drawn. |
| void endDraw() | Finish a draw mode sequence. |

The `render()` implementation below uses draw mode to render the image shown in Figure 4.3, right:

```
   public void render (Renderer renderer, int flags) {
      // the corners of the square
      Vector3d p0 = new Vector3d (0, 0, 0);
      Vector3d p1 = new Vector3d (1, 0, 0);
      Vector3d p2 = new Vector3d (1, 0, 1);
      Vector3d p3 = new Vector3d (0, 0, 1);

      renderer.setShading (Shading.NONE);  // turn off lighting

      renderer.setPointSize (6);

      renderer.beginDraw (DrawMode.POINTS);
```

```
      renderer.setColor (Color.RED);
      renderer.addVertex (p0);
      renderer.addVertex (p1);
      renderer.addVertex (p2);
      renderer.addVertex (p3);
      renderer.endDraw();

      renderer.setLineWidth (3);
      renderer.setColor (Color.BLUE);
      renderer.beginDraw (DrawMode.LINE_LOOP);
      renderer.addVertex (p0);
      renderer.addVertex (p1);
      renderer.addVertex (p2);
      renderer.addVertex (p3);
      renderer.endDraw();

      renderer.setShading (Shading.FLAT);  // restore lighting  // sphere centers
   }
```

Finally, a `Renderer` contains methods for the rendering of *render objects*, which are collections of vertex, normal and color information that can be rendered quickly; it may be more efficient to use render objects for complex rendering involving large numbers of primitives. Full details on this, and other features of the rendering interface, are given in the "Rendering" section of the Maspack Reference Manual.

### 4.4.2 Implementing custom rendering

There are two easy ways to add custom rendering to a model:

1. Override the root model's `render()` method;
2. Define a custom rendering component, with its own `render()` method, and add it to the model. This approach is more modular and makes it easier to reuse the rendering between models.

To override the root model render method, one simply adds the following declaration to the model's definition:

```
      void render (Renderer renderer, int flags) {
         super.render (renderer, flags);

         ... custom rendering code goes here ...

      }
```

A call to `super.render()` is recommended to ensure that any rendering done by the model's base class will still occur. Subsequent statements should then use the `renderer` object to perform whatever rendering is required, using methods such as described in Section 4.4.1 or in the "Rendering" section of the Maspack Reference Manual.

To create a custom rendering component, one can simply declare a subclass of RenderableComponentBase with a custom `render()` method:

```
   import artisynth.core.modelbase.*;
   import maspack.render.*;

   class MyRenderComp extends RenderableComponentBase {

      void render (Renderer renderer, int flags) {

         ... custom rendering code goes here ...

      }
   }
```

There is no need to call `super.render()` since RenderableComponentBase does not provide an implementation of `render()`. It does, however, provide default implementations of everything else required by the Renderable interface. This includes exporting render properties via the composite property renderProps, which the `render()` method may use to control the rendering appearance in a manner consistent with Section 4.3.2. It also includes an implementation of `prerender()` which by default does nothing. As discussed more in Section 4.4.4, this is often acceptable *unless*:

1. rendering will be adversely affected by quantities being updated asynchronously within the simulation;
2. the component contains subcomponents that also need to be rendered.

Once a custom rendering component has been defined, it can be created and added to the model within the `build()` method:

```
MechModel mech;
...

MyRenderComp rcomp = new MyRenderComp();
mech.addRenderable (rcomp);
```

In this example, the component is added to the `MechModel`'s subcomponent list `renderables`, which ensures that it will be found by the rendering code. Methods for managing this list include:

| | |
|---|---|
| void addRenderable(Renderable r) | Adds a renderable to the model. |
| boolean removeRenderable(Renderable r) | Removes a renderable from the model. |
| void clearRenderables() | Removes all renderables from the model. |
| ComponentListView<RenderableComponent> renderables() | Returns the list of all renderables. |

### 4.4.3 Example: rendering body forces

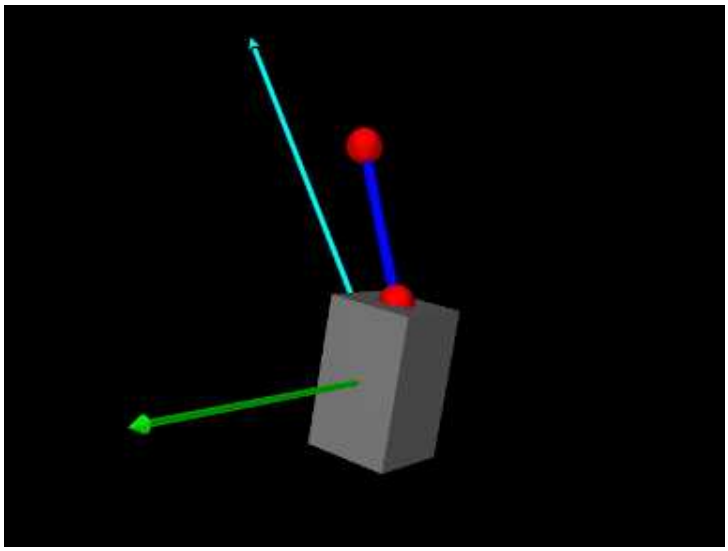


Figure 4.4: `BodyForceRendering` being run in ArtiSynth.

The application model

```
artisynth.demos.tutorial.BodyForceRendering
```

gives an example of custom rendering to draw the force and moment vectors acting on a rigid body as cyan and green arrows, respectively. The model extends `RigidBodySpring` (Section 3.2.2), defines a custom renderable class named `ForceRenderer`, and then uses this to render the forces on the box in the original model. The code, with the include files omitted, is listed below:

```
1  public class BodyForceRendering extends RigidBodySpring {
2
3     // Custom rendering component to draw forces acting on a rigid body
4     class ForceRenderer extends RenderableComponentBase {
5
6        RigidBody myBody;     // body whose forces are to be rendered
7        double myForceScale;  // scale factor for force vector
8        double myMomentScale; // scale factor for moment vector
9
10       ForceRenderer (RigidBody body, double forceScale, double momentScale) {
11          myBody = body;
12          myForceScale = forceScale;
13          myMomentScale = momentScale;
14       }
15
16       public void render (Renderer renderer, int flags) {
17          if (myForceScale > 0) {
18             // render force vector as a cyan colored arrow
19             Vector3d pnt = myBody.getPosition();
20             Vector3d dir = myBody.getForce().f;
21             renderer.setColor (Color.CYAN);
22             double radius = myRenderProps.getLineRadius();
23             renderer.drawArrow (pnt, dir, myForceScale, radius,/*capped=*/false);
24          }
25          if (myMomentScale > 0) {
26             // render moment vector as a green arrow
27             Vector3d pnt = myBody.getPosition();
28             Vector3d dir = myBody.getForce().m;
29             renderer.setColor (Color.GREEN);
30             double radius = myRenderProps.getLineRadius();
31             renderer.drawArrow (pnt, dir, myMomentScale, radius,/*capped=*/false);
32          }
33       }
34    }
35
36    public void build (String[] args) {
37       super.build (args);
38
39       // get the MechModel from the superclass
40       MechModel mech = (MechModel)findComponent ("models/mech");
41       RigidBody box = mech.rigidBodies().get("box");
42
43       // create and add the force renderer
44       ForceRenderer frender = new ForceRenderer (box, 0.1, 0.5);
45       mech.addRenderable (frender);
46
47       // set line radius property to control radius of the force arrows
48       RenderProps.setLineRadius (frender, 0.01);
49    }
50 }
```

The force renderer is defined at lines (4-34). For attributes, it contains a reference to the rigid body, plus scale factors for rendering the force and moment. Within the `render()` method (lines 16-33), the force and moment vectors are draw as cyan and green arrows, starting at the current body position, and scaled by their respective factors. This scaling is needed to ensure the arrows have a size appropriate to the viewer, and separate scale factors are needed because the force and moment vectors have different units. The arrow radii are given by the renderer's lineRadius render property (lines 22 and 30).

The `build()` method starts by calling the super class `build()` method to create the original model (line 36), and then uses `findComponent()` to retrieve its `MechModel`, within which the "`box`" rigid body is located (lines 40-41). A `ForceRenderer` is then created for this body, with force and moment scale factors of 0.1 and 0.5, and added to the `MechModel` (lines 44-45). The renderer's lineRadius render property is then set to 0.01 (line 48).

To run this example in ArtiSynth, select All demos > tutorial > BodyForceRenderer from the Models menu. When run,

the model should appear as in Figure 4.4, showing the force and moment vectors acting on the box.

### 4.4.4 The `prerender()` method

Component `render()` methods are called within ArtiSynth's graphics thread, and so are called asynchronously with respect to the simulation thread(s). This means that the simulation may be updating component attributes, such as positions or forces, at the same time they are being accessed within `render()`, leading to inconsistent results in the viewer. While these inconsistencies may not be significant, particularly if the attributes are changing slowly between time steps, some applications may wish to avoid them. For this, renderable components also implement a prerender() method, with the signature

```
void prerender (RenderList list);
```

that is called in synchronization with the simulation prior to rendering. Its purpose is to:

- Make copies of simulation-varying attributes so that they can be used without conflict in the `render()` method;
- Identify to the system any additional subcomponents that also need to be rendered.

The first task is usually accomplished by copying simulation-varying attributes into cache variables stored within the component itself. For example, if a component is responsible for rendering the forces of a rigid body (as per Section 4.4.3), it may wish to make a local copy of these forces within prerender:

```
RigidBody myBody;                      // body whose forces are being rendered
Wrench myRenderForce = new Wrench();   // copy of forces used for rendering

void prerender (RenderList list) {
   myRenderForce.set (myBody.getForce());
}

void render (Renderer renderer, int flags) {
   // do rendering with myRenderForce
   ...
}
```

The second task, identifying renderable subcomponents, is accomplished using the addIfVisible() and addIfVisibleAll() methods of the RenderList argument, as in the following examples:

```
// additional sub components that need to be rendered
RenderableComponent mySubCompA;
RenderableComponent mySubCompB;

void prerender (RenderList list) {
   list.addIfVisible (mySubCompA);
   list.addIfVisible (mySubCompB);
   ...

}
```

```
// list of child frame components that also need to be rendered
RenderableComponentList<Frame> myFrames;

void prerender (RenderList list) {
   list.addIfVisibleAll (myFrames);
   ...

}
```

It should be noted that some ArtiSynth components already support cached copies of attributes that can be used for rendering. In particular, Point (whose subclasses include `Particle` and `FemNode3d`) uses `prerender()` to cache its position as an array of `float[]` which can be obtained using getRenderCoords(), and Frame (whose subclasses include `RigidBody`) caches its pose as a `RigidTransform3d` that can be obtained with getRenderFrame().

## 4.5 Point-to-point muscles, tendons and ligaments

Point-to-point muscles are a simple type of component in biomechanical models that provide muscle-activated forces acting along a line between two points. ArtiSynth provides this through Muscle, which is a subclass of AxialSpring that generates an active muscle force in response to its `excitation` property. The excitation property can be set and queried using the methods

```
setExcitation (double excitation)
double getExcitation()
```

As with `AxialSpring`s, `Muscle` components use subclasses of AxialMaterial to compute the applied force $f(l,\dot{l},a)$ in response to the muscle's length $l$, length velocity $\dot{l}$, and excitation signal $a$, which is assumed to lie in the interval $a \in [0,1]$. Special *muscle* subclasses of `AxialMaterial` exist that compute forces that vary in response to the excitation. As with other axial materials, this is done by the material's `computeF()` method, first described in Section 3.1.4:

```
double computeF (l, ldot, l0, excitation)
```

Usually the force is the sum of a *passive* component plus an *active* component that arises in response to the excitation signal.

Once a muscle material is created, it can be assigned to a muscle or queried using `Muscle` methods

```
setMaterial (AxialMaterial mat)
AxialMaterial getMaterial()
```

> Muscle materials can also be assigned to axial springs, although the resulting force will always be computed with 0 excitation.

### 4.5.1 Simple muscle materials

A number of simple muscle materials are described below. All compute force as a function of $a$ and $l$, with an optional damping force that is proportional to $\dot{l}$. More complex Hill-type muscles, with force velocity curves, pennation angle, and a tendon component in series, are also available and described in Section 4.5.3.

> For historical reasons, the materials `ConstantAxialMuscle`, `LinearAxialMuscle` and `PeckAxialMuscle`, described below, contain a property called forceScaling that uniformly scales the computed force. This property is now deprecated, and should therefore have a value of 1. However, creating these materials with constructors *not* indicated in the documentation below will cause forceScaling to be set to a default value of 1000, thus requiring that the maximum force and damping values be correspondingly reduced.

#### 4.5.1.1 SimpleAxialMuscle

SimpleAxialMuscle is the default `AxialMaterial` for `Muscle`, and is essentially an activated version of LinearAxialMaterial. It computes a simple force according to

$$f(l,\dot{l}) = k(l-l_0) + d\dot{l} + f_{max}\,a, \tag{4.1}$$

where $l_0$ is the muscle rest length, $k$ and $d$ are stiffness and damping terms, and $f_{max}$ is the maximum excitation force. $l_0$ is specified by the restLength property of the muscle component, $a$ by the excitation property of the `Muscle`, and $k$, $d$ and $f_{max}$ by the following properties of `SimpleAxialMuscle`:

| | Property | Description | Default |
|---|---|---|---|
| $k$ | stiffness | stiffness term | 0 |
| $d$ | damping | damping term | 0 |
| $f_{max}$ | maxForce | maximum activation force that can be imparted by $a$ | 1 |

`SimpleAxialMuscle`s can be created with the following constructors:

```
SimpleAxialMuscle()
SimpleAxialMuscle (stiffness, damping, fmax)
```

where `stiffness`, `damping`, and `fmax` specify $k$, $d$ and $f_{max}$, and properties are otherwise set to their defaults.

#### 4.5.1.2 ConstantAxialMuscle

ConstantAxialMuscle is a simple muscle material that has a contractile force proportional to its activation, a constant passive tension, and linear damping. The resulting force is described by:

$$f(\dot{l}) = f_{max}(a + f_p) + d\dot{l}. \tag{4.2}$$

The parameters $f_{max}$, $f_p$ and $d$ are specified as properties of `ConstantAxialMuscle`:

| | Property | Description | Default |
|---|---|---|---|
| $f_{max}$ | maxForce | maximum contractile force | 1 |
| $f_p$ | passiveFraction | proportion of $f_{max}$ to apply as passive tension | 0 |
| $d$ | damping | damping parameter | 0 |

`ConstantAxialMuscle`s can be created with the following factory methods and constructors:

```
ConstantAxialMuscle.create()
ConstantAxialMuscle.create (fmax)
ConstantAxialMuscle (fmax, pfrac)
ConstantAxialMuscle (fmax, pfrac, damping)
```

where `fmax`, `pfrac`, and `damping` specify $f_{max}$, $f_p$ and $d$, and properties are otherwise set to their defaults.

> Creating a `ConstantAxialMuscle` with the no-args constructor, or another constructor *not* listed above, will cause its forceScaling property (Section 4.5.1) to be set to 1000 instead of 1, thus requiring that `fmax` and `damping` be corresponding reduced.

#### 4.5.1.3 LinearAxialMuscle

LinearAxialMuscle is a simple muscle material that has a linear relationship between length and tension, as well as linear damping. Given a normalized length described by

$$\hat{l} = \frac{l - l_{opt}}{l_{max} - l_{opt}}, \quad \text{with } 0 \le \hat{l} \le 1 \text{ enforced,} \tag{4.3}$$

the force generated by this material is:

$$f(l, \dot{l}) = f_{max}\left(a\hat{l} + f_p\hat{l}\right) + d\dot{l}. \tag{4.4}$$

The parameters are specified as properties of `LinearAxialMaterial`:

| | Property | Description | Default |
|---|---|---|---|
| $f_{max}$ | maxForce | maximum contractile force | 1 |
| $f_p$ | passiveFraction | proportion of $f_{max}$ that forms the maximum passive force | 0 |
| $l_{max}$ | maxLength | length beyond which maximum passive and active forces are generated | 1 |
| $l_{opt}$ | optLength | length below which zero active force is generated | 0 |
| $d$ | damping | damping parameter | 0 |

`LinearAxialMuscle`s can be created with the following factory methods and constructors:

```
LinearAxialMuscle.create()
LinearAxialMuscle.create (fmax, lrest)
LinearAxialMuscle (fmax, lopt, lmax, pfrac)
LinearAxialMuscle (fmax, lopt, lmax, pfrac, damping)
```

where `fmax`, `lopt`, `lmax`, `pfrac` and `damping` specify $f_{max}$, $l_{opt}$, $l_{max}$, $f_p$ and $d$, `lrest` specifies $l_{opt}$ and $l_{max}$ via $l_{opt} =$ `lrest` and $l_{max} = 3/2$ `lrest`, and other properties are set to their defaults.

Creating a `LinearAxialMuscle` with the no-args constructor, or another constructor *not* listed above, will cause its forceScaling property (Section 4.5.1) to be set to 1000 instead of 1, thus requiring that `fmax` and `damping` be corresponding reduced.

#### 4.5.1.4 PeckAxialMuscle

The PeckAxialMuscle material generates a force as described in [19]. It has a typical Hill-type active force-length relationship (modeled as a cosine), but the passive force-length properties are linear. This muscle model was empirically verified for jaw muscles during wide jaw opening.

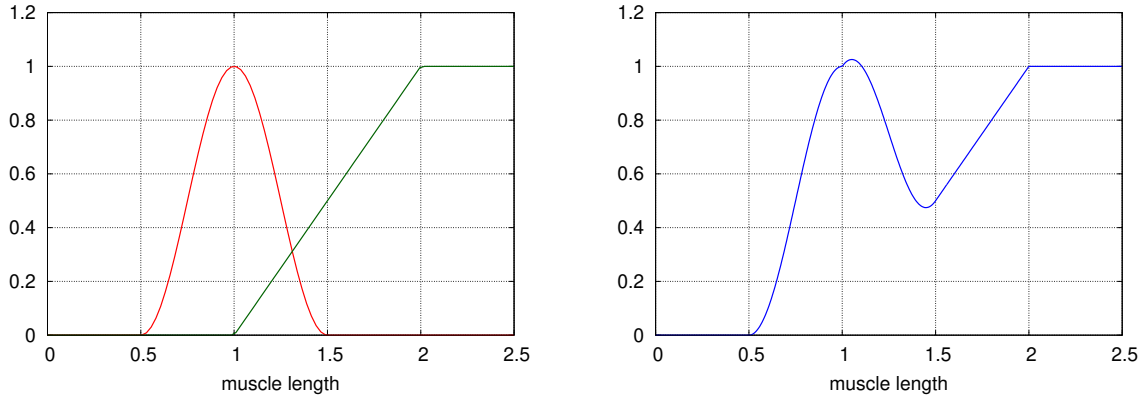



Figure 4.5: Left: active (red) and passive force length curve (green) for a `PeckAxialMuscle` with $f_{max} = 1$, $f_p = 1$, $T_r = 0$, $l_{opt} = 1$, $l_{max} = 2$, and $a = 1$. Note that the passive force curve is linear for $l \in [l_{opt}, l_{max}]$ and saturates for $l > l_{max}$. Right: combined active and passive force (blue).

Given a normalized fibre length described by

$$\hat{l}_f = \frac{l - l_{opt} T_r}{l_{opt}(1 - T_r)}, \quad \text{with } 1/2 \leq \hat{l} \leq 3/2 \text{ enforced,} \tag{4.5}$$

and a normalized muscle length

$$\hat{l}_m = \frac{l - l_{opt}}{l_{max} - l_{opt}}, \quad \text{with } 0 \leq \hat{l} \leq 1 \text{ enforced,} \tag{4.6}$$

the force generated by this material is:

$$f(l, \dot{l}) = f_{max} \left( a \frac{1 + \cos(2\pi \hat{l}_f)}{2} + f_p \hat{l}_m \right) + d\dot{l}. \tag{4.7}$$

The parameters are specified as properties of `PeckAxialMuscle`:

| | Property | Description | Default |
|---|---|---|---|
| $f_{max}$ | maxForce | maximum contractile force | 1 |
| $f_p$ | passiveFraction | proportion of $f_{max}$ that forms the maximum passive force | 0 |
| $T_r$ | tendonRatio | tendon to fibre length ratio | 0 |
| $l_{max}$ | maxLength | length at which maximum passive force is generated | 1 |
| $l_{opt}$ | optLength | length at which maximum active force is generated | 0 |
| $d$ | damping | damping parameter | 0 |

Figure 4.5 illustrates the force length relationship for a `PeckAxialMuscle`.

`PeckAxialMuscle`s can be created with the following factory methods:

```
PeckAxialMuscle.create()
PeckAxialMuscle.create (fmax, lopt, lmax, tratio, pfrac)
PeckAxialMuscle.create (fmax, lopt, lmax, tratio, pfrac, damping)
```

where `fmax`, `lopt`, `lmax`, `tratio`, `pfrac` and `damping` specify $f_{max}$, $l_{opt}$, $l_{max}$, $T_r$, $f_p$ and $d$, and other properties are set to their defaults.

> Creating a `PeckAxialMuscle` with the no-args constructor, or another constructor *not* listed above, will cause its forceScaling property (Section 4.5.1) to be set to 1000 instead of 1, thus requiring that `fmax` and `damping` be corresponding reduced.

#### 4.5.1.5 BlemkerAxialMuscle

The BlemkerAxialMuscle material generates a force as described in [5]. It is the axial muscle equivalent to the constitutive equation along the muscle fiber direction specified in the BlemkerMuscle FEM material.

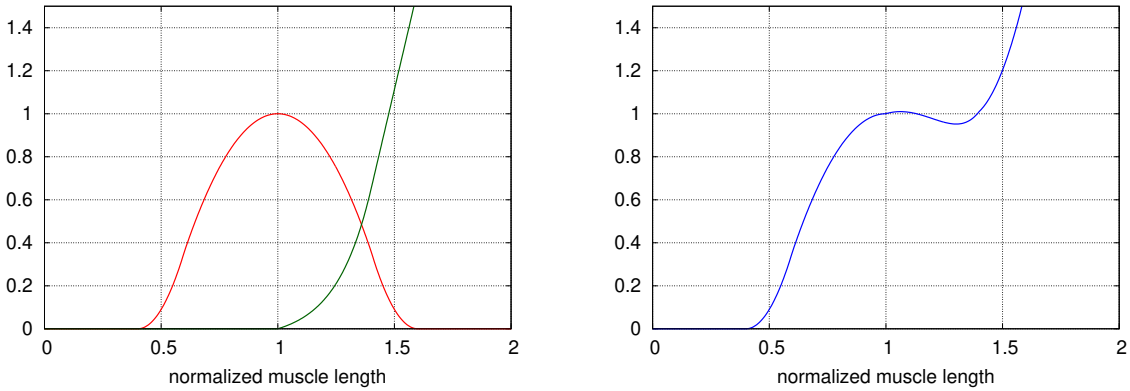



Figure 4.6: Left: active force curve $f_L(\hat{l})$ (red) and passive force curve $f_P(\hat{l})$ (green) for a `BlemkerAxialMuscle` with $l_{opt} = 1$, $l_{max} = 1.4$, $P_1 = 0.05$, and $P_2 = 6.6$. Right: total force length curve (blue) with $f_{max} = 1$ and $a = 1$.

The force produced is a combination of active and passive terms, plus a damping term, given by

$$f(l,\dot{l}) = f_{max}\left(a f_L(\hat{l}) + f_P(\hat{l})\right) + d\dot{l}, \tag{4.8}$$

where $f_{max}$ is the maximum force, $\hat{l} \equiv l/l_{opt}$ is the normalized muscle length, and $f_L(\hat{l})$ and $f_P(\hat{l})$ are the active and passive force length curves, given by

$$f_L(\hat{l}) = \begin{cases} 9\,(\hat{l}-0.4), & \hat{l} \in [0.4, 0.6] \\ 1-4(1-\hat{l}), & \hat{l} \in [0.6, 1.4] \\ 9\,(\hat{l}-1.6), & \hat{l} \in [1.4, 1.6] \\ 0, & \text{otherwise,} \end{cases} \tag{4.9}$$

and

$$f_P(\hat{l}) = \begin{cases} 0, & \hat{l} < 1 \\ P_1(e^{P_2(\hat{l}-1)} - 1), & \hat{l} \in [l_{opt}, l_{max}/l_{opt}] \\ P_3\hat{l} + P_4, & \hat{l} > l_{max}/l_{opt}. \end{cases} \tag{4.10}$$

For the passive force length curve, $P_3$ and $P_4$ are computed to provide linear extrapolation for $\hat{l} > l_{max}/l_{opt}$. The other parameters are specified by properties of `BlemkerAxialMuscle`:

| | Property | Description | Default |
|---|---|---|---|
| $f_{max}$ | maxForce | the maximum contractile force | $3 \times 10^5$ |
| $P_1$ | expStressCoeff | exponential stress coefficient | 0.05 |
| $P_2$ | uncrimpingFactor | fibre uncrimping factor | 6.6 |
| $l_{max}$ | maxLength | length at which passive force becomes linear | 1.4 |
| $l_{opt}$ | optLength | length at which maximum active force is generated | 1 |
| $d$ | damping | damping parameter | 0 |

Figure 4.6 illustrates the force length relationship for a `BlemkerAxialMuscle`.

`BlemkerAxialMuscle`s can be created with the following constructors,

```
BlemkerAxialMuscle()
BlemkerAxialMuscle (lmax, lopt, fmax, ecoef, uncrimp)
```

where `lmax`, `lopt`, `fmax`, `ecoef`, and `uncrimp` specify $l_{max}$, $l_{opt}$, $f_{max}$, $P_1$, $P_2$ and $d$, and properties are otherwise set to their defaults.

### 4.5.2 Example: muscle attached to a rigid body

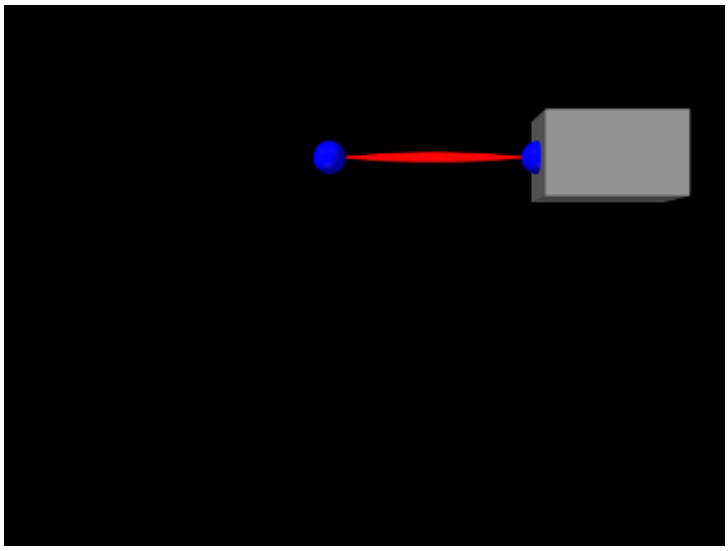


Figure 4.7: SimpleMuscle model loaded into ArtiSynth.

A simple model showing a single muscle connected to a rigid body is defined in

```
artisynth.demos.tutorial.SimpleMuscle
```

This model is identical to `RigidBodySpring` described in Section 3.2.2, except that the code to create the spring is replaced with code to create a muscle with a `SimpleAxialMuscle` material:

```
// create the muscle:
muscle = new Muscle ("mus", /*restLength=*/0);
muscle.setPoints (p1, mkr);
muscle.setMaterial (
   new SimpleAxialMuscle (/*stiffness=*/20, /*damping=*/10, /*fmax=*/10));
```

Also, so that the muscle renders differently, the rendering style for lines is set to `SPINDLE` using the convenience method

```
RenderProps.setSpindleLines (muscle, 0.02, Color.RED);
```

To run this example in ArtiSynth, select All demos > tutorial > SimpleMuscle from the Models menu. The model should load and initially appear as in Figure 4.7. Running the model (Section 1.5.3) will cause the box to fall and sway under gravity. To see the effect of the `excitation` property, select the muscle in the viewer and then choose Edit properties ... from the right-click context menu. This will open an editing panel that allows the muscle's properties to be adjusted interactively. Adjusting the `excitation` property using the adjacent slider will cause the muscle force to vary.

### 4.5.3 Equilibrium muscles

ArtiSynth supplies several muscle materials that simulate a pennated muscle in series with a tendon. Both the muscle and tendon generate forces which are functions of their respective lengths $l_m$ and $l_t$, and because these components are

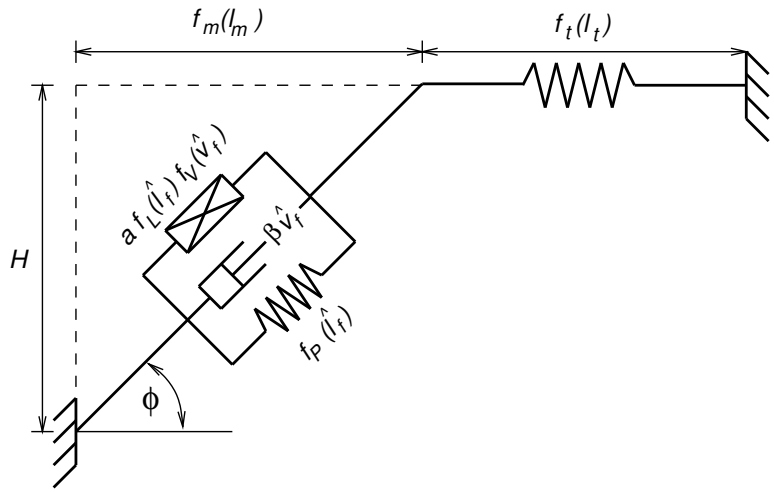



Figure 4.8: Schematic illustration of an pennated muscle in series with a tendon.

in series, their respective forces must be equal when the system is in equilibrium. Given an overall muscle-tendon length $l \equiv l_m + l_t$, ArtiSynth solves for $l_m$ at each time step to ensure that this equilibrium condition is met.

A general muscle-tendon system is illustrated by Figure 4.8, where $l_m$ and $l_t$ are the muscle and tendon lengths. These two components generate forces given by $f_m(l_m, v_m)$ and $f_t(l_t)$, where $v_m \equiv \dot{l}_m$, and the fact that these components are in series implies that their forces must be in equilibrium:

$$f_m(l_m, v_m) = f_t(l_t). \tag{4.11}$$

The muscle force is in turn produced by a *fibre force* $f_f$ acting at an *pennation angle* $\phi$ with respect to the principal muscle direction, such that

$$f_m = \cos(\phi) f_f(l_f, v_f),$$

where $l_f$ is the fibre length, which satisfies $l_m = \cos(\phi) l_f$, and $v_f \equiv \dot{l}_f$.

The fibre force is usually computed from the normalized fibre length $\hat{l}_f$ and normalized fibre velocity $\hat{v}_f$, defined by

$$\hat{l}_f = \frac{l_f}{l_o}, \quad \hat{v}_f = \frac{v_f}{l_o V_m}, \tag{4.12}$$

where $l_o$ is the *optimal fibre length* and $V_m$ is the *maximum contraction velocity*. It is composed of three components in parallel, such that

$$f_f(\hat{l}_f, \hat{v}_f) = F_o \left( a f_L(\hat{l}_f) f_V(\hat{v}_f) + f_P(\hat{l}_f) + \beta \hat{v}_f \right), \tag{4.13}$$

where $F_o$ is the *maximum isometric force*, $a f_L(\hat{l}_f) f_V(\hat{v}_f)$ is the active force term induced by an activation level $a$ and modulated by the *active force length curve* $f_L(\hat{l}_f)$ and the *force velocity curve* $f_V(\hat{v}_f)$, $f_P(\hat{l}_f)$ is the *passive force length curve*, and $\beta \hat{v}_f$ is an optional damping term induced by a *fibre damping* parameter $\beta$.

The tendon force $f_t(\hat{l}_t)$ is computed from

$$f_t(l_t) = F_o f_T(\hat{l}_t),$$

where $F_o$ is (again) the maximum isometric force, $f_T(\hat{l}_t)$ is the *tendon force length curve* and $\hat{l}_t$ is the normalized tendon length, defined by dividing $l_t$ by the *tendon slack length* $T$:

$$\hat{l}_t = \frac{l_t}{T}.$$

As the muscle moves, it is assumed that the height $H$ from the fibre origin to the main muscle line of action remains constant. This height is defined by

$$H = l_o \sin \phi_o,$$

where $\phi_o$ is the *optimal pennation angle* at the optimal fibre length when $l_f = l_o$.

### 4.5.4 Equilibrium muscle materials

The equilibrium muscle materials supplied by ArtiSynth include Thelen2003AxialMuscle and Millard2012AxialMuscle. These are all controlled by properties which specify the parameters presented in Section 4.5.3:

| | Property | Description | Default |
|---|---|---|---|
| $F_o$ | maxIsoForce | maximum isometric force | 1000 |
| $l_o$ | optFibreLength | optimal fibre length | 0.1 |
| $T$ | tendonSlackLength | length beyond which tendon exerts force | 0.2 |
| $\phi_o$ | optPennationAngle | pennation angle at optimal fibre length (radians) | 0 |
| $V_m$ | maxContractionVelocity | maximum contraction velocity | 10 |
| $\beta$ | fibreDamping | damping parameter for normalized fibre velocity | 0 |

The materials differ from each other with respect to their active, passive and tendon force length curves ($f_L(\hat{l}_f)$, $f_P(\hat{l}_f)$, and $f_T(\hat{l}_t)$) and their force velocity curves ($f_V(\hat{v}_f)$).

For a given muscle instance, it is typically only necessary to specify $F_o$, $l_o$, $T$ and $\phi_o$, where $F_o$, $l_o$ and $T$ are given in the model's basic force and length units. $V_m$ is given in units of $l_o$ per second and has a default value of 10; changing this value will stretch or contract the domain of the force velocity curve $f_V(\hat{v}_f)$. The damping parameter $\beta$ imparts a damping force proportional to $\hat{v}_f$ and has a default value of 0 (i.e., no damping). It is often not necessary to add fibre damping (since damping can be readily applied to the model in other ways), but $\beta$ is supplied for formulations that do specify damping. If non-zero, $\beta$ is usually set to a value less than or equal to 0.1.

In addition to the above parameters, equilibrium muscle materials also export the following properties to adjust their behavior:

| Property | Description | Default |
|---|---|---|
| rigidTendon | forces the tendon to be rigid | `false` |
| ignoreForceVelocity | ignore the force velocity curve $f_V(\hat{v}_f)$ | `false` |

If `rigidTendon` is `true`, the tendon will be assumed to be rigid with a length given by the tendon slack length parameter $T$. This simplifies the muscle computations since $l_m$ is then given by $l_m = l - T$ and there is no need to compute an equilibrium position. If `ignoreForceVelocity` is `true`, then force velocity effects are ignored by replacing the force velocity curve $f_V(\hat{v}_f)$ with 1.

The different equilibrium muscle materials are now summarized:

#### 4.5.4.1 Millard2012AxialMuscle

Millard2012AxialMuscle implements the default version of the `Millard2012EquilibriumMuscle` model supplied by OpenSim [7] and described in [15].

The active, passive and tendon force length curves ($f_L(\hat{l}_f)$, $f_P(\hat{l}_f)$, and $f_T(\hat{l}_t)$) and force velocity curve ($f_V(\hat{v}_f)$) are implemented using cubic Hermite spline curves to conform closely to the default curve values provided by OpenSim's `Millard2012EquilibriumMuscle`. Plots of these curves are shown in Figures 4.9 and 4.10. Both the passive and tendon force curves are linearly extrapolated (and hence exhibit constant stiffness) past $\hat{l}_f = 1.7$ and $\hat{l}_t = 1.049$, respectively, with stiffness values of 2.857 and 29.06.

> OpenSim requires that the `Millard2012EquilibriumMuscle` always exhibits a small bias activation, even when the activation should be zero, in order to avoid a singularity in the computation of the muscle length. ArtiSynth computes the muscle length in a manner that makes this unnecessary.

`Millard2012AxialMuscles` can be created using the constructors

```
Millard2012AxialMuscle()
Millard2012AxialMuscle(fmax, lopt, tslack, optPenAng)
```

where `fmax`, `lopt`, `tslack`, and `optPenAng` specify $F_o$, $l_o$, $T$, and $\phi_o$, and other properties are set to their defaults.

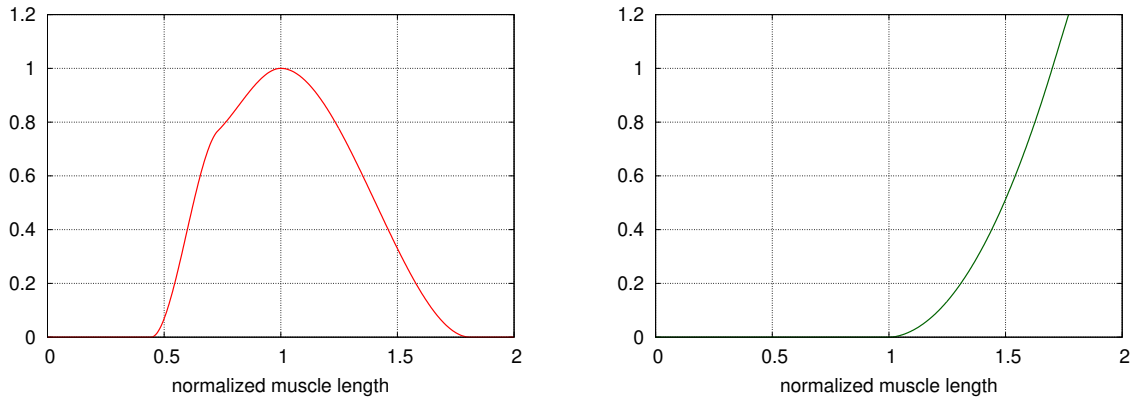



Figure 4.9: Default active force length curve $f_L(\hat{l}_f)$ (left) and passive force length curve $f_P(\hat{l}_f)$ (right) for the Millard 2012 muscle material.

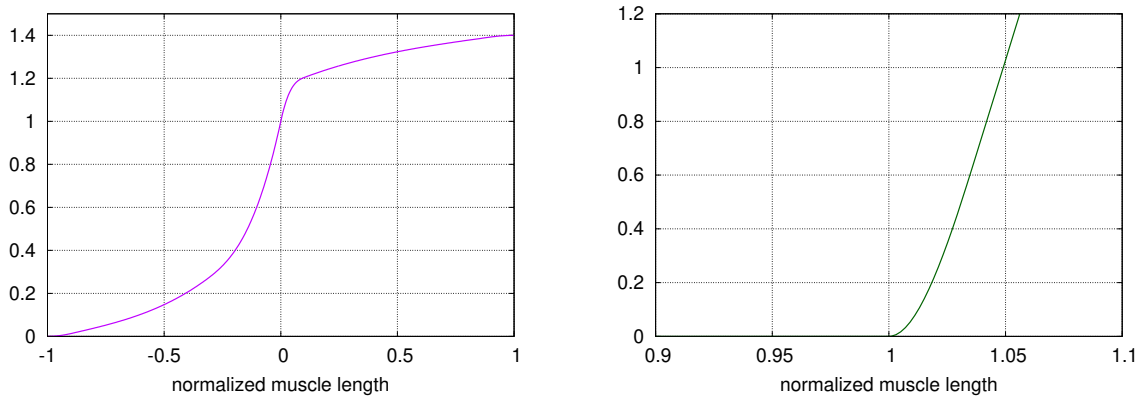



Figure 4.10: Default force velocity curve $f_V(\hat{v}_f)$ (left) and tendon force length curve $f_T(\hat{l}_t)$ (right) for the Millard 2012 muscle material. Note that the tendon force curve is about 10 times stiffer than the passive force curve, as seen by that fact the tendon curve is shown with a horizontal range of $[0.9, 1.1]$ vs. $[0, 2]$ for the passive curve.

#### 4.5.4.2 Thelen2003AxialMuscle

Thelen2003AxialMuscle implements the `Thelen2003Muscle` model supplied by OpenSim [7] and introduced in [26].

The active and passive force length curves are described by

$$f_L(\hat{l}_f) = e^{-(\hat{l}_f-1)^2/\gamma} \tag{4.14}$$

and

$$f_P(\hat{l}_f) = \frac{e^{k^{PE}(\hat{l}_f-1)/\varepsilon_0^M} - 1}{e^{k^{PE}} - 1}, \text{‘} \tag{4.15}$$

where $\gamma$, $k^{PE}$, and $\varepsilon_0^M$ are parameters, described in [26], that control the curve shapes. These are exposed as properties of `Thelen2003AxialMuscle` and are described in Table 4.3.

The tendon force length curve is described by

$$f_T(\hat{l}_t) = \begin{cases} 0, & \hat{l}_t < 1 \\ \dfrac{F_{toe}}{e^{k_{toe}} - 1}(e^{k_{toe}(\hat{l}_t-1)/\varepsilon_{toe}^T} - 1), & \hat{l}_t \le 1 + \varepsilon_{toe}^T \\ k_{lin}(\hat{l}_t - 1 - \varepsilon_{toe}^T) + F_{toe}, & \hat{l}_t > 1 + \varepsilon_{toe}^T, \end{cases} \tag{4.16}$$

where $F_{toe}$, $k_{toe}$, $k_{lin}$ and $\varepsilon_{toe}^T$ are parameters, described in [26], that control the curve shape. The values of these are either fixed, or derived from the *maximum isometric tendon strain* $\varepsilon_0^T$ (controlled by the fmaxTendonStrain property,

Table 4.3), according to

$$F_{toe} = 0.33, \quad k_{toe} = 3.0, \quad \varepsilon^T_{toe} = \frac{99\varepsilon^T_0 e^3}{166e^3 - 67}, \quad k_{lin} = \frac{0.67}{\varepsilon^T_0 - \varepsilon^T_{toe}}. \tag{4.17}$$

| | Property | Description | Default |
|---|---|---|---|
| $\gamma$ | kShapeActive | shape factor for active force length curve | 0.45 |
| $k^{PE}$ | kShapePassive | shape factor for active force length curve | 5.0 |
| $\varepsilon^M_0$ | fmaxMuscleStrain | passive muscle strain at maximum isometric muscle force | 0.6 |
| $\varepsilon^T_0$ | fmaxTendonStrain | tendon strain at maximum isometric muscle force | 0.04 |
| $A_f$ | af | force velocity shape factor | 0.25 |
| $\bar{F}^M_{len}$ | flen | maximum normalized lengthening force | 1.4 |
| | fvLinearExtrapThreshold | $\hat{v}_f$ beyond which $f_V(\hat{v}_f)$ is linearly extrapolated | 0.95 |

Table 4.3: Properties of `Thelen2003AxialMuscle` that control the shapes of the force curves.

The force velocity curve is determined from equation (6) in [26]. In the notation used by that equation and the accompanying equation (7), $V^M/V^M_{\max} = \hat{v}_f$, $f_l = f_L(\hat{l}_f)$, and $\bar{F}^M = a f_L(\hat{l}_f) f_V(\hat{v}_f)$. Inverting equation (6) yields the force velocity curve:

$$f_v = \begin{cases} \dfrac{\alpha + \bar{v}^M}{\alpha - \bar{v}^M/A_f}, & \bar{v}^M \le 0 \\ \dfrac{\beta + \bar{v}^M \bar{F}^M_{len}}{\beta + \bar{v}^M}, & \bar{v}^M > 0, \end{cases}$$

where

$$\alpha \equiv 0.25 + 0.75a, \qquad \beta \equiv \alpha \frac{\bar{F}^M_{len} - 1}{2 + 2/A_f},$$

and $a$ is the activation level. Parameters include the *maximum normalized lengthening force* $\bar{F}^M_{len}$, and force velocity shape factor $A_f$, described in Table 4.3.

Plots of the curves resulting from the above equations are shown, for default values, in Figures 4.11 and 4.12.

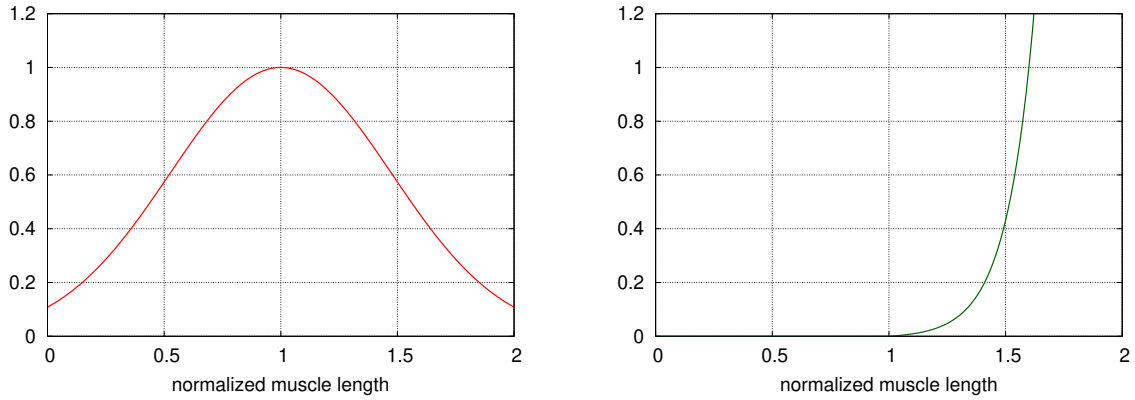



Figure 4.11: Default active force length curve $f_L(\hat{l}_f)$ (left) and passive force length curve $f_P(\hat{l}_f)$ (right) for the Thelen 2003 muscle material. Note that the passive curve is exponential and does *not* transition to a constant slope for high values of $\hat{l}_f$.

`Thelen2003AxialMuscles` can be created using the constructors

```
Thelen2003AxialMuscle()
Thelen2003AxialMuscle(fmax, lopt, tslack, optPenAng)
```

where `fmax`, `lopt`, `tslack`, and `optPenAng` specify $F_o$, $l_o$, $T$, and $\phi_o$, and other properties are set to their defaults.

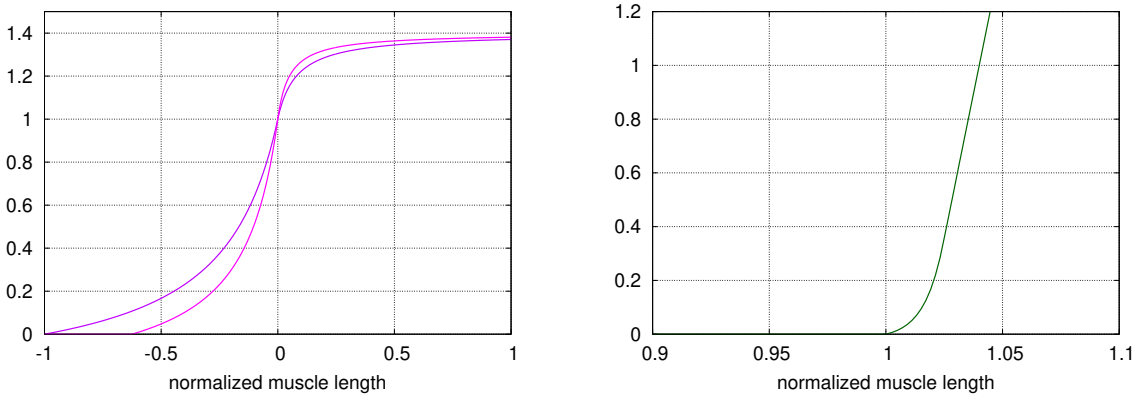



Figure 4.12: Default force velocity curve $f_V(\hat{v}_f)$ (left) and tendon force length curve $f_T(\hat{l}_t)$ (right) for the Thelen 2003 muscle model. Since the force velocity curve also depends on the activation, its plot shows the curve for two activation values: 1.0 (dark magenta), and 0.5 (light magenta).

### 4.5.5 Tendons and ligaments

Special point-to-point spring materials are also available to model tendons and ligaments. Because these are passive materials, they would normally be assigned to `AxialSpring` instead of `Muscle` components, although they will also work correctly in the latter. The materials include:

#### 4.5.5.1 Millard2012AxialTendon

Millard2012AxialTendon implements the default tendon material of the `Millard2012AxialMuscle` (Section 4.5.4.1), with a force length relationship given by

$$f(l) = F_o f_T(\hat{l}_t), \quad \hat{l}_t \equiv \frac{l}{T}, \tag{4.18}$$

where $F_o$ is the maximum isometric force, $f_T()$ is the *tendon force length curve* (shown in Figure 4.10, right), and $T$ is the tendon slack length. $F_o$ and $T$ are specified by the following properties of `Millard2012AxialTendon`:

| | Property | Description | Default |
|---|---|---|---|
| $F_o$ | maxIsoForce | maximum isometric force | 1000 |
| $T$ | tendonSlackLength | length beyond which tendon exerts force | 0.2 |

`Millard2012AxialTendon`s can be created with the constructors

```
Millard2012AxialTendon()
Millard2012AxialTendon (fmax, tslack)
```

where `fmax` and `tslack` specify $F_o$ and $T$, and other properties are set to their defaults.

#### 4.5.5.2 Thelen2003AxialTendon

Thelen2003AxialTendon implements the default tendon material of the `Thelen2003AxialMuscle` (Section 4.5.4.2), with a force length relationship given by

$$f(l) = F_o f_T(\hat{l}_t), \quad \hat{l}_t \equiv \frac{l}{T}, \tag{4.19}$$

where $F_o$ is the maximum isometric force, $f_T()$ is the *tendon force length curve* described by equation (4.16), and $T$ is the tendon slack length. $F_o$, $T$, and the *maximum isometric tendon strain* $\varepsilon_0^T$ (used to determine the parameters of (4.16), according to (4.17)), are specified by the following properties of `Thelen2003AxialTendon`:

| | Property | Description | Default |
|---|---|---|---|
| $F_o$ | maxIsoForce | maximum isometric force | 1000 |
| $T$ | tendonSlackLength | length beyond which tendon exerts force | 0.2 |
| $\varepsilon_0^T$ | fmaxTendonStrain | tendon strain at maximum isometric muscle force | 0.04 |

`Thelen2003AxialTendon`s can be created with the constructors

```
Thelen2003AxialTendon()
Thelen2003AxialTendon (fmax, tslack)
```

where `fmax` and `tslack` specify $F_o$ and $T$, and other properties are set to their defaults.

#### 4.5.5.3 Blankevoort1991AxialLigament

Blankevoort1991AxialLigament implements the `Blankevoort1991Ligament` model supplied by OpenSim [7] and described in [4, 23].

With the ligament strain and its derivative defined by

$$\varepsilon \equiv \frac{l - l_0}{l_0}, \quad \dot{\varepsilon} \equiv \frac{\dot{l}}{l_0}, \tag{4.20}$$

the ligament force is given by

$$f(l, \dot{l}) = f_e(\varepsilon) + f_d(\dot{\varepsilon}), \tag{4.21}$$

where

$$f_e(\varepsilon) = \begin{cases} 0, & \varepsilon < 0 \\ \frac{1}{2\varepsilon_t} k\varepsilon^2, & 0 \le \varepsilon \le \varepsilon_t \\ k(\varepsilon - \frac{\varepsilon_t}{2}), & \varepsilon > \varepsilon_t, \end{cases}$$

and

$$f_d(\dot{\varepsilon}) = \begin{cases} d\dot{\varepsilon}, & \varepsilon > 0 \text{ and } \dot{\varepsilon} > 0 \\ 0, & \text{otherwise.} \end{cases}$$

The parameters $l_0$, $\varepsilon_t$, $k$, and $d$ are specified by the following properties of `Blankevoort1991Ligament`:

| | Property | Description | Default |
|---|---|---|---|
| $l_0$ | slackLength | ligament slack length | 0 |
| $\varepsilon_t$ | transitionStrain | strain at transition from toe to linear | 0.06 |
| $k$ | linearStiffness | maximum stiffness of force-length curve | 100 |
| $d$ | damping | damping parameter | 0.003 |

`Blankevoort1991Ligament`s can be created with the constructors

```
Blankevoort1991Ligament()
Blankevoort1991Ligament (stiffness, slackLen, damping)
```

where `stiffness`, `slackLen` and `damping` specify $k$, $l_o$ and $d$, and other properties are set to their defaults.

### 4.5.6 Example: muscles with separate tendons

An alternate way to model the equilibrium muscles of Section 4.5.3 is to use `two` separate point-to-point muscles, attached in series via a connecting particle with a small mass (and possibly damping), thus implementing the muscle and tendon components separately. This approach allows the muscle equilibrium length to be determined automatically by the physical response of the connecting particle, in a manner similar to that employed by OpenSim's `Millard2012AccelerationMuscle` (described in [14]). For the muscle material, one can use any of the tendonless materials of Section 4.5.1, *or* the materials of Section 4.5.3 with the properties rigidTendon and tendonSlackLength set to `true` and 0, respectively, while the tendon material can be one described in Section 4.5.5 or some other suitable passive material. The implicit integrators used by ArtiSynth should permit relatively stable simulation of the arrangement.

While it is usually easier to employ the equilibrium muscle materials of Section 4.5.4, especially for models involving wrapping and/or via points (Chapter 9) where it may be difficult to handle the connecting particle correctly, in some situations the separate muscle/tendon method may offer a more versatile solution. It can also be used to validate the equilibrium muscles, as illustrated by the application model defined in

```
artisynth.demos.tutorial.EquilibriumMuscleDemo
```

which provides a direct comparison of the methods. It creates two simple instances of a Millard 2012 muscle, one above the other in the x-z plane, with the top instance implemented using the separate muscle/tendon method and the bottom instance implemented using an equilibrium muscle material (Figure 4.13). The application model uses control and monitoring components introduced in Chapter 5 to drive the simulation and record its results: a *controller* (Section 5.3) is used to uniformly extend the length of both muscles; *output probes* (Section 5.4) are used to record the resulting tension forces; and a *control panel* (Section 5.1) is created to allow the user to interactively adjust some of the muscle parameters.

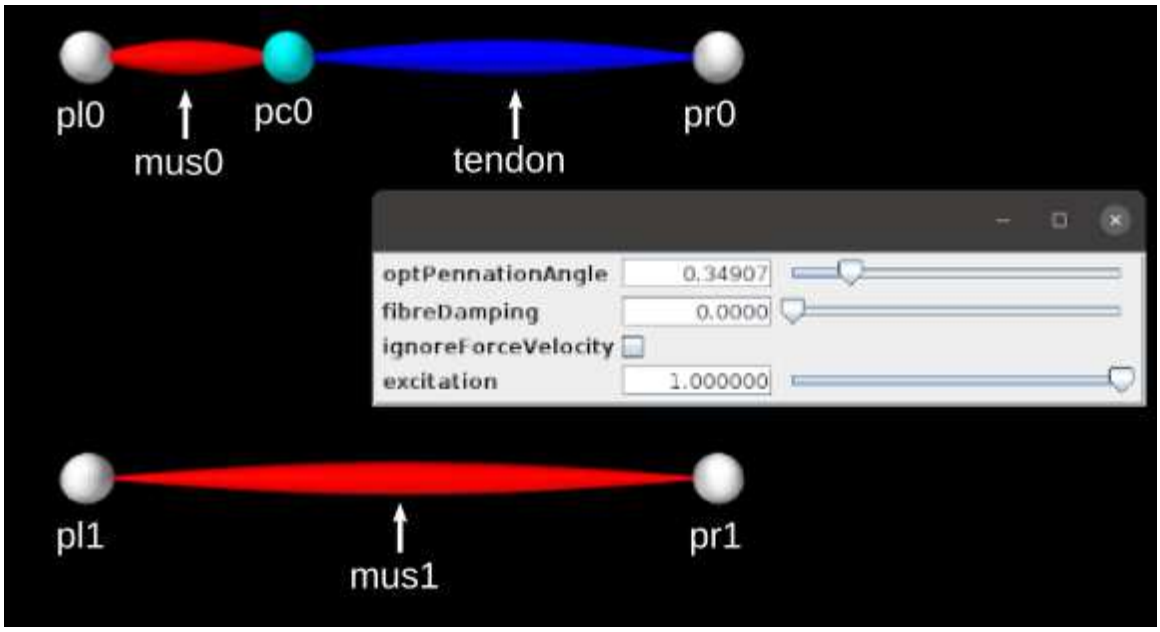



Figure 4.13: `EquilibriumMuscleDemo` loaded into ArtiSynth, with different particles and muscle components labeled. The separate muscle/tendon muscle is at the top, the equilibrium muscle at the bottom, and the control panel is overlaid on the viewer.

The code for the `EquilibriumMuscleDemo` class attributes and `build()` method is shown below:

```
1    Particle pr0, pr1; // right end point particles; used by controller
2
3    // default muscle parameter settings
4    private double myOptPennationAng = Math.toRadians(20.0);
5    private double myMaxIsoForce = 10.0;
6    private double myTendonSlackLen = 0.5;
7    private double myOptFibreLen = 0.5;
8
9    // initial total length of the muscles:
10   private double len0 = 0.25 + myTendonSlackLen;
11
12   public void build (String[] args) {
13      // create a mech model with zero gravity
14      MechModel mech = new MechModel ("mech");
15      addModel (mech);
16      mech.setGravity (0, 0, 0);
17
18      // build first muscle, consisting of a tendonless muscle, attached to a
19      // tendon via a connecting particle pc0 with a small mass.
20      Particle pl0 = new Particle ("pl0", 1.0, 0.0, 0, 0); // left end point
21      pl0.setDynamic (false); // point is fixed
22      mech.addParticle (pl0);
23
24      // create connecting particle. x coordinate will be set later.
25      Particle pc0 = new Particle ("pc0", /*mass=*/1e-5, 0, 0, 0);
```

```
        mech.addParticle (pc0);

        pr0 = new Particle ("pr0", 1.0, len0, 0, 0); // right end point
        pr0.setDynamic (false); // point will be positioned by length controller
        mech.addParticle (pr0);

        // create muscle and attach it between pl0 and pc0
        Muscle mus0 = new Muscle("mus0"); // muscle
        Millard2012AxialMuscle mat0 = new Millard2012AxialMuscle (
           myMaxIsoForce, myOptFibreLen, myTendonSlackLen, myOptPennationAng);
        mat0.setRigidTendon (true); // set muscle to rigid tendon with zero length
        mat0.setTendonSlackLength (0);
        mus0.setMaterial (mat0);
        mech.attachAxialSpring (pl0, pc0, mus0);

        // create explicit tendon and attach it between pc0 and pr0
        AxialSpring tendon = new AxialSpring(); // tendon
        tendon.setMaterial (
           new Millard2012AxialTendon (myMaxIsoForce, myTendonSlackLen));
        mech.attachAxialSpring (pc0, pr0, tendon);

        // build second muscle, using combined muscle/tendom material, and attach
        // it between pl1 and pr1.
        Particle pl1 = new Particle (1.0, 0, 0, -0.5); // left end point
        pl1.setDynamic (false);
        mech.addParticle (pl1);

        pr1 = new Particle ("pr1", 1.0, len0, 0, -0.5); // right end point
        pr1.setDynamic (false);
        mech.addParticle (pr1);

        Muscle mus1 = new Muscle("mus1");
        Millard2012AxialMuscle mat1 = new Millard2012AxialMuscle (
           myMaxIsoForce, myOptFibreLen, myTendonSlackLen, myOptPennationAng);
        mus1.setMaterial (mat1);
        mech.attachAxialSpring (pl1, pr1, mus1);

        // initialize both muscle excitations to 1, and then adjust the muscle
        // lengths to the corresponding (zero velocity) equilibrium position
        mus0.setExcitation (1);
        mus1.setExcitation (1);
        // compute equilibrium muscle length with for 0 velocity
        double lm = mat1.computeLmWithConstantVm (
           len0, /*vel=*/0, /*excitation=*/1);
        // set muscle length of mat1 and x coord of pc0 to muscle length:
        mat1.setMuscleLength (lm);
        pc0.setPosition (new Point3d (lm, 0, 0));

        // set render properties:
        // render markers as white spheres, and muscles as red spindles
        RenderProps.setSphericalPoints (mech, 0.03, Color.WHITE);
        RenderProps.setSpindleLines (mech, 0.02, Color.RED);
        // render tendon in blue and the juntion point in cyan
        RenderProps.setLineColor (tendon, Color.BLUE);
        RenderProps.setPointColor (pc0, Color.CYAN);

        // create a control panel to adjust material parameters and excitation
        ControlPanel panel = new ControlPanel();
        panel.addWidget ("material.optPennationAngle", mus0, mus1);
        panel.addWidget ("material.fibreDamping", mus0, mus1);
        panel.addWidget ("material.ignoreForceVelocity", mus0, mus1);
        panel.addWidget ("excitation", mus0, mus1);
        addControlPanel (panel);

        // add a controller to extend/contract the muscle end points, and probes
```

```
91        // to record both muscle forces
92        addController (new LengthController());
93        addForceProbe ("muscle/tendon force", mus0, 2);
94        addForceProbe ("equilibrium force", mus1, 2);
95     }
```

Lines 4-10 declare attributes for muscle parameters and the initial length of the combined muscle-tendon. Within the `build()` method, a `MechModel` is created with zero gravity (lines 14-16).

Next, the separate muscle/tendon muscle is assembled, consisting of three particles (`pl0`, `pc0`, and `pr0`, lines 20-30), a muscle `mus0` connected between `pr0` and `pc0` (lines 33-39), and an `tendon` connected between `pc0` and `pr0` (lines 42-45). Particles `pl0` and `pr0` are both set to be non-dynamic, since `pl0` will be fixed and `pr0` will be moved parametrically, while the connecting particle `pc0` has a small mass and its position is updated by the simulation and so automatically maintains force equilibrium between the muscle and tendon. The material `mat0` used for `mus0` is an instance of `Millard2012AxialMuscle`, with the tendon removed by setting the tendonSlackLength and rigidTendon properties to 0 and `true`, respectively, while the material for `tendon` is an instance of `Millard2012AxialTendon`.

The equilibrium muscle is then assembled, consisting of two particles (`pl1` and `pr1`, lines 49-55) and a muscle `mus1` connected between them (lines 57-61). `pl1` and `pr1` are both set to be non-dynamic, since `pl1` will be fixed and `pr1` will be moved parametrically, and the material `mat1` for `mus1` is an instance of `Millard2012AxialMuscle` in its default equilibrium mode. Excitations for both `mus0` and `mus1` are initialized to 1, and the zero-velocity equilibrium muscle length is computed using `mat1.computeLmWithConstantVm()` (lines 65-69). This is used to update the muscle position, for `mus0` by setting the x coordinate of `pc0` and for `mus1` by setting the internal muscle length variable of `mat1` (lines 71-72).

Render properties for different components are set at lines 76-80, and then a *control panel* is created to allow interactive adjustment of the muscle material properties optPennationAngle, fibreDamping, and ignoreForceVelocity, as well the muscle excitations (lines 83-88). As explained in Sections 5.1.1 and 5.1.3, `addWidget()` methods are used to create widgets that control each of these properties for both `mus0` and `mus1`.

At line 92, a *controller* is added to the root model to move the right side particles `p0r` and `p1r` during the simulation, thus changing the muscles' lengths and hence their tension forces. Controllers are explained in Section 5.3, and the controller itself is defined by lines 101-127 of the code listing below. At the start of each simulation time step, the controller's `apply()` method is called, with `t0` and `t1` denoting the step's start and end times. It increases the x positions of `p0r` and `p1r` uniformly with a speed of `mySpeed` until time `myRunTime/2`, and then reverses direction until time `myRunTime`. Velocities are also updated since these are needed to determine $\dot{l}$ for the muscles' `computeF()` methods.

Each muscle's tension force is recorded by an *output probe* connected to its forceNorm property. Probes are explained in Section 5.4; in this example, they are created using the convenience method `addForceProbe()`, defined by lines 130-140 (below) and called at lines 93-94 in the `build()` method, with probes for the muscle/tendon and equilibrium muscles named "muscle/tendon force" and "equilibrium force", respectively.

```
97     /**
98      * A controller to extend and the contract the muscle length by moving the
99      * rightmost muscle end points.
100     */
101    public class LengthController extends ControllerBase {
102
103       double myRunTime = 1.5; // total extensions/contraction time
104       double mySpeed = 1.0;   // speed of the end point motion
105
106       public LengthController() {
107          // need null args constructor if this model is read from a file
108       }
109
110       public void apply (double t0, double t1) {
111          double xlen = len0; // x position of the end points
112          double xvel = 0; // x velocity of the end points
113          if (t1 <= myRunTime/2) { // extend
114             xlen += mySpeed*t0;
115             xvel = mySpeed;
116          }
117          else if (t1 <= myRunTime) { // contract
118             xlen += mySpeed*(2*myRunTime/2 - t1);
```

```
            xvel = -mySpeed;
         }
         // update end point positions and velocities
         pr0.setPosition (new Point3d (xlen, 0, 0));
         pr1.setPosition (new Point3d (xlen, 0, -0.5));
         pr0.setVelocity (new Vector3d (xvel, 0, 0));
         pr1.setVelocity (new Vector3d (xvel, 0, 0));
      }
   }

   // Create and add an output probe to record the tension force of a muscle
   void addForceProbe (String name, Muscle mus, double stopTime) {
      try {
         NumericOutputProbe probe =
            new NumericOutputProbe (mus, "forceNorm", 0, stopTime, -1);
         probe.setName (name);
         addOutputProbe (probe);
      }
      catch (Exception e) {
         e.printStackTrace();
      }
   }
```

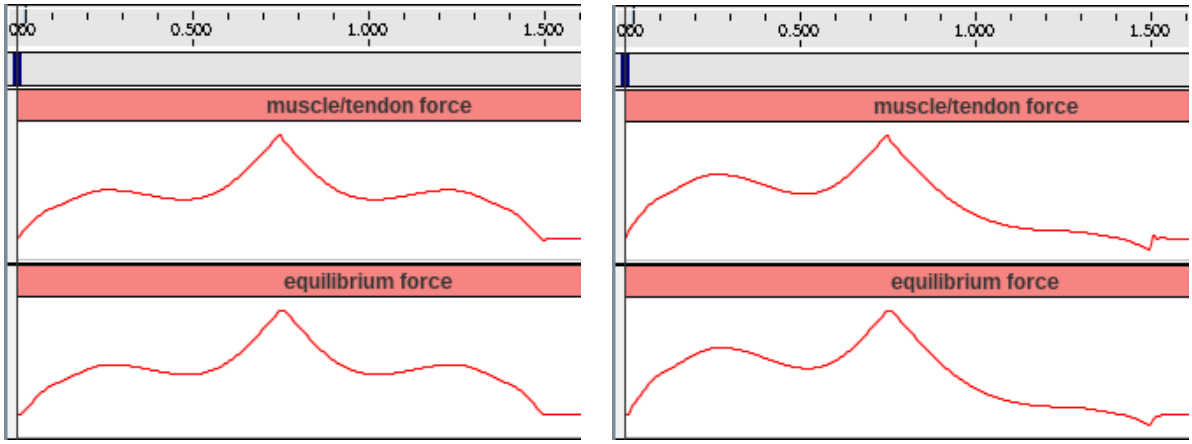



Figure 4.14: Tension forces produced by `EquilibriumMuscleDemo`, displayed by the output probes, as both muscles are extended and then contracted. Left and right images show results with the muscle material property ignoreForceVelocity set to `true` and `false`, respectively.

To run this example in ArtiSynth, select All demos > tutorial > EquilibriumMuscleDemo from the Models menu. The model should load and initially appear as in Figure 4.13. As explained above, running the model will cause the muscles to be extended and contracted by moving particles `p0r` and `p1r` right and back again. As this happens, the resulting tension forces can be examined by expanding the output probes in the timeline (see section "The Timeline" in the ArtiSynth User Interface Guide). The results are shown in Figure 4.14, with the left and right images showing results with the muscle material property ignoreForceVelocity set to `true` and `false`, respectively.

As should be expected, the results for the two muscle implementations are very similar. In both cases, the forces exhibit a local peak near time 0.25 when the muscle lengths are at the maximum of the active force length curve. As time increases, forces then decrease and then rise again as the passive force increases, peaking at time 0.75 when the muscle starts to contract. In the left image, the forces during contraction are symmetrical with those during extension, while in the right image the post-peak forces are more attenuated, because in the latter case the force velocity relationship is enabled as this reduces forces when a muscle is contracting.

## 4.6 Distance Grids and Components

Distance grids, implemented by the class DistanceGrid, are currently used in ArtiSynth for both collision handling (Section 8.4.2) and spring and muscle wrapping around general surfaces (Section 9.3). A distance grid is a regular three

dimensional grid that is used to interpolate a scalar distance field and its associated normals. For collision handling and wrapping purposes, this distance function is assumed to be signed and is used to estimate the penetration depth and associated normal direction of a point within some 3D object.

A distance grid consists of $n_x \times n_y \times n_z$ evenly spaced vertices partitioning a volume into $r_x \times r_y \times r_z$ cuboid cells, with

$$r_x = n_x - 1, \quad r_y = n_y - 1, \quad r_z = n_z - 1.$$

The grid has overall widths $w_x$, $w_y$, $w_z$ along each axis, so that the widths of each cell are $w_x/r_x$, $w_y/r_y$, $w_z/r_z$.

Scalar distances and normal vectors are stored at each vertex, and interpolated at points within each cell using trilinear interpolation. Distance values (although not normals) can also be interpolated *quadratically* over a composite *quadratic* cell composed of $2 \times 2 \times 2$ regular cells. This provides a smoother result than trilinear interpolation and is currently used for muscle wrapping. To ensure that all points within the grid can be assigned a unique quadratic cell, the grid resolution is restricted so that $r_x$, $r_y$, $r_z$ are always even. Distance grids are typically generated automatically from mesh data. More details on the actual functionality of distance grids is contained in the DistanceGrid API documentation.

When used within ArtiSynth, distance grids are generally contained within an encapsulating DistanceGridComp component, which maintains a mesh (or list of meshes) used to generate the grid, and exports properties for controlling its resolution, mesh fit, and visualization. For any object that implements CollidableBody (which includes RigidBody), its distance grid component can be obtained using the method

```
DistanceGridComp getDistanceGridComp();
```

A distance grid maintains its own local coordinate system, which is related to world coordinates by a *local-to-world* transform that can be queried and set via the component methods setLocalToWorld() and getLocalToWorld(). For grids associated with a rigid body, the local coordinate system is synchronized with the rigid body's coordinate system (which is queried and set via setPose() and getPose()). The grid's axes are nominally aligned with the local coordinate system, although they can be subject to an additional orientation offset (e.g., see the description of the property fitWithOBB below).

By default, a `DistanceGridComp` generates its grid automatically from its mesh data, within an axis-aligned bounding volume with the resolutions $r_x$, $r_y$, $r_z$ chosen automatically. However, in some cases it may be necessary to control the resolution explicitly. `DistanceGridComp` exports the following properties to control both the grid's resolution and how it is fit around the mesh data:

**resolution** A vector of 3 integers that specifies the resolutions $r_x$, $r_y$, $r_z$. If any value in this vector is set $\leq 0$, then all values are set to zero and the maxResolution property is used to determine the grid divisions instead.

**maxResolution** Sets the default maximum cell resolution that should be used when constructing the grid. This is the number of cells that should be used along the *longest* bounding volume axis, with the cell counts along the other axes adjusted to maintain a uniform cell size. If all three values of resolution are $> 0$, those will be used to specify the cell resolutions instead.

**fitWithOBB** If `true`, offsets the orientation of the grid's x, y, and z axes (with respect to local coordinates) to align with an oriented bounding box fit to the mesh. Otherwise, the grid axes are aligned with those of the local coordinate frame.

**marginFraction** Specifies the fractional amount by which the mesh should be extended with respect to a bounding box fit around the mesh(es). The default value is `0.1`.

As with all properties, these can be set either interactively (either using a custom control panel or by selecting the component and then choosing Edit properties ... from the right-click context menu), or through their set/get accessor methods. For example, resolution and maxResolution can be set and queried via the methods:

```
void setResolution (Vector3i res)
Vector3i getResolution()

void setMaxResolution (int max)
int getMaxResolution()
```

When used for either collisions or wrapping, the distance values interpolated by a distance grid are used to determine whether a point is inside or outside some 3D object, with the inside/outside boundary being the isosurface surface corresponding to a distance value of 0. This isosurface surface (which differs depending on whether trilinear or

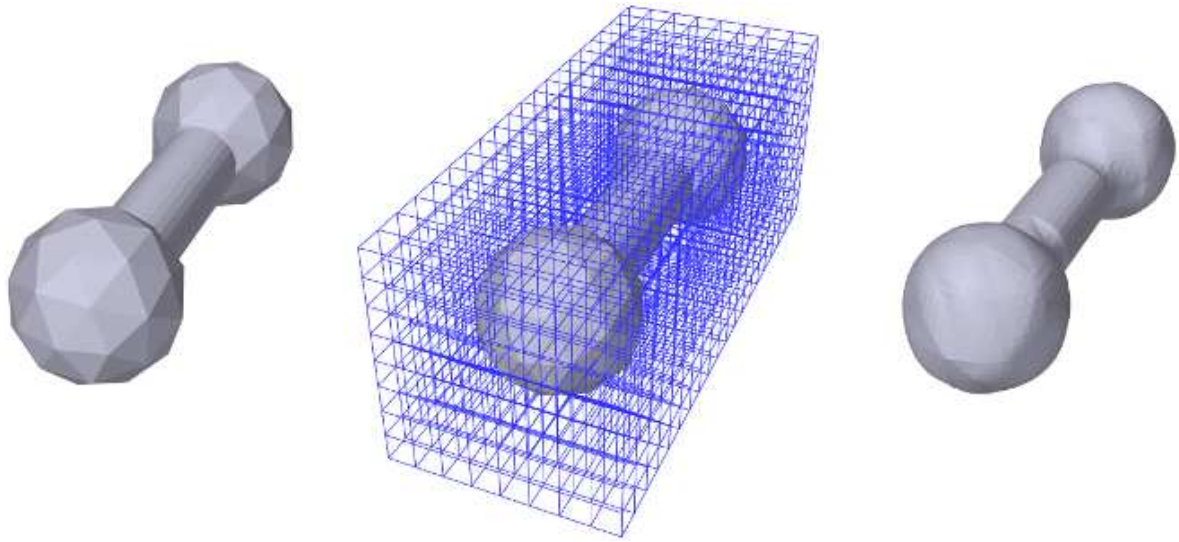


Figure 4.15: A mesh used to generate a distance grid, (left), along with a visualization of the grid itself (middle) and the corresponding quadratic isosurface (right). Notice how in this case the quadratic isosurface is smoother than the coarser features of the original mesh.

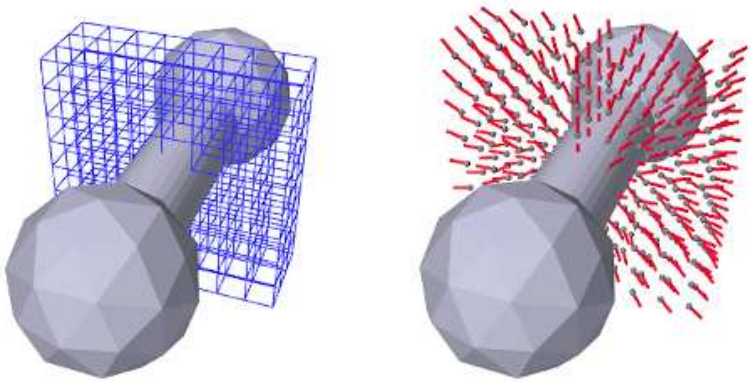


Figure 4.16: Rendering a subportion of a distance grid restricted along the x axis by setting renderRanges to `"9:12 * *"`, with render properties set to show the grid cells (left), and the vertices and normals (right).

quadratic distance interpolation is used) therefore represents the effective collision boundary, and it may be somewhat different from the mesh surface used to generate it. It may be smoother, or may have discretization artifacts, depending on both the smoothness and complexity of the mesh and the grid's resolution (Figure 4.15). It is therefore important to be able to visualize both the trilinear and quadratic isosurfaces. `DistanceGridComp` provides a number of properties to control this along with other aspects of grid visualization:

**renderProps** Render properties that control the colors, styles, and sizes (or widths) used to render faces, lines and points (Section 4.3). Point, line and edge properties are used for rendering grid vertices, normals, and cell edges, while face properties are used for rendering the isosurface. One can select which components are visible by zeroing appropriate render properties: zeroing `pointRadius` (or `pointSize`, if the `pointStyle` is `POINT`) disables drawing of the vertices; zeroing `lineRadius` (or `lineWidth`, if the `lineStyle` is `LINE`) disables drawing of the normals; and zeroing `edgeWidth` disables drawing of the cell edges. For an illustration, see Figure 4.16.

**renderGrid** If set to `true`, causes the grid's vertices, normals and cell edges to be rendered, using the render properties as described above.

**renderRanges** Can be used to restrict which part of the distance grid is drawn. The value is a string which specifies the vertex ranges along the $x$, $y$, and $z$ axes. In general, the grid will have $n_x \times n_y \times n_z$ vertices along the $x$, $y$, and $z$ axes, where $n_x$, $n_y$, and $n_z$ are each one greater than the cell resolutions $r_x$, $r_y$, and $r_z$. The range string

should contain three range specifications, one for each axis, where each specification is either `*` (all vertices), `n:m` (vertices in the index range `n` to `m`, inclusive), or `n` (vertices only at index `n`). A range specification of `"* * *"` (or `"*"`) means draw all vertices, which is the default behavior. Other examples include:

```
"* 7 *"      all vertices along x and z, and those at index 7 along y
"0 2 3"      a single vertex at indices (0, 2, 3)
"0:3 4:5 *"  all vertices between 0 and 3 along x, and 4 and 5 along y
```

For an illustration, see Figure 4.16.

**renderSurface** If set to `true`, renders the grid's isosurface, as determined by the surfaceType property. See Figure 4.15, right.

**surfaceType** Controls the interpolation used to form the isosurface rendered in response to the renderSurface property. `QUADRATIC` (the default) specifies a quadratic isosurface, while `TRILINEAR` specifies a trilinear isosurface.

**surfaceDistance** Controls the level set value used to determine the isosurface. To render the isosurface used for collision handling or muscle wrapping, this value should be 0.

When visualizing the isosurface for a distance grid, it is generally convenient to also turn *off* visualization for the meshes used to generate the grid. For `RigidBody` objects, this can be accomplished easily using the convenience property gridSurfaceRendering. If set `true`, it will cause the isosurface to be rendered *instead* of its mesh components. The isosurface type will be that indicated by the grid component's surfaceType property, and the rendering will occur independently of the visibility settings for the meshes or the grid component.

## 4.7 Transforming geometry

Certain ArtiSynth components, including `MechModel`, implement the interface TransformableGeometry, which allows the geometric transformation of the component's attributes (such as meshes, points, frame locations, etc.), along with its descendant components. The interface provides the method

```
public void transformGeometry (AffineTransform3dBase X);
```

where `X` is an AffineTransform3dBase that may be either a RigidTransform3d or a more general AffineTransform3d (Section 2.2).

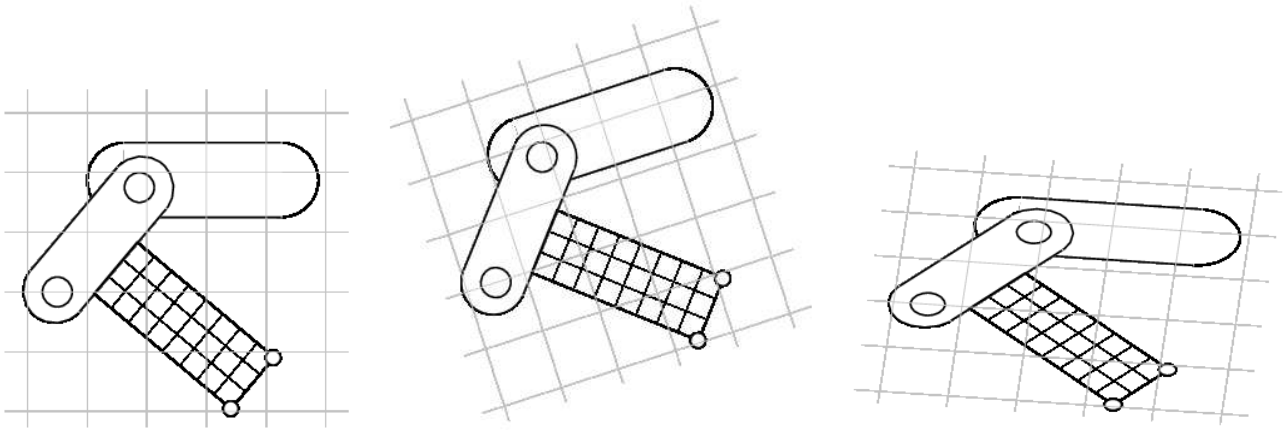


Figure 4.17: Simple illustration of a model (left) undergoing a rigid transformation (middle) and an affine transformation (right).

transformGeometry(X) can be used to translate, rotate, shear or scale components. It can be applied to an entire model or individual components. Unlike scaleDistance(), it actually changes the physical geometry and so may change the simulation behavior. For example, applying `transformGeometry()` to a RigidBody will cause the shape of its mesh to change, which will change its mass if its inertiaMethod property is set to `DENSITY`. Figure 4.17 shows a simplified illustration of both rigid and affine transformations being applied to a model.

The example below shows how to apply a transformation to a model in code. In it, a `MechModel` is first scaled by the factors 1.5, 2, and 3 along the x, y, and z axes, and then flipped upside down using a `RigidTransform3d` that rotates it by 180 degrees about the x axis:

```
MechModel mech;

... build mech model ...

AffineTransform3d X = new AffineTransform3d();
X.applyScaling (1.5, 2, 3);
mech.transformGeometry (X);

RigidTransform3d T =
   new RigidTransform3d (/*x,y,z=*/0, 0, 0, /*r,p,y=*/0, 0, Math.PI);
mech.transformGeometry (T);
```

The transform specified to `transformGeometry` is in *world* coordinates. If the component being transformed has its own local coordinate frame (such as a rigid body), and one wants to specify the transform in that local frame, then one needs to convert the transform from local to world coordinates. Let $\mathbf{T}_{CW}$ be the local coordinate frame, and let $\mathbf{X}_W$ and $\mathbf{X}_B$ be the transform in world and body coordinates, respectively. Then

$$\mathbf{X}_W \mathbf{T}_{CW} = \mathbf{T}_{CW} \mathbf{X}_B$$

and so

$$\mathbf{X}_W = \mathbf{T}_{CW} \mathbf{X}_B \mathbf{T}_{CW}^{-1}.$$

(See Appendix A.2 and A.3).

As an example of converting a transform to world coordinates, suppose we wish to scale a rigid body by $a$, $b$, and $c$ along its local axes. The transformation could then be done as follows:

```
AffineTransform3d XB = new AffineTransform3d(); // transform in body coords
AffineTransform3d XW = new AffineTransform3d(); // transform in world coords
RigidTransform3d TBW = body.getPose(): // coordinate frame of the body

XB.setScaling (a, b, c);      // set scaling in body coords ...
XW.mul (TBW, XB);             // ... convert to world coords ...
XW.mulInverseRight (XW, TBW);
body.transformGeometry (XW);  // ... and apply to the body
```

### 4.7.1 Nonlinear transformations

The TransformableGeometry interface also supports general, nonlinear geometric transforms. This can be done using a GeometryTransformer, which is an abstract class for performing general transformations. To apply such a transformation to a component, one can create and initialize an appropriate subclass of `GeometryTransformer` to perform the desired transformation, and then apply it using the static `transform` method of the utility class TransformGeometryContext:

```
ModelComponent comp;     // component to be transformed
GeometryTransformer gtr; // transformer to do the transforming

... instantiate and initialize the transformer ...

TransformGeometryContext.transform (comp, gtr, /*flags=*/0);
```

At present, the following subclasses of `GeometryTransformer` are available:

**RigidTransformer**

Implements rigid 3D transformations.

**AffineTransformer**

Implements affine 3D transformations.

**FemGeometryTransformer**

Implements a general transformation, using the deformation field induced by a finite element model.

TransformGeometryContext also supplies the following convenience methods to apply transformations to components or collections of components:

```
void transform (Iterable<TransformableGeometry>, GeometryTransformer, int);
void transform (TransformableGeometry[], GeometryTransformer, int);

void transform (TransformableGeometry, AffineTransform3dBase, int);
void transform (Iterable<TransformableGeometry>, AffineTransform3dBase, int);
void transform (TransformableGeometry[], AffineTransform3dBase, int);
```

The last three of these methods create an instance of either RigidTransformer or AffineTransformer for the supplied AffineTransform3dBase. In fact, most `TransformableGeometry` components implement their transformGeometry(X) method as follows:

```
public void transformGeometry (AffineTransform3dBase X) {
   TransformGeometryContext.transform (this, X, 0);
}
```

The FemGeometryTransformer class is derived from the class DeformationTransformer, which uses the single method getDeformation() to obtain deformation field information at a specified reference position:

```
void getDeformation (Vector3d p, Matrix3d F, Vector3d r)
```

If the deformation field is described by $\mathbf{x}' = f(\mathbf{x})$, then for a given reference position $\mathbf{r}$ (in undeformed coordinates), this method should return the deformed position $\mathbf{p} = f(\mathbf{r})$ and the deformation gradient

$$\mathbf{F} \equiv \frac{\partial f}{\partial \mathbf{x}} \tag{4.22}$$

evaluated at $\mathbf{r}$.

FemGeometryTransformer obtains $f(\mathbf{x})$ and $\mathbf{F}$ from a FemModel3d (see Section 6) whose elemental rest positions enclose the components to be transformed, using the fact that a finite element model creates an implied piecewise-smooth deformation field as it deviates from its rest position. For each reference point $\mathbf{r}$ needed by the transformation process, `FemGeometryTransformer` finds the FEM element whose rest volume encloses $\mathbf{r}$, and then uses the corresponding shape function coordinates to compute $f(\mathbf{x})$ and $\mathbf{F}$ from the element's deformation. If the FEM model does *not* enclose $\mathbf{r}$, the nearest element is used to determine the shape function coordinates (however, this calculation becomes less accurate and meaningful the farther $\mathbf{r}$ is from the FEM model). Transformations based on FEM models are further illustrated in Section 4.7.2, and by Figure 4.19. Full details on ArtiSynth finite element models are given in Section 6.

Besides FEM models, there are numerous other ways to create deformation fields, such as radial basis functions, thin plate splines, etc. Some of these may be more appropriate for a particular application and can provide deformations that are globally smooth (as opposed to piecewise smooth). It should be relatively easy for an application to create its own subclass of `DeformationTransformer` to implement the deformation of choice by overriding the single getDeformation() method.

### 4.7.2 Example: the FemModelDeformer class

An FEM-based geometric transformation of a `MechModel` is facilitated by the class FemModelDeformer, which one can add to an existing `RootModel` to transform the geometry of a `MechModel` already located within that `RootModel`. `FemModelDeformer` subclasses FemModel3d to include a FemGeometryTransformer, and provides some utility methods to support the transformation process.

A `FemModelDeformer` can be added to a `RootModel` by adding the following code fragment to the end of the `build()` method:

```
public void build (String[] args) {

   ... build the model ...

   FemModelDeformer deformer =
      new FemModelDeformer ("deformer", this, /*maxn=*/10);
   addModel (deformer);
   // add a control panel (this is optional)
   addControlPanel (deformer.createControlPanel());
}
```

When the deformer is created, its constructor searches the specified `RootModel` to locate the first top-level `MechModel`. It then creates a hexahedral FEM grid around this model, with `maxn` specifying the number of cells along the maximum dimension. Material and mass properties of the model are computed automatically from the underlying `MechModel` dimensions (but can be altered if necessary after construction). When added to the `RootModel`, the deformer becomes another top-level model that can be deformed independently of the `MechModel` to create the required deformation field, as described below. It also supplies application-defined menu items that appear under the Application menu in the ArtiSynth menu bar (see Section 5.5). The deformer's `createControlPanel()` can also be used to create a `ControlPanel` (Section 5.1) that controls the visibility of the FEM model and the dynamic behavior of both it and the `MechModel`.

An example is defined in

```
artisynth.demos.tutorial.DeformedJointedCollide
```

where the `JointedCollide` example of Section 8.1.3 is extended to include a `FemModelDeformer` using the code described above.

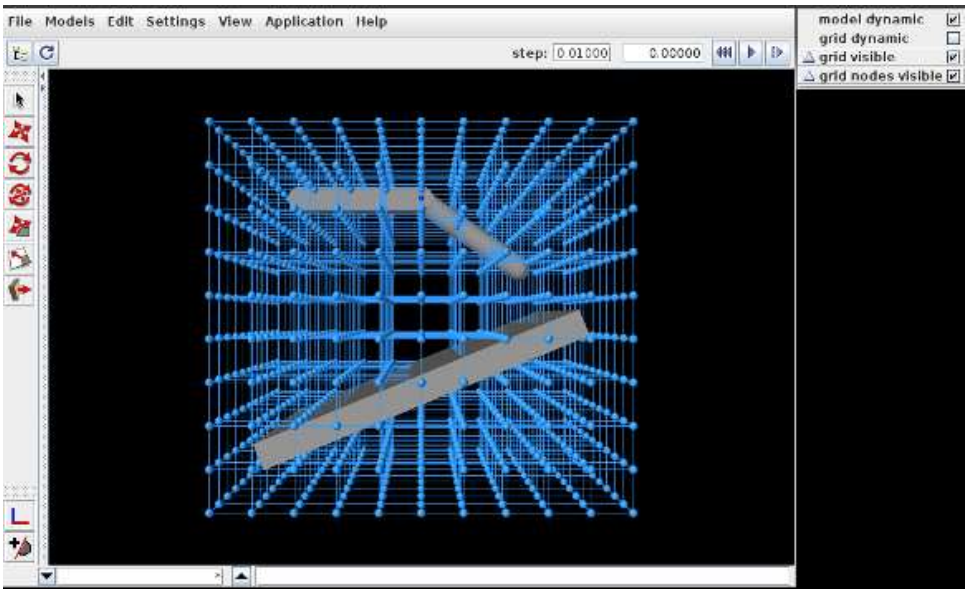



Figure 4.18: The `DeformedJointedCollide` example initially loaded into ArtiSynth.

To load this example in ArtiSynth, select All demos > tutorial > DeformedJointedCollide from the Models menu. The model should load and initially appear as in Figure 4.18, where the control panel appears on the right.

The underlying `MechModel` (or "model") can now be transformed by first deforming the FEM model (or "grid") and then using the resulting deformation field to effect the transformation:

1. Make the model non-dynamic and the grid dynamic by unchecking model dynamic and checking grid dynamic in the control panel. This means that when simulation is run, the model will be inert while the grid will respond physically.

2. Deform the grid using simulation. One easy way to do this is to fix certain nodes, generally on or near the grid boundary, and then move some of these using the translation or transrotator tool while simulation is running. To fix a set of nodes, select them in the viewer, choose Edit properties ... from the right-click context menu, and then uncheck their dynamic property. To easily select a large number of nodes without also selecting model components or grid elements, one can specify `FemNode` in the selection filter widget. (See the sections "Transformer Tools" and "Selection filtering" in the ArtiSynth User Interface Guide.)

3. After the grid has been deformed, choose deform from the Application menu in the ArtiSynth toolbar to transform the model. Afterwards, the transformation can be undone by choosing undo, and the grid can be reset by choosing reset grid.

4. To run the deformed model after the transformation, it should again be made dynamic by checking model dynamic in the control panel. The itself grid can be made non-dynamic, and it and/or its nodes can be made invisible by unchecking grid visible and/or grid nodes visible in the control panel.

The result of a possible deformation is shown in Figure 4.19.

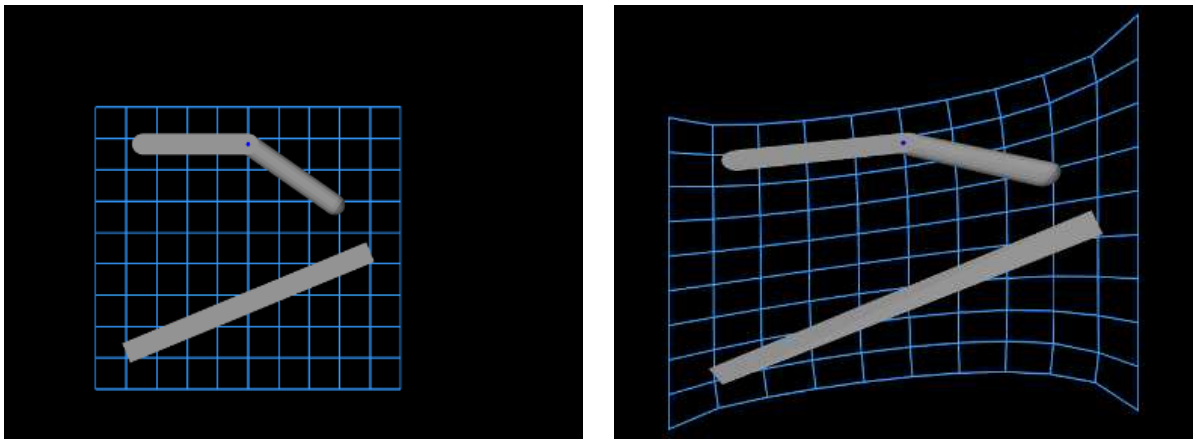


Figure 4.19: Deformation achieved in `DeformedJointedCollide`, showing both the model and grid (using an orthographic view) before and after the deformation.

> Note: `FemModelDeformer` is not intended to provide a general purpose solution to nonlinear geometric transformations. Rather, it is mainly intended to illustrate the capabilities of GeometryTransformer and the TransformableGeometry interface.

### 4.7.3 Implementation and behavior

As indicated above, the management of transforming the geometry for one or more components is handled by the TransformGeometryContext class. The transform operations themselves are carried out by this class's apply() method, which (a) assembles all the components that need to be transformed, (b) performs the actual transform operations, (c) invokes any required updating actions on other components, and finally (d) notifies parent components of the change using a GeometryChangeEvent.

To support this, ArtiSynth components which implement TransformableGeometry must also supply the methods

```
public void addTransformableDependencies (
   TransformGeometryContext context, int flags);

public void transformGeometry (
   GeometryTransformer gtr, TransformGeometryContext context, int flags);
```

The first method, addTransformableDependencies(context,flags), is called in step (a) to add to the context any additional components which should be transformed along with this component. This includes any descendants which should be transformed, since the transformation of these should not generally be done within

`transformGeometry(gtr,context,flags)`.

The second method, transformGeometry(gtr,context,flags), is called in step (b) to perform the actual transformation on this component. It should use the supplied geometry transformer `gtr` to transform its attributes, as well as `context` to query what other components are also being transformed and to request any needed updating actions to be called in step (c). The `flags` argument specifies conditions associated with the transformation, which at the moment may currently include:

**TG_SIMULATING**

The system is currently simulating, and therefore it may not be desirable to transform all attributes;

**TG_ARTICULATED**

Rigid body articulation constraints should be enforced as the transform proceeds.

Full details for all this are given in the documentation for TransformGeometryContext.

The transforming behavior of a component is up to its implementing method, but the following rules are generally observed:

1. Transformable descendants are also transformed, by using `addTransformableDependencies()` to add them to the context as described above;
2. When the nodes of an FEM model (Section 6) are transformed, the rest positions are also transformed if the system is not simulating (i.e., if the `TG_SIMULATING` flag is not set). This also causes the mass of the adjacent nodes to be recomputed from the densities of the adjacent elements;
3. When dynamic components are transformed, any attachments and constraints associated with them are updated appropriately, but only if the system is not simulating. Non-transforming dynamic components that are attached to transforming components as slaves are generally updated so as to follow the transforming components to which they are attached.

### 4.7.4 Use in model registration

Transforming model geometry can obviously be used as part of the process of creating subject-specific biomechanical and anatomical models. However, registration will generally require more that geometric transformation, since other properties, such as material stiffnesses, densities, and maximum forces will generally need to be adjusted as well. As a specific example, when applying a geometric transform to a model containing `AxialSprings`, the `restLength` properties of the springs will be unchanged, whereas the initial lengths may be, resulting in a different applied forces and physical behavior.

## 4.8 General component arrangements

As discussed in Section 1.1.5 and elsewhere, a `MechModel` provides a number of predefined child components for storing particles, rigid bodies, springs, constraints, and other components. However, applications are not required to store their components in these containers, and may instead create any sort of component arrangement desired.

For example, suppose that one wishes to create a biomechanical model of both the right and left human arms, consisting of bones, point-to-point muscles, and joints. The standard containers supplied by MechModel would require that all the components be placed within the following containers:

```
rigidBodies          // all bones
axialSprings         // all point-to-point muscles
connectors           // all joints
```

Instead of this, one may wish to set up a more appropriate component hierarchy, such as

```
leftArm              // left-arm components
   bones             //   left bones
   muscles           //   left muscles
   joints            //   left joints
rightArm             // right-arm components
   bones             //   right bones
   muscles           //   right muscles
   joints            //   right joints
```

To do this, the application `build()` method can create the necessary hierarchy and then populate it with whatever components are desired. Before simulation begins (or whenever the model structure is changed), the `MechModel` will recursively traverse the component hierarchy and update whatever internal structures are needed to run the simulation.

### 4.8.1 Container components

The generic class ComponentList can be used as a container for model components of a specific type. It can be created using a declaration of the form

```
ComponentList<Particle> list = new ComponentList<Type> (Type.class, name);
```

where `Type` is the class type of the components and `name` is the name for the container. Once the container is created, it should be added to the `MechModel` (or another internal container) and populated with child components of the specified type. For example,

```
MechModel mech;
...
ComponentList<Particle> parts =
   new ComponentList<Particle> (Particle.class, "parts");
ComponentList<Frame> frames =
   new ComponentList<Frame> (Frame.class, "frames");

// add containers to the mech model
mech.add (parts);
mech.add (frames);
```

creates two containers named `"parts"` and `"frames"` for storing components of type `Particle` and `Frame`, respectively, and adds them to a `MechModel` referenced by `mech`.

In addition to `ComponentList`, applications may use several "specialty" container types which are subclasses of `ComponentList`:

**RenderableComponentList**

A subclass of `ComponentList`, that has its *own* set of render properties which can be inherited by its children. This can be useful for compartmentalizing render behavior. Note that it is *not* necessary to store renderable components in a `RenderableComponentList`; components stored in a `ComponentList` will be rendered too.

**PointList**

A `RenderableComponentList` that is optimized for rendering points, and also contains its own `pointDamping` property that can be inherited by its children.

**PointSpringList**

A `RenderableComponentList` designed for storing point-based springs. It contains a `material` property that specifies a default axial material that can be used by its children.

**AxialSpringList**

A `PointSpringList` that is optimized for rendering two-point axial springs.

If necessary, it is relatively easy to define one's own customized list by subclassing one of the other list types. One of the main reasons for doing so, as suggested above, is to supply default properties to be inherited by the list's descendants.

A component list which declares `ModelComponent` as its type can be used to store `any` type of component, including other component lists. This allows the creation of arbitrary component hierarchies. Generally either `ComponentList<ModelComponent>` or `RenderableComponentList<ModelComponent>` are best suited to implement hierarchical groupings.

### 4.8.2 Example: a net formed from balls and springs

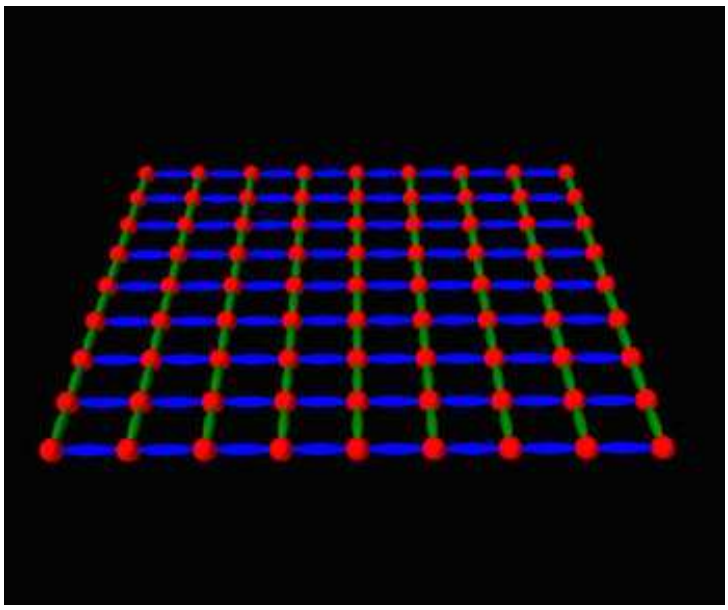


Figure 4.20: NetDemo model loaded into ArtiSynth.

A simple example showing an arrangement of balls and springs formed into a net is defined in

```
artisynth.demos.tutorial.NetDemo
```

The `build()` method and some of the supporting definitions for this example are shown below.

```
   protected double stiffness = 1000.0;   // spring stiffness
   protected double damping = 10.0;       // spring damping
   protected double maxForce = 5000.0;    // max force with excitation = 1
   protected double mass = 1.0;           // mass of each ball
   protected double widthx = 20.0;        // width of the net along x
   protected double widthy = 20.0;        // width of the net along y
   protected int numx = 8;                // num balls along x
   protected int numy = 8;                // num balls along y

   // custom component containers
   protected MechModel mech;
   protected PointList<Particle> balls;
   protected ComponentList<ModelComponent> springs;
   protected RenderableComponentList<AxialSpring> greenSprings;
   protected RenderableComponentList<AxialSpring> blueSprings;

   private AxialSpring createSpring (
      PointList<Particle> parts, int idx0, int idx1) {
      // create a "muscle" spring connecting particles indexed by 'idx0' and
      // 'idx1' in the list 'parts'
      Muscle spr = new Muscle (parts.get(idx0), parts.get(idx1));
      spr.setMaterial (new SimpleAxialMuscle (stiffness, damping, maxForce));
      return spr;
   }

```

```
26    public void build (String[] args) {
27
28       // create MechModel and add to RootModel
29       mech = new MechModel ("mech");
30       mech.setGravity (0, 0, -980.0);
31       mech.setPointDamping (1.0);
32       addModel (mech);
33
34       int nump = (numx+1)*(numy+1); // nump = total number of balls
35
36       // create custom containers:
37       balls = new PointList<Particle> (Particle.class, "balls");
38       springs = new ComponentList<ModelComponent>(ModelComponent.class,"springs");
39       greenSprings = new RenderableComponentList<AxialSpring> (
40          AxialSpring.class, "greenSprings");
41       blueSprings = new RenderableComponentList<AxialSpring> (
42          AxialSpring.class, "blueSprings");
43
44       // create balls in a grid pattern and add to the list 'balls'
45       for (int i=0; i<=numx; i++) {
46          for (int j=0; j<=numy; j++) {
47             double x = widthx*(-0.5+i/(double)numx);
48             double y = widthy*(-0.5+j/(double)numy);
49             Particle p = new Particle (mass, x, y, /*z=*/0);
50             balls.add (p);
51             // fix balls along the edges parallel to y
52             if (i == 0 || i == numx) {
53                p.setDynamic (false);
54             }
55          }
56       }
57
58       // connect balls by green springs parallel to y
59       for (int i=0; i<=numx; i++) {
60          for (int j=0; j<numy; j++) {
61             greenSprings.add (
62                createSpring (balls, i*(numy+1)+j, i*(numy+1)+j+1));
63          }
64       }
65       // connect balls by blue springs parallel to x
66       for (int j=0; j<=numy; j++) {
67          for (int i=0; i<numx; i++) {
68             blueSprings.add (
69                createSpring (balls, i*(numy+1)+j, (i+1)*(numy+1)+j));
70          }
71       }
72
73       // add containers to the mechModel
74       springs.add (greenSprings);
75       springs.add (blueSprings);
76       mech.add (balls);
77       mech.add (springs);
78
79       // set render properties for the components
80       RenderProps.setLineColor (greenSprings, new Color(0f, 0.5f, 0f));
81       RenderProps.setLineColor (blueSprings, Color.BLUE);
82       RenderProps.setSphericalPoints (mech, widthx/50.0, Color.RED);
83       RenderProps.setCylindricalLines (mech, widthx/100.0, Color.BLUE);
84    }
```

The `build()` method begins by creating a `MechModel` in the usual way (lines 29-30). It then creates a net composed of a set of balls arranged as a uniform grid in the x-y plane, connected by a set of green colored springs running parallel to the y axis and a set of blue colored springs running parallel to the x axis. These are arranged into a component hierarchy of the form

```
    balls
    springs
       greenSprings
       blueSprings
```

using containers created at lines 37-42. The balls are then created and added to `balls` (lines 45-56), the springs are created and added to `greenSprings` and `blueSprings` (lines 59-71), and the containers are added to the `MechModel` at lines 74-77. The balls along the edges parallel to the y axis are fixed. Render properties are set at lines 80-83, with the colors for `greenSprings` and `blueSprings` being explicitly set to dark green and blue.

> `MechModel`, along with other classes derived from `ModelBase`, enforces *reference containment*. That means that all components referenced by components within a `MechModel` must themselves be contained within the `MechModel`. This condition is checked whenever a component is added directly to a `MechModel` or one of its ancestors. This means that the components must be added to the `MechModel` in an order that ensures any referenced components are already present. For example, in the `NetDemo` example above, adding the particle list *after* the spring list would generate an error.

To run this example in ArtiSynth, select All demos > tutorial > NetDemo from the Models menu. The model should load and initially appear as in Figure 4.20. Running the model will cause the net to fall and sway under gravity. When the ArtiSynth navigation panel is opened and expanded, the component hierarchy will appear as in Figure 4.21. While the standard `MechModel` containers are still present, they are not displayed by default because they are empty.

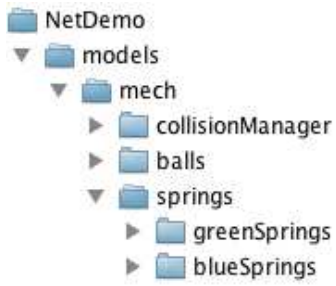



Figure 4.21: NetDemo components displayed in the ArtiSynth navigation panel.

### 4.8.3 Adding containers to other models

In addition to `MechModel`, application-defined containers can be added to any model that inherits from ModelBase. This includes `RootModel` and `FemModel`. However, at the present time, components added to such containers won't do anything, other than be rendered in the viewer if they are Renderable.

## 4.9 Custom Joints

If desired, it is also possible for applications to create their own custom joints. This involves creating two custom classes: a *coupling* class that does the constraint computations, and a *joint* class that wraps around it and allows it to connect connectable bodies. Details on how to create these classes are given in Sections 4.9.3 and 4.9.4, after some explanation of the constraint mechanism that underlies joint operation.

This section assumes that the reader is highly familiar with spatial kinematics and dynamics.

### 4.9.1 Joint constraints

To create a custom joint, it is necessary to understand how joints are implemented. The basic function of a joint is to constraint the set of poses allowed by the joint transform $\mathbf{T}_{CD}$ that relates frame C to D. To do this, the joint imposes

restrictions on the six-dimensional spatial velocity $\hat{\mathbf{v}}_{CD}$ that describes how frame C is moving with respect to D. This restriction is done using a set of bilateral constraints $\mathbf{G}_k$ and (in some cases) unilateral constraints $\mathbf{N}_l$, each of which is a $1 \times 6$ matrix that acts to restrict a single degree of freedom in $\hat{\mathbf{v}}_{CD}$ (see Section A.5 for a review of spatial velocities and forces). Bilateral constraints take the form of an equality,

$$\mathbf{G}_k \hat{\mathbf{v}}_{CD} = 0, \tag{4.23}$$

while unilateral constraints take the form of an inequality:

$$\mathbf{N}_l \hat{\mathbf{v}}_{CD} \geq 0. \tag{4.24}$$

These constraints are defined with respect to frame C, and their total number equals the number of DOFs that the joint *removes*. A joint's main computational task is to specify these constraints. ArtiSynth then uses its own knowledge of how frames C and D are connected to bodies A and B (or ground, if there is no body B) to map the individual $\mathbf{G}_k$ and $\mathbf{N}_l$ onto the joint's full bilateral and unilateral constraint matrices $\mathbf{G}_\text{J}$ and $\mathbf{N}_\text{J}$ (see (3.10) and (3.11)) that restrict the body velocities.

As a simple example, consider a cylindrical joint, in which C is free to rotate about and translate along the $z$ axis of D but other motions are restricted. Letting $\mathbf{v}$ and $\boldsymbol{\omega}$ denote the translational and rotational components of $\hat{\mathbf{v}}_{CD}$, such that

$$\hat{\mathbf{v}}_{CD} = \begin{pmatrix} \mathbf{v} \\ \boldsymbol{\omega} \end{pmatrix},$$

we see that the constraints must enforce

$$v_x = v_y = \omega_x = \omega_y = 0. \tag{4.25}$$

This can be accomplished using four constraints defined as follows:

$$\begin{aligned} \mathbf{G}_0 &= (1,0,0,0,0,0) \\ \mathbf{G}_1 &= (0,1,0,0,0,0) \\ \mathbf{G}_2 &= (0,0,0,1,0,0) \\ \mathbf{G}_3 &= (0,0,0,0,1,0). \end{aligned}$$

Constraining velocities is a necessary but insufficient condition for constraint enforcement. Because of numerical errors, as well as the fact that constraints are often nonlinear, the joint transform $\mathbf{T}_{CD}$ will tend to drift away from the joint restrictions as the simulation proceeds, leading to the error $\mathbf{T}_{err}$ described at the end of Section 3.3.1. These errors are corrected during a *position correction* at the end of every simulation time step: the joint first *projects* $\mathbf{T}_{CD}$ onto the nearest valid constraint surface to form $\mathbf{T}_{GD}$, and $\mathbf{T}_{err}$ is then computed from

$$\mathbf{T}_{err} = \mathbf{T}_{CG} = \mathbf{T}_{GD}^{-1} \mathbf{T}_{CD}. \tag{4.26}$$

Because $\mathbf{T}_{err}$ is (usually) small, we can approximate it as a twist $\hat{\boldsymbol{\delta}}_{err}$ representing a small displacement from frame G (which lies on the constraint surface) to frame C. During the position correction, ArtiSynth adjusts the pose of C relative to D in order to try and bring $\hat{\boldsymbol{\delta}}_{err}$ to zero. To do this, it uses an estimate of the *distance* $d_k$ along each constraint to the constraint surface, which it computes from the dot product of $\mathbf{G}_k$ and $\hat{\boldsymbol{\delta}}_{err}$:

$$d_k = \mathbf{G}_k \hat{\boldsymbol{\delta}}_{err}. \tag{4.27}$$

ArtiSynth assembles these distances into a composite distance vector $\mathbf{d}_g$ for *all* bilateral constraints, and then uses the system solver to find a displacement $\delta\mathbf{q}$ of the system coordinates that satisfies

$$\mathbf{G}(\mathbf{q})\, \delta\mathbf{q} = -\mathbf{d}_g.$$

Adding $\delta\mathbf{q}$ to the system coordinates $\mathbf{q}$ then reduces the constraint errors. While for nonlinear constraints several steps may be required to bring the error to 0, the process usually converges quickly.

Unlike bilateral constraints, unilateral constraints are one-sided, and take effect, or are *engaged*, only when $\mathbf{T}_{CD}$ encounters an inadmissible region. The constraint then acts to prevent further penetration into the region, via the velocity restriction (4.24), and also to push $\mathbf{T}_{CD}$ *out* of the inadmissible region, using a position correction analogous to that used for bilateral constraints.

Whether or not a unilateral constraint is engaged is determined by its *engaged* value $E_l$, which takes one of the three values: $\{0, 1, -1\}$, and is updated by the joint implementation as the simulation proceeds. A value of 0 means that the constraint is not engaged, and will *not* be included in the joint's unilateral constraint matrix $\mathbf{N}_\text{J}$. Otherwise, if $E_l$ is 1

or $-1$, then the constraint is engaged and will be included in $\mathbf{N}_J$, using $\mathbf{N}_l$ if $E_l = 1$, or its negative $-\mathbf{N}_l$ if $E_l = -1$. $E_l$ therefore defines a *sign* for the constraint. General details on how unilateral constraints should be engaged or disengaged are discussed in Section 4.9.2.

A common use of unilateral constraints is to implement limits on joint coordinate values; this also illustrates the utility of $E_l$. For example, the cylindrical joint mentioned above may have two coordinates, $z$ and $\theta$, describing the translation and rotation along and about the D frame's $z$ axis. Now suppose we wish to bound $z$, such that

$$z_{\min} \leq z \leq z_{\max}. \tag{4.28}$$

When these limits are violated, a unilateral constraint can be engaged to limit motion along the $z$ axis. A constraint $\mathbf{N}_z$ that will do this is

$$\mathbf{N}_z = (0,0,1,0,0,0).$$

Whenever $z \leq z_{\min}$, using $\mathbf{N}_z$ in (4.24) will ensure that $\dot{\mathbf{z}} \geq 0$ and hence $z$ will not fall further below the lower bound. On the other hand, when $z \geq z_{\max}$, we want to employ $-\mathbf{N}_z$ in (4.24) to ensure that $\dot{\mathbf{z}} \leq 0$. In other words, lower bounds can be enforced by engaging $\mathbf{N}_l$ with $E_l = 1$, while upper bounds can be enforced with $E_l = -1$.

As with bilateral constraints, constraining velocities is not sufficient; it is also necessary to correct position errors, particularly as unilateral constraints are typically not engaged until the inadmissible region is violated. The position correction procedure is the same: for each engaged unilateral constraint, find a distance $d_l$ along its constraint direction that indicates the distance to the inadmissible region boundary. ArtiSynth will then assemble these $d_l$ into a composite distance vector $\mathbf{d}_n$ for all unilataral constraints, and solve for a system coordinate displacement $\delta\mathbf{q}$ that satisfies

$$\mathbf{N}(\mathbf{q})\,\delta\mathbf{q} \geq -\mathbf{d}_n. \tag{4.29}$$

Because of the inequality direction in (4.29), distances $d_l$ representing penetration into a inadmissible region must be *negative*. For coordinate bounds such as (4.28), we need to use $d_l = z - z_{\min}$ for the lower bound and $d_l = z_{\max} - z$ for the upper bound. Alternatively, if the unilateral constraint has been included into the projection of C onto G and hence into the error term $\hat{\delta}_{err}$, $d_l$ can be computed from

$$d_l = E_l \mathbf{N}_l\, \hat{\delta}_{err}. \tag{4.30}$$

Note that unilateral constraints for coordinate limits are *not* usually incorporated into the $G$ projection; more on this details are given in Section 4.9.4.

As simulation proceeds, the velocity limits imposed by (4.23) and (4.24) are enforced by bilateral and unilateral constraint forces $\mathbf{f}_k$ and $\mathbf{f}_l$ whose magnitudes are given by

$$\mathbf{f}_k = \mathbf{G}_k^T \lambda_k, \qquad \mathbf{f}_l = \mathbf{N}_l^T \theta_l, \tag{4.31}$$

where $\lambda_k$ and $\theta_l$ are the Lagrange multipliers computed by the mechanical system solver (and are components of $\lambda$ or $\theta$ in (1.8) and (1.6)). $\mathbf{f}_k$ and $\mathbf{f}_l$ are 6 DOF spatial force vectors, or *wrenches* (Section A.5), which like $\mathbf{G}_k$ and $\mathbf{N}_l$ are expressed in frame C. Because $\mathbf{G}_k^T$ and $\mathbf{N}_l^T$ are proportional to spatial wrenches, they are often themselves referred to as *constraint wrenches*, and within the ArtiSynth codebase are described by a Wrench object.

### 4.9.2 Unilateral constraint engagement

As mentioned above, joints which implement unilateral constraints must monitor $\mathbf{T}_{CD}$ and the joint coordinates as the simulation proceeds and decide when to engage or disengage them.

Engagement is usually easy: a constraint is engaged whenever $\mathbf{T}_{CD}$ or a joint coordinate hits an inadmissible region. The constraint $\mathbf{N}_l$ is itself a spatial vector that is (locally) perpendicular to the inadmissible region boundary, and $E_l$ is chosen to be either 1 or $-1$ so that $E_l\mathbf{N}_l$ is directed *away* from the inadmissible region. In the remainder of this section, we shall assume $E_l = 1$.

To disengage, we usually want to ensure that the joint configuration is out of the inadmissible region. If we have a constraint $\mathbf{N}_l$, with a local distance $d_l$ defined such that $d_l < 0$ implies the joint is inside the region, then we are out of the region when $d_l > 0$. However, if we use only this as the disengagement criterion, we may encounter a problem known as *chattering*, illustrated in Figure 4.22 (left). An inadmissible region is shown in gray, with a unilateral constraint $N_1$ perpendicular to its boundary. As simulation proceeds, the joint lands inside the region at an initial point A at the lower left, at a (negative) distance $d_1$ from the boundary. Ideally the position correction step will move the configuration by $-d_1$ so that it lands right on the region boundary. However, numerical errors and nonlinearities may

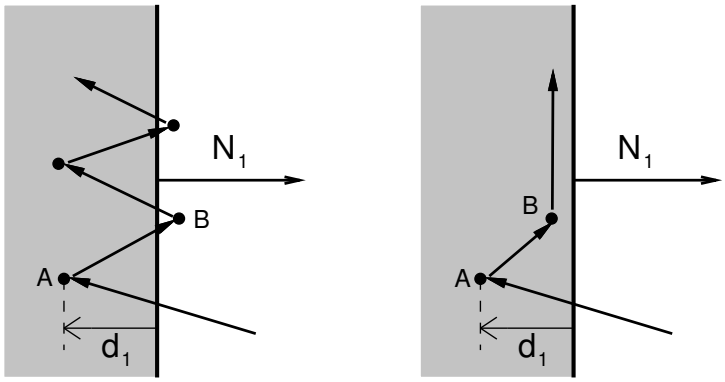



Figure 4.22: Left: A joint configuration at A inside an inadmissible region (gray) is pushed further outside the region than intended B, so that it reenters the region during the next simulation step, resulting in chattering. Right: a deadband solution, in which the position correction is reduced sufficiently so that the joint configuration remains inside the region.

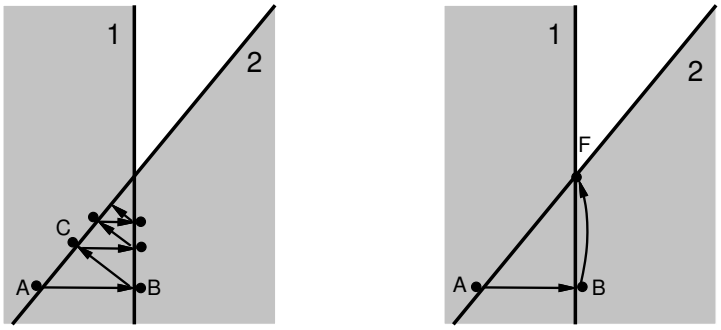



Figure 4.23: Left: constraint oscillation, in which a joint configuration starting at A oscillates between two overlapping inadmissible regions 1 and 2 whose boundaries are not perpendicular. Right: ensuring that constraints stay engaged for more than one simulation step allows the solver to quickly determine a stable solution at F, where the region boundaries intersect.

mean that in fact it lands *outside* the region, at point B. Then on the next step it reenters the region, only to again be pushed out, etc.

ArtiSynth implements two solutions to chattering. One is to implement a *deadband*, so that instead of correcting the position by $-d_1$, we correct it by $-d_1 - p_{\text{tol}}$, where $p_{\text{tol}}$ is a *penetration tolerance*. This means that the correction will try to leave the joint inside the region by a small amount (Figure 4.22, right) so that chattering is suppressed. The penetration tolerance used depends on the constraint type. Those that are primarily linear use the value of the penetrationTol property, while those that are primarily rotary use the value of the rotaryLimitTol property; both of these are exported as inheritable properties by both MechModel and BodyConnector, with default values computed from the model's overall dimensions.

The second chattering solution is to disengage only when the joint is actively moving away from the region, as determined by $\dot{d}_l > 0$. The disengagement criteria then become

$$d_l > 0 \quad \text{and} \quad \dot{d}_l > 0. \tag{4.32}$$

$\dot{d}_l$ is called the *contact speed* and can be computed from

$$\dot{d}_l = \mathbf{N}_l \hat{\mathbf{v}}_{CD}. \tag{4.33}$$

Another problem, which we call *constraint oscillation*, can occur when we are near two or more overlapping inadmissible regions whose boundaries are not perpendicular. See Figure 4.23 (left), which shows two overlapping regions 1 and 2. The joint starts at point A, inside region 1 but just outside region 2. Since only constraint 1 is engaged, the position correction moves it toward the boundary of 1, overshooting and landing at point B outside of 1 but inside region 2. Constraint 2 now engages, moving the joint to C, where it is past the boundary of 2 but inside 1 again. While the example in the figure converges to the corner where the boundaries of 1 and 2 meet, convergence may be slow and may

be prevented entirely by external forcing. While the mechanisms that prevent chattering *may* also prevent oscillation, we find that an additional measure is useful, which is to simply require that *a constraint must be engaged for at least two simulation steps*. The result is shown in Figure 4.23 (right), where after the joint arrives at B, constraint 1 remains engaged along with constraint 2, and the subsequent solution takes the joint directly to point F at the corner where 1 and 2 meet.

### 4.9.3 Implementing a custom joint

All of the work of computing joint constraints and coordinates, as described in the previous sections, is done within a "coupling" class which is a subclass of RigidBodyCoupling. An instance of this is then embedded within a "joint" class (which is a subclass of JointBase) that supports connections with other bodies, provides rendering, exports various properties, and allows the joint to be attached to a `MechModel`.

For purposes of this discussion, we will assume that these two custom classes are called `CustomCoupling` and `CustomJoint`, respectively. The implementation of `CustomJoint` can be as simple as this:

```
import artisynth.core.mechmodels.ConnectableBody;
import artisynth.core.mechmodels.JointBase;
import maspack.matrix.RigidTransform3d;

public class CustomJoint extends JointBase {

   public CustomJoint() {
      setCoupling (new CustomCoupling());
   }

   public CustomJoint (
      ConnectableBody bodyA, ConnectableBody bodyB, RigidTransform3d TDW) {
      this(); // call the default constructor
      setBodies(bodyA, bodyB, TDW);
   }
}
```

This creates an instance of `CustomCoupling` and sets it to the (inherited) `myCoupling` attribute inside the default constructor (which is where this normally should be done). Another constructor is provided which uses `setBodies()` to create a joint that is attached to two bodies with the D frame specified in world coordinates. In practice, a joint may also export some properties (such as joint coordinates), provide additional constructors, and implement rendering; one should examine the source code for some existing joints.

### 4.9.4 Implementing a custom coupling

Implementing a custom coupling constitutes most of the effort in creating a custom joint, since the coupling is responsible for maintaining the constraints $\mathbf{G}_k$ and $\mathbf{N}_l$ that enforce the joint behavior.

Before proceeding, we discuss the coordinate frame in which these constraints are situated. It is often convenient to describe joint constraints with respect to frame C, since rotations are frequently centered there. However, the joint transform $\mathbf{T}_{CD}$ usually contains errors (Section 3.3.1) due to a combination of simulation error and possible joint compliance. To determine these errors, we project C onto another frame G, defined to be the nearest to C that is consistent with the bilateral (and possibly some unilateral) constraints. (This is done by the `projectToConstraints()` method, described below). The result is a joint transform $\mathbf{T}_{GD}$ that is "error free" with respect to bilateral constraints and also consistent with the coordinates (if supported). This makes it convenient to formulate constraints with respect to frame G instead of C, and so this is the convention ArtiSynth uses. In particular, the `updateConstraints()` method, described below, uses $\mathbf{T}_{GD}$, together with the spatial velocity $\hat{\mathbf{v}}_{GD}$ describing the motion of G with respect to C.

An actual custom coupling implementation involves subclassing RigidBodyCoupling and then implementing five abstract methods, the outline of which looks like this:

```
import maspack.matrix.*;
import maspack.spatialmotion.*;
import maspack.util.*;

class CustomCoupling extends RigidBodyCoupling {
```

```
   public CustomCoupling() {
      super();
   }

   // Initialize the constraints and coordinates.
   public void initializeConstraints () {
      ...
   }

   // If coordinates are implemented, set TCD from the supplied coordinates.
   public void coordinatesToTCD (RigidTransform3d TCD, VectorNd coords) {
      ...
   }

   // If coordinates are implemented, set their values from TCD.
   public void TCDToCoordinates (VectorNd coords, RigidTransform3d TCD) {
      ...
   }

   // Project TCD to the nearest transform TGD admissible to the
   // bilateral constraints, and maybe some unilateral constraints.
   public void projectToConstraints (
      RigidTransform3d TGD, RigidTransform3d TCD, boolean updateCoords) {
      ...
   }

   // Update the constraint wrenches, and maybe the engaged
   // and distance settings for some unilateral constraints.
   public void updateConstraints (
      RigidTransform3d TGD, RigidTransform3d TCD, Twist errC,
      Twist velGD, boolean updateEngaged) {
      ...
   }
}
```

The implementations of these methods are now described in detail.

#### initializeConstraints()

This method has the signature

```
public void initializeConstraints ()
```

and is called in the coupling's superclass constructor (i.e., the constructor for `RigidBodyCoupling`). It is responsible for initializing the coupling's constraints and (if supported) coordinates.

Constraints are added using one of the two superclass methods:

```
RigidBodyConstraint addConstraint (int flags)

RigidBodyConstraint addConstraint (int flags, Wrench wrench)
```

Each creates a new RigidBodyConstraint and adds it to the coupling's constraint list. *flags* is an or-ed combination of the following flags defined in RigidBodyConstraint:

**BILATERAL**

Constraint is bilateral (i.e., an equality). If `BILATERAL` is not specified, the constraint is considered unilateral.

**ROTARY**

Constraint primarily restricts rotary motion. If it is unilateral, the joint's rotaryLimitTol property is used for its penetration tolerance.

**LINEAR**

Constraint primarily restricts translational motion. If it is unilateral, the joint's penetrationTol property is used for its penetration tolerance.

**CONSTANT**

Constraint is constant with respect to frame G. This flag is set automatically if the constraint is created using `addConstraint(flags,wrench)`.

**LIMIT**

Constraint is used to enforce limits for a coordinate. This flag is set automatically if the constraint is specified as the limit constraint for a coordinate.

The method `addConstraint(flags,wrench)` takes an additional Wrench argument specifying the (presumed constant) value of the constraint with respect to frame G, and sets the `CONSTANT` flag just described.

Coordinates are added similarly using using one of the two superclass methods:

```
   CoordinateInfo addCoordinate ()

   CoordinateInfo addCoordinate (
      double min, double max, int flags, RigidBodyConstraint limCon)
```

Each creates a new `CoordinateInfo` object (which is an inner class of RigidBodyCoupling), and adds it to the coupling's coordinate list. In the second method, `min` and `max` give the initial range limits, and `limCon`, if non-`null`, specifies a unilateral constraint (previously created using `addConstraint`) for enforcing the limits and causes that constraint's `LIMIT` to be set. The argument `flags` is reserved for future use and should be set to 0. If not specified, the default coordinate limits are $(-\inf, \inf)$.

The implementation of `initializeConstraints()` for a coupling that implements a hinge type joint might look like this:

```
public void initializeConstraints () {
   addConstraint (BILATERAL|LINEAR, new Wrench (1, 0, 0, 0, 0, 0));
   addConstraint (BILATERAL|LINEAR, new Wrench (0, 1, 0, 0, 0, 0));
   addConstraint (BILATERAL|LINEAR, new Wrench (0, 0, 1, 0, 0, 0));
   addConstraint (BILATERAL|ROTARY, new Wrench (0, 0, 0, 1, 0, 0));
   addConstraint (BILATERAL|ROTARY, new Wrench (0, 0, 0, 0, 1, 0));
   addConstraint (ROTARY, new Wrench (0, 0, 0, 0, 0, 1));

   addCoordinate (-Math.PI, Math.PI, 0, getConstraint(5));
}
```

Six constraints are specified, with the sixth being a unilateral constraint that enforces the limits on the single coordinate describing the rotation angle. Each constraint and coordinate has an integer index giving the location in its list, in the order it was added. This index can be used to later retrieve the `RigidBodyConstraint` or `CoordinateInfo` object for the constraint or coordinate, using the methods `getConstraint(idx)` or `getCoordinateInfo(idx)`.

> Because `initializeConstraints()` is called in the superclass constructor, member attributes for the custom coupling will not yet be initialized when it is first called. Therefore, the method should not depend on the initial values of non-static member variables. `initializeConstraints()` can also be called later to rebuild the constraints if some defining setting is changed.

#### coordinatesToTCD()

This method has the signature

```
   public void coordinatesToTCD (RigidTransform3d TCD, VectorNd coords)
```

and is called when needed by the system. If coordinates are supported, then the transform $\mathbf{T}_{CD}$ should be set from the coordinate values supplied in `coords`, and returned in the argument `TCD`. Otherwise, this method should do nothing.

#### TCDToCoordinates()

This method has the signature

```
public void TCDToCoordinates (VectorNd coords, RigidTransform3d TCD)
```

and is called when needed by the system. It is the inverse of `coordinatesToTCD()`: if coordinates are supported, then their values should be set from the joint transform $\mathbf{T}_{CD}$ supplied by `TCD` and returned in `coords`. Otherwise, this method should do nothing.

When calling this method, it is assumed that `TCD` is "legal" with respect to the joint's constraints (as defined by `projectToConstraints()`, described next). If this is not the case, then `projectToConstraints()` should be called instead.

One issue that can arise is when a coordinate represents an angle $\phi$ that has a range greater than $2\pi$. In that case, a common strategy is to compute a nominal value for $\phi$, and then add or subtract $2\pi$ from it until the resulting value is as close as possible to the *current* value for the angular coordinate. This allows the angle to wrap through its entire range. To implement this, one can use the method

```
double nearestAngle (double phi)
```

in the coordinate's `CoordinateInfo` object, which finds the angle equivalent to `phi` that is nearest to the current coordinate value.

> Coordinate values computed by this method should *not* be clipped to their ranges.

#### projectToConstraints()

This method has the signature

```
public void projectToConstraints (
  RigidTransform3d TGD, RigidTransform3d TCD, VectorNd coords)
```

and is called when needed by the system. It is responsible for projecting the joint transform $\mathbf{T}_{CD}$ (supplied by `TCD`) onto the nearest transform $\mathbf{T}_{GD}$ that is valid for the *bilateral* constraints, and returning this in `TGD`. If coordinates are supported and `coords` is non-`null`, then the coordinate values corresponding to $\mathbf{T}_{GD}$ should also be computed and returned in `coords`. The easiest way to do this is to simply call `TCDToCoordinates(TGD,coords)`, although in some cases it may be computationally cheaper to compute both the coordinates and the projection at the same time.

Optionally, the coupling may also extend the projection to include unilateral constraints that are *not* associated with coordinate limits. In particular, this should be done for constraints for which is it desired to have the constraint error included in $\mathbf{T}_{err}$ and the corresponding argument `errC` that is passed to `updateConstraints()`.

#### updateConstraints()

This method has the signature

```
public void updateConstraints (
  RigidTransform3d TGD, RigidTransform3d TCD, Twist errC,
  Twist velGD, boolean updateEngaged)
```

and is usually called once per simulation time step. It is responsible for:

- Updating the values of all non-constant constraint wrenches, along with their derivatives;
- If `updateEngaged` is `true`, updating the engaged and distance attributes for all unilateral constraints not associated with a coordinate limit.

The method supplies several arguments:

- `TGD`, containing the idealized joint transform $\mathbf{T}_{GD}$ from frame G to D produced by calling `projectToConstraints()`.
- `TCD`, containing the joint transform $\mathbf{T}_{CD}$ from frame C to D and supplied for legacy reasons.
- `errC`, representing the (hopefully small) error transform $\mathbf{T}_{err}$ from frame C to G as a spatial twist vector.
- `velGD`, giving the spatial velocity $\hat{\mathbf{v}}_{GD}$ of frame G with respect to D, as seen in G; this is needed to compute wrench derivatives.
- `updateEngaged`, which requests the updating of unilateral engaged and distance attributes as describe above.

If the coupling supports coordinates, their values will be updated before the method is called so as to correspond to $\mathbf{T}_{GD}$. If needed, a coordinate's value may be obtained from the value attribute of its `CoordinateInfo` object, which may in turn be obtained using `getCoordinateInfo(idx)`. Likewise, `ConstraintInfo` objects for each constraint may be obtaining using `getConstraint(idx)`.

Constraint wrenches correspond to $\mathbf{G}_k$ and $\mathbf{N}_l$ in Section 4.9.1. These, along with their derivatives $\dot{\mathbf{G}}_k$ and $\dot{\mathbf{N}}_l$, are described by the wrenchG and dotWrenchG attributes of each constraint's RigidBodyConstraint object, and may be managed by a variety of methods:

```
Wrench getWrenchG()    // return the reference to wrenchG
void setWrenchG (
   double fx, double fy, double fx, double mx, double my, double mx)
void setWrenchG (Vector3d f, Vector3d m) // either f or m may be null
void setWrenchG (Wrench wr)
void negateWrenchG()
void zeroWrenchG()

Wrench getDotWrenchG() // return the reference to dotWrenchG
void setDotWrenchG (
   double fx, double fy, double fx, double mx, double my, double mx)
void setDotWrenchG (Vector3d f, Vector3d m) // either f or m may be null
void setDotWrenchG (Wrench wr)
void negateDotWrenchG()
void zeroDotWrenchG()
```

dotWrenchG is used in computing the time derivative terms **g** and **n** that appear in (3.9) and (1.6). While these improve the computational accuracy of the simulation, their effect is often small, and so in practice one may be able to omit computing dotWrenchG and instead leave its value as 0.

Wrench information must also be computed for unilateral constraints which implement coordinate limits. While it is not necessary to compute the distance and engaged attributes for these constraints (this is done automatically), it *is* necessary to ensure that the wrench's magnitude is compatible with the coordinate's speed. More precisely, if the coordinate is given by $\phi$, then the limit wrench $\mathbf{N}_l$ must have a magnitude such that

$$\dot{\phi} = \mathbf{N}_l \hat{\mathbf{v}}_{GD}. \tag{4.34}$$

As mentioned above, if `updateEngaged` is `true`, the engaged and distance attributes for unilateral constraints not associated with coordinate limits must be updated. These correspond to $E_l$ and $d_l$ in Section 4.9.1, and are contained in the constraint's RigidBodyConstraint object and may be queried using the methods

```
double getDistance()
void setDistance (double d)

int getEngaged()
void setEngaged (int engaged)
```

It is up to `updateConstraints()` to compute the distance, with a negative value denoting penetration into the inadmissible region. If `projectToConstraints()` is implemented so as to account for the constraint, then $\mathbf{T}_{GD}$ will be projected out of the inadmissible region and the distance will be implicitly present $\mathbf{T}_{err}$ and so can be recovered by taking the dot product of the constraint wrench and `velGD`:

```
RigidBodyConstraint cons = getConstraint (3); // assume constraint index is 3
...
double dist = cons.getWrench().dot (velGD);
```

Otherwise, if the constraint is not accounted for in `projectToConstraints()`, the distance must be obtained by other means.

To update engaged, one may use the general convenience method

```
void updateEngaged (
   RigidBodyConstraint cons, double dist,
   double dmin, double dmax, Twist velGD)
```

which sets engaged according to the rules of Section 4.9.2, for an inadmissible region corresponding to `dist` < `dmin` or `dist` > `dmax`. The upper or lower bounds may be removed by setting `dmin` to `-inf` or `max` to `inf`, respectively.

### 4.9.5 Example: a simple custom joint

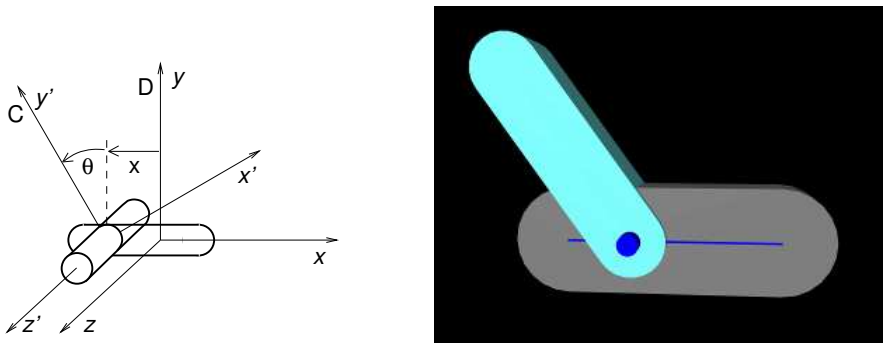



Figure 4.24: Coordinate frames for the `CustomJoint` (left) and the associated `CustomJointDemo` (right).

An simple model illustrating custom joint creation is provided by

```
artisynth.demos.tutorial.CustomJointDemo
```

This implements a joint class defined by `CustomJoint` (also in the package `artisynth.demos.tutorial`), which is actually just a simple implementation of SlottedHingeJoint (Section 3.4.4). Certain details are omitted, such as exporting coordinate values and ranges as properties, and other things are simplified, such as the rendering code. One may consult the source code for `SlottedHingeJoint` to obtain a more complete example.

This section will focus on the implementation of the joint coupling, which is created as an inner class of `CustomJoint` called `CustomCoupling` and which (like all couplings) extends RigidBodyCoupling. The joint itself creates an instance of the coupling in its default constructor, exactly as described in Section 4.9.3.

The coupling allows two DOFs (Figure 4.24, left): translation along the $x$ axis of D (described by the coordinate $x$), and rotation about the $z$ axis of D (described by the coordinate $\theta$), with $\mathbf{T}_{CD}$ related to the coordinates by (3.25). It implements `initializeConstraints()` as follows:

```
public void initializeConstraints () {
   addConstraint (BILATERAL|LINEAR);
   addConstraint (BILATERAL|LINEAR, new Wrench(0, 0, 1, 0, 0, 0));
   addConstraint (BILATERAL|ROTARY, new Wrench(0, 0, 0, 1, 0, 0));
   addConstraint (BILATERAL|ROTARY, new Wrench(0, 0, 0, 0, 1, 0));
   addConstraint (LINEAR);
   addConstraint (ROTARY, new Wrench(0, 0, 0, 0, 0, 1));

   addCoordinate (-1, 1, 0, getConstraint(4)); // x
   addCoordinate (-2*Math.PI, 2*Math.PI, 0, getConstraint(5)); // theta
}
```

Six constraints are added using `addConstraint()`: two linear bilaterals to restrict translation along the $y$ and $z$ axes of D, two rotary bilaterals to restrict rotation about the $x$ and $y$ axes of D, and two unilaterals to enforce limits on $x$ and $\theta$. Four of the constraints are constant in frame G, and so are initialized with a wrench value. The other two are not constant in G and so will need to be updated in `updateConstraints()`. The coordinates for $x$ and $\theta$ are added at the end, using `addCoordinate()`, with default joint limits and a reference to the constraint that will enforce the limit.

The implementations for `coordinatesToTCD()` and `TCDToCoordinates()` simply use (3.25) to compute $\mathbf{T}_{CD}$ from the coordinates, or vice versa:

```
public void coordinatesToTCD (RigidTransform3d TCD, VectorNd coords) {
   double x = coords.get (X_IDX);
   double theta = coords.get (THETA_IDX);
   TCD.setIdentity();
   TCD.p.x = x;
   double c = Math.cos (theta);
   double s = Math.sin (theta);
   TCD.R.m00 = c;
   TCD.R.m11 = c;
   TCD.R.m01 = -s;
   TCD.R.m10 = s;
}

public void TCDToCoordinates (VectorNd coords, RigidTransform3d TGD) {
   coords.set (X_IDX, TGD.p.x);
   double theta = Math.atan2 (TGD.R.m10, TGD.R.m00);
   coords.set (THETA_IDX, getCoordinateInfo(THETA_IDX).nearestAngle (theta));
}
```

`X_IDX` and `THETA_IDX` are constants defining the coordinate indices for $x$ and $\theta$. In `TCDToCoordinates()`, note the use of the `CoordinateInfo` method `nearestAngle()`, as discussed in Section 4.9.4.

Projecting $\mathbf{T}_{CD}$ onto the error-free $\mathbf{T}_{GD}$ is done by `projectToConstraints()`, implemented as follows:

```
public void projectToConstraints (
   RigidTransform3d TGD, RigidTransform3d TCD, VectorNd coords) {
   TGD.R.set (TCD.R);
   TGD.R.rotateZDirection (Vector3d.Z_UNIT);
   TGD.p.x = TCD.p.x;
   TGD.p.y = 0;
   TGD.p.z = 0;
   if (coords != null) {
      TCDToCoordinates (coords, TGD);
   }
}
```

The translational projection is easy - the y and z components of the translation vector `p` are simply zeroed out. To project the rotation `R`, we use its `rotateZDirection()` method, which applies the shortest rotation aligning its $z$ axis with $(0,0,1)$. The residual rotation will be a rotation in the $x$-$y$ plane. If `coords` is non-`null` and needs to be computed, we simply call `TCDToCoordinates()`.

Lastly, the implementation for `projectToConstraints()` is as follows:

```
public void updateConstraints (
   RigidTransform3d TGD, RigidTransform3d TCD, Twist errC,
   Twist velGD, boolean updateEngaged) {
   RigidBodyConstraint cons = getConstraint(0); // constraint along y
   double s = TGD.R.m10;  // sin (theta)
   double c = TGD.R.m00;  // cos (theta)
   // constraint wrench along y is constant in D but needs to be
   // transformed to G
   cons.setWrenchG (s, c, 0, 0, 0, 0);
   // derivative term:
   double dotTheta = velGD.w.z;
   cons.setDotWrenchG (c*dotTheta, -s*dotTheta, 0, 0, 0, 0);

   // update x limit constraint if necessary
```

```
      cons = getConstraint(4);
      if (cons.getEngaged() != 0) {
         // constraint wrench along x, transformed to G, is (-c, s, 0)
         cons.setWrenchG (c, -s, 0, 0, 0, 0);
         cons.setDotWrenchG (-s*dotTheta, -c*dotTheta, 0, 0, 0, 0);
      }
      // theta limit constraint is constant; no need to do anything
   }
```

Only constraints 0 and 4 need to have their wrenches updated, since the rest are constant, and we obtain their constraint objects using `getConstraint(idx)`. Constraint 0 restricts motion along the $y$ axis in D, and while this is constant in D, it is *not* constant in G, which is where the wrench must be situated. The $y$ axis of D as seen in G is the given by the second row of the rotation matrix of $\mathbf{T}_{GD}$, which from (3.25) we see is $(s,c,0)^T$, where $s \equiv \sin(\theta)$ and $c \equiv \cos(\theta)$. We obtain $s$ and $c$ directly from `TGD`, since this has been projected to lie on the constraint surface; alternatively, we could compute them from $\theta$. To obtain the wrench derivative, we note that $\dot{s} = c\dot{\theta}$ and $\dot{c} = -c\dot{\theta}$, and that $\dot{\theta}$ is simply the $z$ component of the angular velocity of $G$ with respect to $D$, or `velGD.w.z`. The wrench and its derivative are set using the constraint's `setWrenchG()` and `setDotWrenchG()` methods.

The other non-constant constraint is the limit constraint for the $x$ coordinate, which is the $x$ axis of D as seen in G. This is updated similarly, although we only need to do so if the limit constraint is engaged. Since all unilateral constraints are coordinate limits, there is no need to update their distance or engaged attributes as this is done automatically by the system.

# Chapter 5

# Simulation Control

This section describes different devices which an application may use to control the simulation. These include *control panels* to allow for the interactive adjustment of properties, as well as *agents* which are applied every time step. Agents include *controllers* and *input probes* to supply and modify input parameters at the beginning of each time step, and *monitors* and *output probes* to observe and record simulation results at the end of each time step.

## 5.1 Control Panels

A *control panel* is an editing panel that allows for the interactive adjustment of component properties.

It is always possible to adjust component properties through the GUI by selecting one or more components and then choosing Edit properties ... in the right-click context menu. However, it may be tedious to repeatedly select the required components, and the resulting panels present the user with *all* properties common to the selection. A control panel allows an application to provide a customized editing panel for selected properties.

> If an application wishes to adjust an attribute that is not exported as a property of some ArtiSynth component, it is often possible to create a *custom property* for the attribute in question. Custom properties are described in Section 5.2.

### 5.1.1 General principles

Control panels are implemented by the ControlPanel model component. They can be set up within a model's `build()` method by creating an instance of `ControlPanel`, populating it with widgets for editing the desired properties, and then adding it to the root model using the latter's addControlPanel() method. A typical code sequence looks like this:

```
ControlPanel panel = new ControlPanel ("controls");
... add widgets ...
addControlPanel (panel);
```

There are various `addWidget()` methods available for adding widgets and components to a control panel. Two of the most commonly used are:

```
addWidget (HasProperties host, String propPath)
addWidget (HasProperties host, String propPath, double min, double max)
```

The first method creates a widget to control the property located by `propPath` with respect to the property's `host` (which is usually a model component or a composite property). Property paths are discussed in Section 1.4.2, and can consist of a simple property name, a composite property name, or, for properties located in descendant components, a component path followed by a colon '`:`' and then a simple or compound property name.

The second method creates a slider widget to control a property whose value is a single number, with an initial numeric range given by `max` and `min`.

> The first method will *also* create a slider widget for a property whose value is a number, if the property has a default range and slider widgets are not disabled in the property's declaration. However, the second method allows the slider range to be explicitly specified.

Both methods also return the widget component itself, which is an instance of LabeledComponentBase, and assign the widget a text label that is the same as the property's name. In some situations, it is useful to assign a different text label (such as when creating two widgets to control the same property in two different components). For those cases, the methods

```
addWidget (
   String label, HasProperties host, String propPath)
addWidget (
   String label, HasProperties host, String propPath, double min, double max)
```

allow the widget's text label to be explicitly specified.

Sometimes, it is desirable to create a widget that controls that same property across two or more host components (as illustrated in Section 5.1.3 below). For that, the methods

```
addWidget (String propPath, HasProperties... hosts)
addWidget (String propPath, double min, double max, HasProperties... hosts)

addWidget (
   String label, String propPath, HasProperties... hosts)
addWidget (
   String label, String propPath, double min, double max, HasProperties... hosts)
```

allow multiple hosts to be specified using the variable length argument list `hosts`.

Other flavors of `addWidget()` also exist, as described in the API documentation for ControlPanel. In particular, any type of `Swing` or `awt` component can be added using the method

```
addWidget (Component comp)
```

> Control panels can also be created interactively using the GUI; see the section "Control Panels" in the ArtiSynth User Interface Guide.

### 5.1.2 Example: Creating a simple control panel

An application model showing a control panel is defined in

`artisynth.demos.tutorial.SimpleMuscleWithPanel`

This model simply extends `SimpleMuscle` (Section 4.5.2) to provide a control panel for adjusting gravity, the mass and color of the box, and the muscle excitation. The class definition, excluding `include` statements, is shown below:

```
1  public class SimpleMuscleWithPanel extends SimpleMuscle {
2     ControlPanel panel;
3
4     public void build (String[] args) throws IOException {
5
6        super.build (args);
7
8        // add control panel for gravity, rigid body mass and color, and excitation
9        panel = new ControlPanel("controls");
10       panel.addWidget (mech, "gravity");
11       panel.addWidget (mech, "rigidBodies/box:mass");
12       panel.addWidget (mech, "rigidBodies/box:renderProps.faceColor");
13       panel.addWidget (new JSeparator());
14       panel.addWidget (muscle, "excitation");
```

```
15
16        addControlPanel (panel);
17     }
18  }
```

The `build()` method calls `super.build()` to create the model used by `SimpleMuscle`. It then proceeds to create a `ControlPanel`, populate it with widgets, and add it to the root model (lines 8-15). The panel is given the name `"controls"` in the constructor (line 8); this is its component name and is also used as the title for the panel's window frame. A control panel does not need to be named, but if it is, then that name must be unique among the control panels.

Lines 9-11 create widgets for three properties located relative to the `MechModel` referenced by `mech`. The first is the `MechModel`'s `gravity`. The second is the `mass` of the box, which is a component located relative to `mech` by the path name (Section 1.1.3) `"rigidBodies/box"`. The third is the box's face color, which is the sub-property `faceColor` of the box's `renderProps` property.

Line 12 adds a `JSeparator` to the panel, using the `addWidget()` method that accepts general components, and line 13 adds a widget to control the `excitation` property for `muscle`.

> It should be noted that there are different ways to specify target properties in `addWidget()`. First, component paths may contain numbers instead of names, and so the box's mass property could be specified using `"rigidBodies/0:mass"` instead of `"rigidBodies/box:mass"` since the box's number is 0. Second, if a reference to a subcomponent is available, one can specify properties directly with respect to that, instead of indicating the subcomponent in the property path. For example, if the box was referenced by a variable `body`, then one could use the construction
>
> ```
> panel.addWidget (body, "mass");
> ```
>
> in place of
>
> ```
> panel.addWidget (mech, "rigidBodies/box:mass");
> ```

To run this example in ArtiSynth, select All demos > tutorial > SimpleMuscleWithPanel from the Models menu. The demo will appear with the control panel shown in Figure 5.1, allowing the displayed properties to be adjusted interactively by the user while the model is either stationary or running.


Figure 5.1: Control panel created by the model `SimpleMuscleWithPanel`. Each of the property widgets consists of a text label followed by a field displaying the property's value. A and B identify the icons for the inheritable properties gravity and faceColor, indicating whether their values have has been explicitly set (A) or inherited from an ancestor component (B).

As described in Section 1.4.3, some properties are *inheritable*, meaning that their values can either be set explicitly within their host component, or inherited from the equivalent property in an ancestor component. Widgets for inheritable properties include an icon in the left margin indicating whether the value is explicitly set (square icon) or inherited from an ancestor (triangular icon) (Figure 5.1). These settings can be toggled by clicking on the icon. Changing an *explicit* setting to *inherited* will cause the property's value to be changed to that of the nearest ancestor component, or to the property's default value if no ancestor component contains an equivalent property.

### 5.1.3 Example: Controlling properties in multiple components

It is sometimes useful to create a property widget that adjusts the same property across several different components at the same time. This can be done using the `addWidget()` methods that accept multiple hosts. An application model

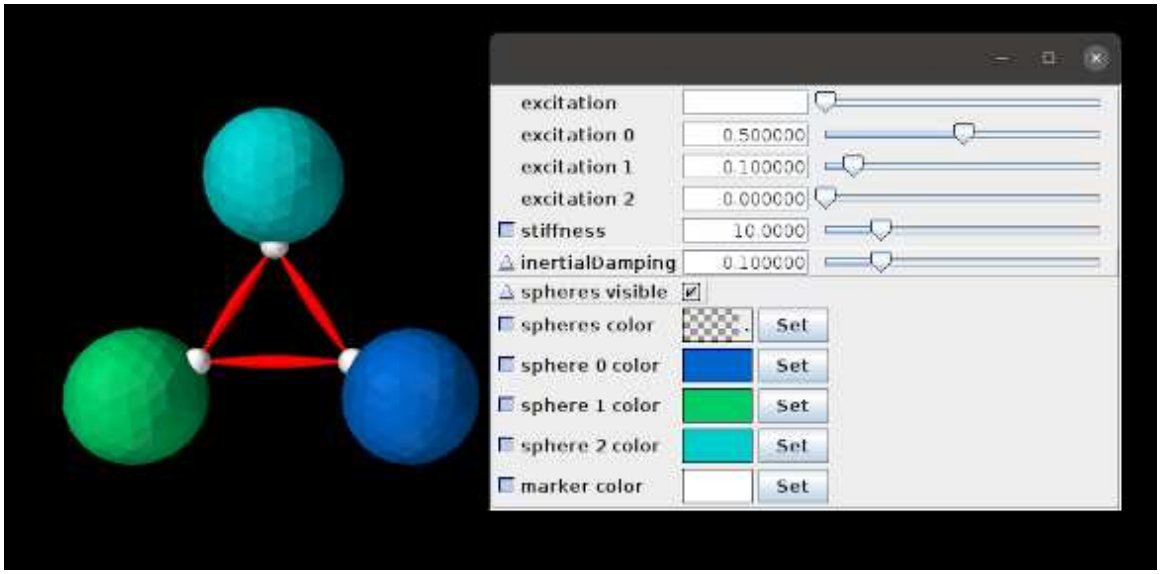



Figure 5.2: Model and control panel created by `ControlPanelDemo`.

demonstrating this is defined in

`artisynth.demos.tutorial.ControlPanelDemo`

and shown in Figure 5.2. The model creates a simple arrangement of three spheres, connected by point-to-point muscles and with collisions enabled (Chapter 8), whose dynamic behavior can be adjusted using the control panel. Selected rendering properties can also be changed using the panel. The class definition, excluding `include` statements, is shown below:

```
public class ControlPanelDemo extends RootModel {

   double myStiffness = 10.0; // default spring stiffness
   double myMaxForce = 100.0; // excitation force multiplier
   double DTOR = Math.PI/180; // degrees to radians

   // Create and attach a simple muscle with default parameters between p0 and p1
   Muscle attachMuscle (String name, MechModel mech, Point p0, Point p1) {
      Muscle mus = new Muscle (name);
      mus.setMaterial (
         new SimpleAxialMuscle (myStiffness, /*damping=*/0, myMaxForce));
      mech.attachAxialSpring (p0, p1, mus);
      return mus;
   }

   public void build (String[] args) {
      // create a mech model with zero gravity
      MechModel mech = new MechModel ("mech");
      addModel (mech);
      mech.setGravity (0, 0, 0);
      mech.setInertialDamping (0.1); // add some damping

      double density = 100.0;
      double particleMass = 50.0;

      // create three spheres, each with a different color, along with a marker
      // to attach a spring to, and arrange them roughly around the origin
      RigidBody sphere0 = RigidBody.createIcosahedralSphere (
         "sphere0", 0.5, density, /*ndivs=*/2);
      FrameMarker mkr0 = mech.addFrameMarker (sphere0, new Point3d(0, 0, 0.5));
      sphere0.setPose (new RigidTransform3d (1, 0, -0.5,  0, -DTOR*60, 0));
```

```
32        RenderProps.setFaceColor (sphere0, new Color(0f, 0.4f, 0.8f));
33        mech.addRigidBody (sphere0);
34
35        RigidBody sphere1 = RigidBody.createIcosahedralSphere (
36           "sphere1", 0.5, 1.5*density, /*ndivs=*/2);
37        FrameMarker mkr1 = mech.addFrameMarker (sphere1, new Point3d(0, 0, 0.5));
38        sphere1.setPose (new RigidTransform3d (-1, 0, -0.5,  0, DTOR*60, 0));
39        RenderProps.setFaceColor (sphere1, new Color(0f, 0.8f, 0.4f));
40        mech.addRigidBody (sphere1);
41
42        RigidBody sphere2 = RigidBody.createIcosahedralSphere (
43           "sphere2", 0.5, 1.5*density, /*ndivs=*/2);
44        FrameMarker mkr2 = mech.addFrameMarker (sphere2, new Point3d(0, 0, 0.5));
45        sphere2.setPose (new RigidTransform3d (0, 0, 1.1,  0, -DTOR*180, 0));
46        RenderProps.setFaceColor (sphere2, new Color(0f, 0.8f, 0.8f));
47        mech.addRigidBody (sphere2);
48
49        // create three muscles to connect the bodies via their markers
50        Muscle muscle0 = attachMuscle ("muscle0", mech, mkr1, mkr0);
51        Muscle muscle1 = attachMuscle ("muscle1", mech, mkr0, mkr2);
52        Muscle muscle2 = attachMuscle ("muscle2", mech, mkr1, mkr2);
53
54        // enable collisions between the spheres
55        mech.setDefaultCollisionBehavior (true, /*mu=*/0);
56
57        // render muscles as red spindles
58        RenderProps.setSpindleLines (mech, 0.05, Color.RED);
59        // render markers as white spheres. Note: unlike rigid bodies, markers
60        // normally have null render properties, and so we need to explicitly set
61        // their render properties for use in the control panel
62        RenderProps.setSphericalPoints (mkr0, 0.1, Color.WHITE);
63        RenderProps.setSphericalPoints (mkr1, 0.1, Color.WHITE);
64        RenderProps.setSphericalPoints (mkr2, 0.1, Color.WHITE);
65
66        // create a control panel to collectively set muscle excitation and
67        // stiffness, inertial damping, sphere visibility and color, and marker
68        // color. Muscle excitations and sphere colors can also be set
69        // individually.
70        ControlPanel panel = new ControlPanel();
71        panel.addWidget ("excitation", muscle0, muscle1, muscle2);
72        panel.addWidget ("excitation 0", "excitation", muscle0);
73        panel.addWidget ("excitation 1", "excitation", muscle1);
74        panel.addWidget ("excitation 2", "excitation", muscle2);
75        panel.addWidget (
76           "stiffness", "material.stiffness", muscle0, muscle1, muscle2);
77        panel.addWidget ("inertialDamping", sphere0, sphere1, sphere2);
78        panel.addWidget (
79           "spheres visible", "renderProps.visible", sphere0, sphere1, sphere2);
80        panel.addWidget (
81           "spheres color", "renderProps.faceColor", sphere0, sphere1, sphere2);
82        panel.addWidget ("sphere 0 color", "renderProps.faceColor", sphere0);
83        panel.addWidget ("sphere 1 color", "renderProps.faceColor", sphere1);
84        panel.addWidget ("sphere 2 color", "renderProps.faceColor", sphere2);
85        panel.addWidget (
86           "marker color", "renderProps.pointColor", mkr0, mkr1, mkr2);
87        addControlPanel (panel);
88     }
89  }
```

First, a `MechModel` is created with zero gravity and a default inertialDamping of 0.1 (lines 18-21). Next, three spheres are created, each with a different color and a frame marker attached to the top. These are positioned around the world coordinate origin, and oriented with their tops pointing toward the origin (lines 26-47), allowing them to be connected, via their markers, with three simple muscles (lines 49-52) created using the support method `attachMuscle()` (lines 8-14). Collisions (Chapter 8) are then enabled between all spheres (line 55).

At lines 62-64, we explicitly set the render properties for each marker; this is done because marker render properties are `null` by default and hence need to be explicitly set to enable widgets to be created for them, as discussed further below.

Finally, a control panel is created for various dynamic and rendering properties. These include the excitation and material stiffness for all muscles (lines 71 and 75); the inertialDamping, rendering visibility and faceColor for all spheres (lines 77, 79, and 81); and the rendering pointColor for all markers (lines 85). The panel also allows muscle excitations and sphere face colors to be set individually (lines 72-74 and 82-84). Some of the `addWidget()` calls explicitly set the label text for their widgets. For example, those controlling individual muscle excitations are labeled as "`excitation 0`", "`excitation 1`", and "`excitation 2`" to denote their associated muscle.

Some widgets are created for subproperties of their components (e.g., material.stiffness and renderProps.faceColor). The following caveats apply in these cases:

1. The parent property (e.g., renderProps for renderProps.faceColor) must be present in the component. While this will generally be true, in some instances the parent property may have a default value of `null`, and must be explicitly set to a non-`null` value before the widget is created (otherwise the subproperty will not be found and the widget creation will fail). This most commonly occurs for the `renderProps` property of smaller components, like particles and markers; in the example, render properties are explicitly assigned to the markers at lines 62-64.
2. If the parent property is *changed* for a particular host, then the widget will no longer be able to access the subproperty. For instance, in the example, if the material property for a muscle is changed (via either code or the GUI), then the widget controlling material.stiffness will no longer be able to access the stiffness subproperty for that muscle.

To run this example in ArtiSynth, select All demos > tutorial > ControlPanelDemo from the Models menu. When the model is run, the spheres will start to be drawn together by the muscles' intrinsic stiffness. Setting non-zero excitation values in the control panel will increase the attraction, while setting stiffness values will likewise affect the dynamics. The panel can also be used to make all the spheres invisible, or to change their colors, either separately or collectively.

> When a single widget is used to control a property across multiple host components, the property's value may not be the same for those components. When this occurs, the widget's value field displays either blank space (for numeric, string and enum values), a "?" (for boolean values), or a checked pattern (for color values). This is illustrated in Figure 5.2 for the "`excitation`" and "`spheres color`" widgets, since their individual values differ.

## 5.2 Custom properties

Because of the usefulness of properties in creating control panels and probes (Sections 5.1) and Section 5.4), model developers may wish to add their own properties, either to the root model, or to a custom component.

This section provides only a brief summary of how to define properties. Full details are available in the "Properties" section of the Maspack Reference Manual.

### 5.2.1 Adding properties to a component

As mentioned in Section 1.4, properties are class-specific, and are exported by a class through code contained in the class's definition. Often, it is convenient to add properties to the `RootModel` subclass that defines the application model. In more advanced applications, developers may want to add properties to a custom component.

The property definition steps are:

**Declare the property list:**

The class exporting the properties creates its own static instance of a PropertyList, using a declaration like

```
static PropertyList myProps = new PropertyList (MyClass.class, MyParent. ↩
 class);

@Override
public PropertyList getAllPropertyInfo() {
   return myProps;
}
```

where `MyClass` and `MyParent` specify the class types of the exporting class and its parent class. The `PropertyList` declaration creates a new property list, with a copy of all the properties contained in the parent class. If one does *not* want the parent class properties, or if the parent class does not have properties, then one would use the constructor PropertyList(MyClass.class) instead. If the parent class is an ArtiSynth model component (including the `RootModel`), then it will always have its own properties. The declaration of the method `getAllPropertyInfo()` exposes the property list to other classes.

**Add properties to the list:**

Properties can then be added to the property list, by calling the `PropertyList`'s add() method:

```
PropertyDesc add (String name, String description, Object defaultValue);
```

where `name` contains the name of the property, `description` is a comment describing the property, and `defaultValue` is an object containing the property's default value. This is done inside a static code block:

```
static {
   myProps.add ("stiffness", "spring stiffness", /*defaultValue=*/1);
   myProps.add ("damping", "spring damping", /*defaultValue=*/0);
}
```

Variations on the `add()` method exist for adding *read-only* or *inheritable* properties, or for setting various property options. Other methods allow the property list to be edited.

**Declare property accessor functions:**

For each property `propXXX` added to the property list, accessor methods of the form

```
void setPropXXX (TypeX value) {
   ...
}

TypeX getPropXXX() {
   TypeX value = ...
   return value;
}
```

must be declared, where `TypeX` is the value associated with the property.

> It is possible to specify different names for the accessor functions in the string argument `name` supplied to the `add()` method. Read-only properties only require a *get* accessor.

### 5.2.2 Example: a visibility property

An model illustrating the exporting of properties is defined in

`artisynth.demos.tutorial.SimpleMuscleWithProperties`

This model extends `SimpleMuscleWithPanel` (Section 4.5.2) to provide a custom property `boxVisible` that is added to the control panel. The class definition, excluding `include` statements, is shown below:

```
1  public class SimpleMuscleWithProperties extends SimpleMuscleWithPanel {
2
3     // internal property list; inherits properties from SimpleMuscleWithPanel
4     static PropertyList myProps =
5        new PropertyList (
6           SimpleMuscleWithProperties.class, SimpleMuscleWithPanel.class);
7
8     // override getAllPropertyInfo() to return property list for this class
9     public PropertyList getAllPropertyInfo() {
10       return myProps;
11    }
12
```

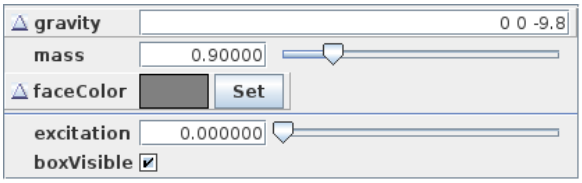



Figure 5.3: Control panel created by the model `SimpleMuscleWithProperties`, showing the newly defined property `boxVisible`.

```
13    // add new properties to the list
14    static {
15       myProps.add ("boxVisible", "box is visible", false);
16    }
17
18    // declare property accessors
19    public boolean getBoxVisible() {
20       return box.getRenderProps().isVisible();
21    }
22
23    public void setBoxVisible (boolean visible) {
24       RenderProps.setVisible (box, visible);
25    }
26
27    public void build (String[] args) throws IOException {
28
29       super.build (args);
30
31       panel.addWidget (this, "boxVisible");
32       panel.pack();
33    }
34 }
```

First, a property list is created for the application class `SimpleMuscleWithProperties.class` which contains a copy of all the properties from the parent class `SimpleMuscleWithPanel.class` (lines 4-6). This property list is made visible by overriding `getAllPropertyInfo()` (lines 9-11). The `boxVisible` property itself is then added to the property list (line 15), and accessor functions for it are declared (lines 19-25).

The `build()` method calls `super.build()` to perform all the model creation required by the super class, and then adds an additional widget for the `boxVisible` property to the control panel.

To run this example in ArtiSynth, select All demos > tutorial > SimpleMuscleWithProperties from the Models menu. The control panel will now contain an additional widget for the property `boxVisible` as shown in Figure 5.3. Toggling this property will make the box visible or invisible in the viewer.

## 5.3 Controllers and monitors

Application models can define custom *controllers* and *monitors* to control input values and monitor output values as a simulation progresses. Controllers are called every time step immediately before the `advance()` method, and monitors are called immediately after (Section 1.1.4). An example of controller usage is provided by ArtiSynth's inverse modeling feature, which uses an internal controller to estimate the actuation signals required to follow a specified motion trajectory.

More precise details about controllers and monitors and how they interact with model advancement are given in the ArtiSynth Reference Manual.

### 5.3.1 Implementation

Applications may declare whatever controllers or monitors they require and then add them to the root model using the methods addController() and addMonitor(). They can be any type of ModelComponent that implements the Controller

or Monitor interfaces. For convenience, most applications simply subclass the default implementations ControllerBase or MonitorBase and then override the necessary methods.

The primary methods associated with both controllers and monitors are:

```
public void initialize (double t0);

public void apply (double t0, double t1);

public boolean isActive();
```

`apply(t0, t1)` is the "business" method and is called once per time step, with `t0` and `t1` indicating the start and end times $t_0$ and $t_1$ associated with the step. `initialize(t0)` is called whenever an application model's state is set (or reset) at a particular time $t_0$. This occurs when a simulation is first started or after it is reset (with $t_0 = 0$), and also when the state is reset at a waypoint or during adaptive stepping.

`isActive()` controls whether a controller or monitor is active; if `isActive()` returns `false` then the `apply()` method will not be called. The default implementations ControllerBase and MonitorBase, via their superclass ModelAgentBase, also provide a `setActive()` method to control this setting, and export it as the property active. This allows controller and monitor activity to be controlled at run time.

> To enable or disable a controller or monitor at run time, locate it in the navigation panel (under the RootModel's controllers or monitors list), chose Edit properties ... from the right-click context menu, and set the active property as desired.

Controllers and monitors may be associated with a particular model (among the list of models owned by the root model). This model may be set or queried using

```
void setModel (Model m);

Model getModel();
```

If associated with a model, `apply()` will be called immediately before (for controllers) or after (for monitors) the model's `advance()` method. If not associated with a model, then `apply()` will be called before or after the advance of *all* the models owned by the root model.

Controllers and monitors may also contain *state*, in which case they should implement the relevant methods from the HasState interface.

Typical actions for a controller include setting input forces or excitation values on components, or specifying the motion trajectory of parametric components (Section 3.1.3). Typical actions for a monitor include observing or recording the motion profiles or constraint forces that arise from the simulation.

When setting the position and/or velocity of a dynamic component that has been set to be parametric (Section 3.1.3), a controller should not set its position or velocity directly, but should instead set its *target position* and/or *target velocity*, since this allows the solver to properly interpolate the position and velocity during the time step. The methods to set or query target positions and velocities for Point-based components are

```
setTargetPosition (Point3d pos);
Point3d getTargetPosition ();       // read-only

setTargetVelocity (Vector3d vel);
Vector3d getTargetVelocity ();      // read-only
```

while for Frame-based components they are

```
setTargetPosition (Point3d pos);
setTargetOrientation (AxisAngle axisAng);
setTargetPose (RigidTransform3d TFW);
Point3d getTargetPosition ();       // read-only
AxisAngle getTargetOrientation ();  // read-only
RigidTransform3d getTargetPose();   // read-only

setTargetVelocity (Twist vel);
Twist getTargetVelocity ();         // read-only
```

### 5.3.2 Example: A controller to move a point

A model showing an application-defined controller is defined in

```
artisynth.demos.tutorial.SimpleMuscleWithController
```

This simply extends `SimpleMuscle` (Section 4.5.2) and adds a controller which moves the fixed particle `p1` along a circular path. The complete class definition is shown below:

```
1  package artisynth.demos.tutorial;
2
3  import java.io.IOException;
4  import maspack.matrix.*;
5
6  import artisynth.core.modelbase.*;
7  import artisynth.core.mechmodels.*;
8  import artisynth.core.gui.*;
9
10 public class SimpleMuscleWithController extends SimpleMuscleWithPanel
11 {
12    private class PointMover extends ControllerBase {
13
14       Point myPnt;      // point to be moved
15       Point3d myPos0;   // initial point position
16
17       public PointMover (Point pnt) {
18          myPnt = pnt;
19          myPos0 = new Point3d (pnt.getPosition());
20       }
21
22       public void apply (double t0, double t1) {
23          double ang = Math.PI*t1/2;            // angle associated with time t1
24          Point3d pos = new Point3d (myPos0);
25          pos.x += 0.5*Math.sin (ang);          // compute position for t1 ...
26          pos.z += 0.5*(1-Math.cos (ang));
27          myPnt.setTargetPosition (pos);        // ... and the set point's target
28       }
29    }
30
31    public void build (String[] args) throws IOException {
32       super.build (args);
33
34       addController (new PointMover (p1));
35       // increase model bounding box for the viewer
36       mech.setBounds (-1, 0, -1, 1, 0, 1);
37    }
38
39 }
```

A controller called `PointMover` is defined by extending `ControllerBase` and overriding the `apply()` method. It stores the point to be moved in `myPnt`, and the initial position in `myPos0`. The `apply()` method computes a target position for the point that starts at `myPos0` and then moves in a circle in the *z*-*x* plane with an angular velocity of $\pi/2$ rad/sec (lines 22-28).

The `build()` method calls `super.build()` to create the model used by `SimpleMuscle`, and then creates an instance of `PointMover` to move particle `p1` and adds it to the root model (line 34). The viewer bounds are updated to make the circular motion more visible (line 36).

To run this example in ArtiSynth, select All demos > tutorial > SimpleMuscleWithController from the Models menu. When the model is run, the fixed particle `p1` will trace out a circular path in the *z*-*x* plane.

## 5.4 Probes

In addition to controllers and monitors, applications can also attach streams of data, known as *probes*, to input and output values associated with the simulation. Probes derive from the same base class ModelAgentBase as controllers and monitors, but differ in that

1. They are associated with an explicit time interval during which they are applied;
2. They can have an attached file for supplying input data or recording output data;
3. They are displayable in the ArtiSynth *timeline* panel.

A probe is applied (by calling its `apply()` method) only for time steps that fall within its time interval. This interval can be set and queried using the following methods:

```
setStartTime (double t0);
setStopTime (double t1);
setInterval (double t0, double t1);

double getStartTime();
double getStopTime();
```

The probe's attached file can be set and queried using:

```
setAttachedFileName (String filePath);
String getAttachedFileName ();
```

where `filePath` is a string giving the file's path name. If `filePath` is relative (i.e., it does not start at the file system root), then it is assumed to be relative to the *ArtiSynth working folder*, which can be queried and set using the methods

```
File ArtisynthPath.getWorkingFolder();
ArtisynthPath.setWorkingFolder (File folder);
```

of ArtisynthPath. The working folder can also be set from the ArtiSynth GUI by choosing File > Set working folder ....

> If not explicitly set within the application, the working folder will default to a system dependent setting, which may be the user's home folder, or the working folder of the process used to launch ArtiSyntn.

Details about the timeline display can be found in the section "The Timeline" in the ArtiSynth User Interface Guide.

There are two types of probe: *input probes*, which are applied at the beginning of each simulation step before the controllers, and *output probes*, which are applied at the end of the step after the monitors.

While applications are free to construct any type of probe by subclassing either InputProbe or OutputProbe, most applications utilize either NumericInputProbe or NumericOutputProbe, which explicitly implement streams of numeric data which are connected to properties of various model components. The remainder of this section will focus on numeric probes.

As with controllers and monitors, probes also implement a `isActive()` method that indicates whether or not the probe is active. Probes that are not active are not invoked. Probes also provide a `setActive()` method to control this setting, and export it as the property active. This allows probe activity to be controlled at run time.

> To enable or disable a probe at run time, locate it in the navigation panel (under the RootModel's inputProbes or outputProbes list), chose Edit properties ... from the right-click context menu, and set the active property as desired.
>
> Probes can also be enabled or disabled in the timeline, by either selecting the probe and invoking activate or deactivate from the right-click context menu, or by clicking the track mute button (which activates or deactivates all probes on that track).

### 5.4.1 Numeric probe structure

Numeric probes are associated with:

- *A vector of temporally-interpolated numeric data*;
- *One or more properties* to which the probe is bound and which are either set by the numeric data (input probes), or used to set the numeric data (output probes).

The numeric data is implemented internally by a NumericList, which stores the data as a series of vector-valued knot points at prescribed times $t_k$ and then interpolates the data for an arbitrary time $t$ using an interpolation scheme provided by Interpolation.

Some of the numeric probe methods associated with the interpolated data include:

```
int getVsize();                       // returns the size of the data vector
setInterpolationOrder (Order order); // sets the interpolation scheme
Order getInterpolationOrder();        // returns the interpolation scheme

VectorNd getData (double t);          // interpolates data for time t
NumericList getNumericList();         // returns the underlying NumericList
```

Interpolation schemes are described by the enumerated type `Interpolation.Order` and presently include:

**Step**

Values at time $t$ are set to the values of the closest knot point $k$ such that $t_k \leq t$.

**Linear**

Values at time $t$ are set by linear interpolation of the knot points $(k, k+1)$ such that $t_k \leq t \leq t_{k+1}$.

**Parabolic**

Values at time $t$ are set by quadratic interpolation of the knots $(k-1, k, k+1)$ such that $t_k \leq t \leq t_{k+1}$.

**Cubic**

Values at time $t$ are set by cubic Catmull interpolation of the knots $(k-1, \ldots, k+2)$ such that $t_k \leq t \leq t_{k+1}$.

Each property bound to a numeric probe must have a value that can be mapped onto a scalar or vector value. Such properties are know as *numeric properties*, and whether or not a value is numeric can be tested using NumericConverter.isNumeric(value).

By default, the total number of scalar and vector values associated with all the properties should equal the size of the interpolated vector (as returned by getVsize()). However, it is possible to establish more complex mappings between the property values and the interpolated vector. These mappings are beyond the scope of this document, but are discussed in the sections "General input probes" and "General output probes" of the ArtiSynth User Interface Guide.

### 5.4.2 Creating probes in code

This section discusses how to create numeric probes in code. They can also be created and added to a model graphically, as described in the section "Adding and Editing Numeric Probes" in the ArtiSynth User Interface Guide.

Numeric probes have a number of constructors and methods that make it relatively easy to create instances of them in code. For NumericInputProbe, there is the constructor

```
NumericInputProbe (ModelComponent c, String propPath, String filePath);
```

which creates a `NumericInputProbe`, binds it to a property located relative to the component `c` by `propPath`, and then attaches it to the file indicated by `filePath` and loads data from this file (see Section 5.4.4). The probe's start and stop times are specified in the file, and its vector size is set to match the size of the scalar or vector value associated with the property.

To create a probe attached to multiple properties, one may use the constructor

```
NumericInputProbe (ModelComponent c, String propPaths[], String filePath);
```

which binds the probe to multiple properties specified relative to `c` by `propPaths`. The probe's vector size is set to the sum of the sizes of the scalar or vector values associated with these properties.

For NumericOutputProbe, one may use the constructor

```
NumericOutputProbe (ModelComponent c, String propPath, String filePath, double
 sample);
```

which creates a `NumericOutputProbe`, binds it to the property `propPath` located relative to `c`, and then attaches it to the file indicated by `filePath`. The argument `sample` indicates the *sample time* associated with the probe, in seconds; a value of 0.01 means that data will be added to the probe every 0.01 seconds. If `sample` is specified as -1, then the sample time will default to the maximum step size associated with the root model.

To create an output probe attached to multiple properties, one may use the constructor

```
NumericOutputProbe (
   ModelComponent c, String propPaths[], String filePath, double sample);
```

> As the simulation proceeds, an output probe will accumulate data, but this data will not be saved to any attached file until the probe's `save()` method is called. This can be requested in the GUI for all probes by clicking on the Save button in the timeline toolbar, or for specific probes by selecting them in the navigation panel (or the timeline) and then choosing Save data in the right-click context menu.

Output probes created with the above constructors have a default interval of [0, 1]. A different interval may be set using `setInterval()`, `setStartTime()`, or `setStopTime()`.

### 5.4.3 Example: probes connected to SimpleMuscle

A model showing a simple application of probes is defined in

```
artisynth.demos.tutorial.SimpleMuscleWithProbes
```

This extends `SimpleMuscle` (Section 4.5.2) to add an input probe to move particle `p1` along a defined path, along with an output probe to record the velocity of the frame marker. The complete class definition is shown below:

```
1  package artisynth.demos.tutorial;
2
3  import java.io.IOException;
4  import maspack.matrix.*;
5  import maspack.util.PathFinder;
6
7  import artisynth.core.modelbase.*;
8  import artisynth.core.mechmodels.*;
9  import artisynth.core.probes.*;
10
11 public class SimpleMuscleWithProbes extends SimpleMuscleWithPanel
12 {
13    public void createInputProbe() throws IOException {
14       NumericInputProbe p1probe =
15          new NumericInputProbe (
16             mech, "particles/p1:targetPosition",
17             PathFinder.getSourceRelativePath (this, "simpleMuscleP1Pos.txt"));
18       p1probe.setName("Particle Position");
19       addInputProbe (p1probe);
20    }
21
22    public void createOutputProbe() throws IOException {
23       NumericOutputProbe mkrProbe =
```

```
24            new NumericOutputProbe (
25               mech, "frameMarkers/0:velocity",
26               PathFinder.getSourceRelativePath (this, "simpleMuscleMkrVel.txt"),
27               0.01);
28         mkrProbe.setName("FrameMarker Velocity");
29         mkrProbe.setDefaultDisplayRange (-4, 4);
30         mkrProbe.setStopTime (10);
31         addOutputProbe (mkrProbe);
32      }
33
34      public void build (String[] args) throws IOException {
35         super.build (args);
36
37         createInputProbe ();
38         createOutputProbe ();
39         mech.setBounds (-1, 0, -1, 1, 0, 1);
40      }
41
42   }
```

The input and output probes are added using the custom methods `createInputProbe()` and `createOutputProbe()`. At line 14, `createInputProbe()` creates a new input probe bound to the `targetPosition` property for the component `particles/p1` located relative to the `MechModel` `mech`. The same constructor attaches the probe to the file `simpleMuscleP1Pos.txt`, which is read to load the probe data. The format of this and other probe data files is described in Section 5.4.4. The method PathFinder.getSourceRelativePath() is used to locate the file relative to the source directory for the application model (see Section 2.6). The probe is then given the name `"Particle Position"` (line 18) and added to the root model (line 19).

Similarly, `createOutputProbe()` creates a new output probe which is bound to the `velocity` property for the component `particles/0` located relative to `mech`, is attached to the file `simpleMuscleMkrVel.txt` located in the application model source directory, and is assigned a sample time of 0.01 seconds. This probe is then named `"FrameMarker Velocity"` and added to the root model.

The `build()` method calls `super.build()` to create everything required for `SimpleMuscle`, calls `createInputProbe()` and `createOutputProbe()` to add the probes, and adjusts the `MechModel` viewer bounds to make the resulting probe motion more visible.

To run this example in ArtiSynth, select All demos > tutorial > SimpleMuscleWithProbes from the Models menu. After the model is loaded, the input and output probes should appear on the timeline (Figure 5.4). Expanding the probes should display their numeric contents, with the knot points for the input probe clearly visible. Running the model will cause particle `p1` to trace the trajectory specified by the input probe, while the velocity of the marker is recorded in the output probe. Figure 5.5 shows an expanded view of both probes after the simulation has run for about six seconds.

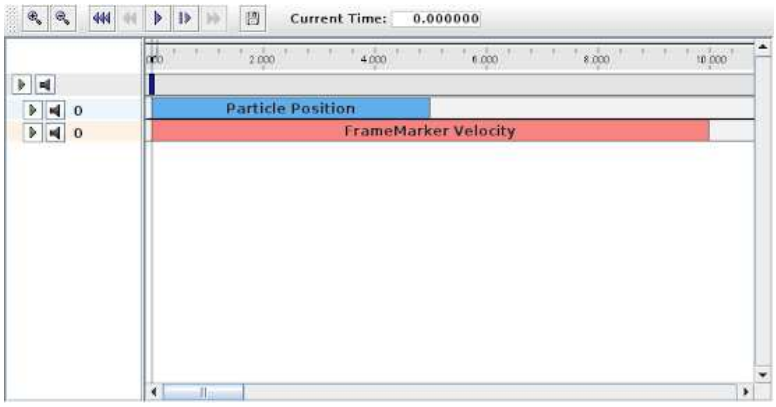



Figure 5.4: Timeline view of the probes created by SimpleMuscleWithProbes.

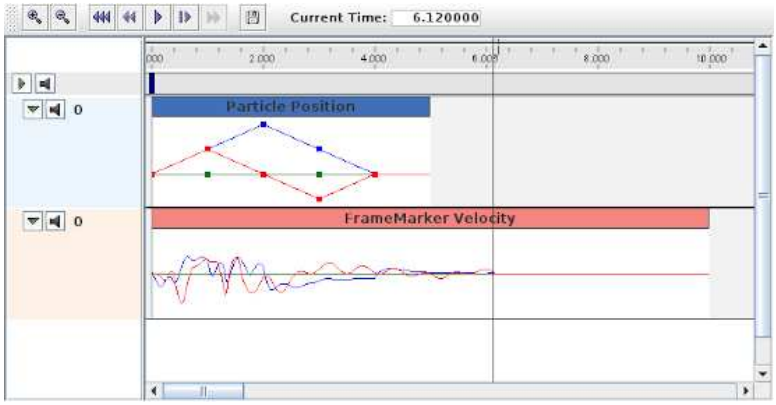



Figure 5.5: Expanded view of the probes after `SimpleMuscleWithProbes` has run for about 6 seconds, showing the data accumulated in the output probe "`FrameMarker Velocity`".

### 5.4.4 Data file format

The data files associated with numeric probes are ASCII files containing two lines of header information followed by a set of knot points, one per line, defining the numeric data. The time value (relative to the probe's start time) for each knot point can be specified explicitly at the start of the each line, in which case the file takes the following format:

```
startTime stopTime scale
interpolation vsize explicit
t0 val00 val01 val02 ...
t1 val10 val11 val12 ...
t0 val20 val21 val22 ...
...
```

Knot point information begins on line 3, with each line being a sequence of numbers giving the knot's time followed by $n$ values, where $n$ is the vector size of the probe (i.e., the value returned by `getVsize()`).

Alternatively, time values can be implicitly specified starting at 0 (relative to the probe's start time) and incrementing by a uniform `timeStep`, in which case the file assumes a second format:

```
startTime stopTime scale
interpolation vsize timeStep
val00 val01 val02 ...
val10 val11 val12 ...
val20 val21 val22 ...
...
```

For both formats, `startTime`, `startTime`, and `scale` are numbers giving the probe's start and stop time in seconds and `scale` gives the scale factor (which is typically 1.0). `interpolation` is a word describing how the data should be interpolated between knot points and is the string value of `Interpolation.Order` as described in Section 5.4.1 (and which is typically `Linear`, `Parabolic`, or `Cubic`). `vsize` is an integer giving the probe's vector size.

The last entry on the second line is either a number specifying a (uniform) time step for the knot points, in which case the file assumes the second format, or the keyword `explicit`, in which case the file assumes the first format.

As an example, the file used to specify data for the input probe in the example of Section 5.4.3 looks like the following:

```
0 4.0 1.0
Linear 3 explicit
0.0   0.0 0.0 0.0
1.0   0.5 0.0 0.5
2.0   0.0 0.0 1.0
3.0  -0.5 0.0 0.5
4.0   0.0 0.0 0.0
```

Since the data is uniformly spaced beginning at 0, it would also be possible to specify this using the second file format:

```
0 4.0 1.0
Linear 3 1.0
 0.0  0.0  0.0
 0.5  0.0  0.5
 0.0  0.0  1.0
-0.5  0.0  0.5
 0.0  0.0  0.0
```

### 5.4.5 Adding probe data in-line

It is also possible to specify input probe data directly in code, instead of reading it from a file. For this, one would use the constructor

```
NumericInputProbe (ModelComponent c, String propPath, double t0, double t1);
```

which creates a `NumericInputProbe` with the specified property and with start and stop times indicated by `t0` and `t1`. Data can then be added to this probe using the method

```
addData (double[] data, double timeStep);
```

where `data` is an array of knot point data. This contains the same knot point information as provided by a file (Section 5.4.4), arranged in row-major order. Times values for the knots are either implicitly specified, starting at 0 (relative to the probe's start time) and increasing uniformly by the amount specified by `timeStep`, or are explicitly specified at the beginning of each knot if `timeStep` is set to the built-in constant `NumericInputProbe.EXPLICIT_TIME`. The size of the `data` array should then be either $n * m$ (implicit time values) or $(n+1) * m$ (explicit time values), where $n$ is the probe's vector size and $m$ is the number of knots.

As an example, the data for the input probe in Section 5.4.3 could have been specified using the following code:

```
NumericInputProbe p1probe =
   new NumericInputProbe (
      mech, "particles/p1:targetPosition", 0, 5);
p1probe.addData (
   new double[] {
      0.0,  0.0, 0.0, 0.0,
      1.0,  0.5, 0.0, 0.5,
      2.0,  0.0, 0.0, 1.0,
      3.0, -0.5, 0.0, 0.5,
      4.0,  0.0, 0.0, 0.0 },
      NumericInputProbe.EXPLICIT_TIME);
```

When specifying data in code, the interpolation defaults to `Linear` unless explicitly specified using `setInterpolationOrder()`, as in, for example:

```
probe.setInterpolationOrder (Order.Cubic);
```

### 5.4.6 Smoothing probe data

Numeric probe data can also be smoothed, which is convenient for removing noise from either input or output data. Different smoothing methods are available; at the time of this writing, they include:

**Moving average**

Applies a mean average filter across the knots, using a moving window whose size is specified by the window size field. The window is centered on each knot, and is reduced in size near the end knots to ensure a symmetric fit. The end knot values are not changed. The window size must be odd and the window size field enforces this.

**Savitzky Golay**

Applies Savitzky-Golay smoothing across the knots, using a moving window of size $w$. Savitzky-Golay smoothing works by fitting the data values in the window to a polynomial of a specified degree $d$, and using this to recompute the value in the middle of the window. The polynomial is also used to interpolate the first and last $w/2$ values, since it is not possible to center the window on these.

The window size $w$ and the polynomial degree $d$ are specified by the window size and polynomial degree fields. $w$ must be odd, and must also be larger than $d$, and the fields enforce these constraints.

These operations may be applied with the following numeric probe methods:

| | |
|---|---|
| void smoothWithMovingAverage (double winSize) | Moving average smoothing over a specified window. |
| void smoothWithSavitzkyGolay ( double winSize, int deg) | Savitzky Golay smoothing with specified window and degree. |

### 5.4.7 Numeric monitor probes

In some cases, it may be useful for an application to deploy an output probe in which the data, instead of being collected from various component properties, is generated by a function within the probe itself. This ability is provided by a NumericMonitorProbe, which generates data using its generateData(vec,t,trel) method. This evaluates a vector-valued function of time at either the absolute time `t` or the probe-relative time `trel` and stores the result in the vector `vec`, whose size equals the vector size of the probe (as returned by getVsize()). The probe-relative time `trel` is determined by

$$\text{trel} = (\text{t} - \text{tstart})/\text{scale} \tag{5.1}$$

where `tstart` and `scale` are the probe's start time and scale factors as returned by getStartTime() and getScale().

As described further below, applications have several ways to control how a `NumericMonitorProbe` creates data:

- Provide the probe with a DataFunction using the setDataFunction(func) method;
- Override the generateData(vec,t,trel) method;
- Override the apply(t) method.

The application is free to generate data in any desired way, and so in this sense a `NumericMonitorProbe` can be used similarly to a `Monitor`, with one of the main differences being that the data generated by a `NumericMonitorProbe` can be automatically displayed in the ArtiSynth GUI or written to a file.

The DataFunction interface declares an `eval()` method,

```
void eval (VectorNd vec, double t, double trel)
```

that for `NumericMonitorProbes` evaluates a vector-valued function of time, where the arguments take the same role as for the monitor's generateData() method. Applications can declare an appropriate `DataFunction` and set or query it within the probe using the methods

```
void setDataFunction (DataFunction func);

DataFunction getDataFunction();
```

The default implementation `generateData()` checks to see if a data function has been specified, and if so, uses that to generate the probe data. Otherwise, if the probe's data function is `null`, the data is simply set to zero.

To create a `NumericMonitorProbe` using a supplied `DataFunction`, an application will create a generic probe instance, using one of its constructors such as

```
NumericMonitorProbe (vsize, filePath, startTime, stopTime, interval);
```

and then define and instantiate a `DataFunction` and pass it to the probe using `setDataFunction()`. It is not necessary to supply a file name (i.e., `filePath` can be `null`), but if one is provided, then the probe's data can be saved to that file.

A complete example of this is defined in

`artisynth.demos.tutorial.SinCosMonitorProbe`

the listing for which is:

```
1  package artisynth.demos.tutorial;
2
3  import maspack.matrix.*;
4  import maspack.util.Clonable;
5
6  import artisynth.core.workspace.RootModel;
7  import artisynth.core.probes.NumericMonitorProbe;
8  import artisynth.core.probes.DataFunction;
9
10 /**
11  * Simple demo using a NumericMonitorProbe to generate sine and cosine waves.
12  */
13 public class SinCosMonitorProbe extends RootModel {
14
15    // Define the DataFunction that generates a sine and a cosine wave
16    class SinCosFunction implements DataFunction, Clonable {
17
18       public void eval (VectorNd vec, double t, double trel) {
19          // vec should have size == 2, one for each wave
20          vec.set (0, Math.sin (t));
21          vec.set (1, Math.cos (t));
22       }
23
24       public Object clone() throws CloneNotSupportedException {
25          return (SinCosFunction)super.clone();
26       }
27    }
28
29    public void build (String[] args) {
30
31       // Create a NumericMonitorProbe with size 2, file name "sinCos.dat", start
32       // time 0, stop time 10, and a sample interval of 0.01 seconds:
33       NumericMonitorProbe sinCosProbe =
34          new NumericMonitorProbe (/*vsize=*/2, "sinCos.dat", 0, 10, 0.01);
35
36       // then set the data function:
37       sinCosProbe.setDataFunction (new SinCosFunction());
38       addOutputProbe (sinCosProbe);
39    }
40 }
```

In this example, the `DataFunction` is implemented using the class `SinCosFunction`, which also implements Clonable and the associated `clone()` method. This means that the resulting probe will also be duplicatable within the GUI. Alternatively, one could implement `SinCosFunction` by extending DataFunctionBase, which implements `Clonable` by default. Probes containing `DataFunction`s which are *not* `Clonable` will not be duplicatable.

When the example is run, the resulting probe output is shown in the timeline image of Figure 5.6.

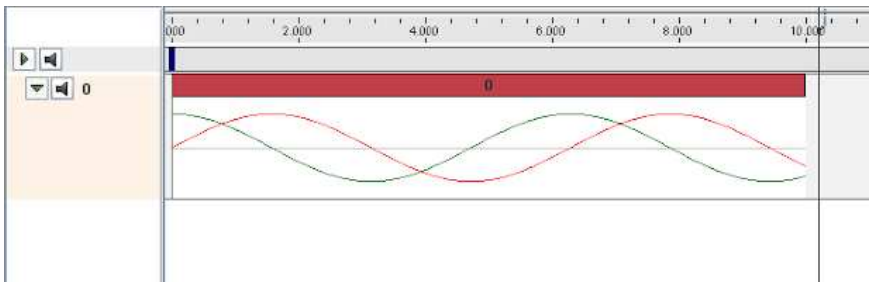



Figure 5.6: Output from a `NumericMonitorProbe` which generates sine and cosine waves.

As an alternative to supplying a DataFunction to a generic NumericMonitorProbe, an application can instead subclass NumericMonitorProbe and override either its generateData(vec,t,trel) or apply(t) methods. As an example of the former, one could create a subclass as follows:

```
class SinCosProbe extends NumericMonitorProbe {

   public SinCosProbe (
      String filePath, double startTime, double stopTime, double interval) {
      super (2, filePath, startTime, stopTime, interval);
   }

   public void generateData (VectorNd vec, double t, double trel) {
      vec.set (0, Math.sin (t));
      vec.set (1, Math.cos (t));
   }
}
```

Note that when subclassing, one must also create constructor(s) for that subclass. Also, NumericMonitorProbes which don't have a DataFunction set are considered to be clonable by default, which means that the clone() method may also need to be overridden if cloning requires any special handling.

### 5.4.8 Numeric control probes

In other cases, it may be useful for an application to deploy an input probe which takes numeric data, and instead of using it to modify various component properties, instead calls an internal method to directly modify the simulation in any way desired. This ability is provided by a NumericControlProbe, which applies its numeric data using its applyData(vec,t,trel) method. This receives the numeric input data via the vector vec and uses it to modify the simulation for either the absolute time t or probe-relative time trel. The size of vec equals the vector size of the probe (as returned by getVsize()), and the probe-relative time trel is determined as described in Section 5.4.7.

A NumericControlProbe is the Controller equivalent of a NumericMonitorProbe, as described in Section 5.4.7. Applications have several ways to control how they apply their data:

- Provide the probe with a DataFunction using the setDataFunction(func) method;
- Override the applyData(vec,t,trel) method;
- Override the apply(t) method.

The application is free to apply data in any desired way, and so in this sense a NumericControlProbe can be used similarly to a Controller, with one of the main differences being that the numeric data used can be automatically displayed in the ArtiSynth GUI or read from a file.

The DataFunction interface declares an eval() method,

```
void eval (VectorNd vec, double t, double trel)
```

that for NumericControlProbes applies the numeric data, where the arguments take the same role as for the monitor's applyData() method. Applications can declare an appropriate DataFunction and set or query it within the probe using the methods

```
void setDataFunction (DataFunction func);

DataFunction getDataFunction();
```

The default implementation applyData() checks to see if a data function has been specified, and if so, uses that to apply the probe data. Otherwise, if the probe's data function is null, the data is simply ignored and the probe does nothing.

To create a NumericControlProbe using a supplied DataFunction, an application will create a generic probe instance, using one of its constructors such as

```
NumericControlProbe (vsize, data, startTime, stopTime, timeStep);

NumericControlProbe (filePath);
```

and then define and instantiate a `DataFunction` and pass it to the probe using `setDataFunction()`. The latter constructor creates the probe and reads in both the data and timing information from the specified file.

A complete example of this is defined in

`artisynth.demos.tutorial.SpinControlProbe`

the listing for which is:

```
1  package artisynth.demos.tutorial;
2
3  import maspack.matrix.RigidTransform3d;
4  import maspack.matrix.VectorNd;
5  import maspack.util.Clonable;
6  import maspack.interpolation.Interpolation;
7
8  import artisynth.core.mechmodels.MechModel;
9  import artisynth.core.mechmodels.RigidBody;
10 import artisynth.core.mechmodels.Frame;
11 import artisynth.core.workspace.RootModel;
12 import artisynth.core.probes.NumericControlProbe;
13 import artisynth.core.probes.DataFunction;
14
15 /**
16  * Simple demo using a NumericControlProbe to spin a Frame about the z
17  * axis.
18  */
19 public class SpinControlProbe extends RootModel {
20
21    // Define the DataFunction that spins the body
22    class SpinFunction implements DataFunction, Clonable {
23
24       Frame myFrame;
25       RigidTransform3d myTFW0; // initial frame to world transform
26
27       SpinFunction (Frame frame) {
28          myFrame = frame;
29          myTFW0 = new RigidTransform3d (frame.getPose());
30       }
31
32       public void eval (VectorNd vec, double t, double trel) {
33          // vec should have size == 1, giving the current spin angle
34          double ang = Math.toRadians(vec.get(0));
35          RigidTransform3d TFW = new RigidTransform3d();
36          TFW.R.mulRpy (ang, 0, 0);
37          myFrame.setPose (TFW);
38       }
39
40       public Object clone() throws CloneNotSupportedException {
41          return super.clone();
42       }
43    }
44
45    public void build (String[] args) {
46
47       MechModel mech = new MechModel ("mech");
48       addModel (mech);
49
50       // Create a parametrically controlled rigid body to spin:
51       RigidBody body = RigidBody.createBox ("box", 1.0, 1.0, 0.5, 1000.0);
```

```
52        mech.addRigidBody (body);
53        body.setDynamic (false);
54
55        // Create a NumericControlProbe with size 1, initial spin data
56        // with time step 2.0, start time 0, and stop time 8.
57        NumericControlProbe spinProbe =
58           new NumericControlProbe (
59              /*vsize=*/1,
60              new double[] { 0.0, 90.0, 0.0, -90.0, 0.0 },
61              2.0, 0.0, 8.0);
62        // set cubic interpolation for a smoother result
63        spinProbe.setInterpolationOrder (Interpolation.Order.Cubic);
64        // then set the data function:
65        spinProbe.setDataFunction (new SpinFunction(body));
66        addInputProbe (spinProbe);
67     }
68  }
```

This example creates a simple box and then uses a `NumericControlProbe` to spin it about the $z$ axis, using a `DataFunction` implementation called `SpinFunction`. A clone method is also implemented to ensure that the probe will be duplicatable in the GUI, as described in Section 5.4.7. A single channel of data is used to control the orientation angle of the box about $z$, as shown in Figure 5.7.

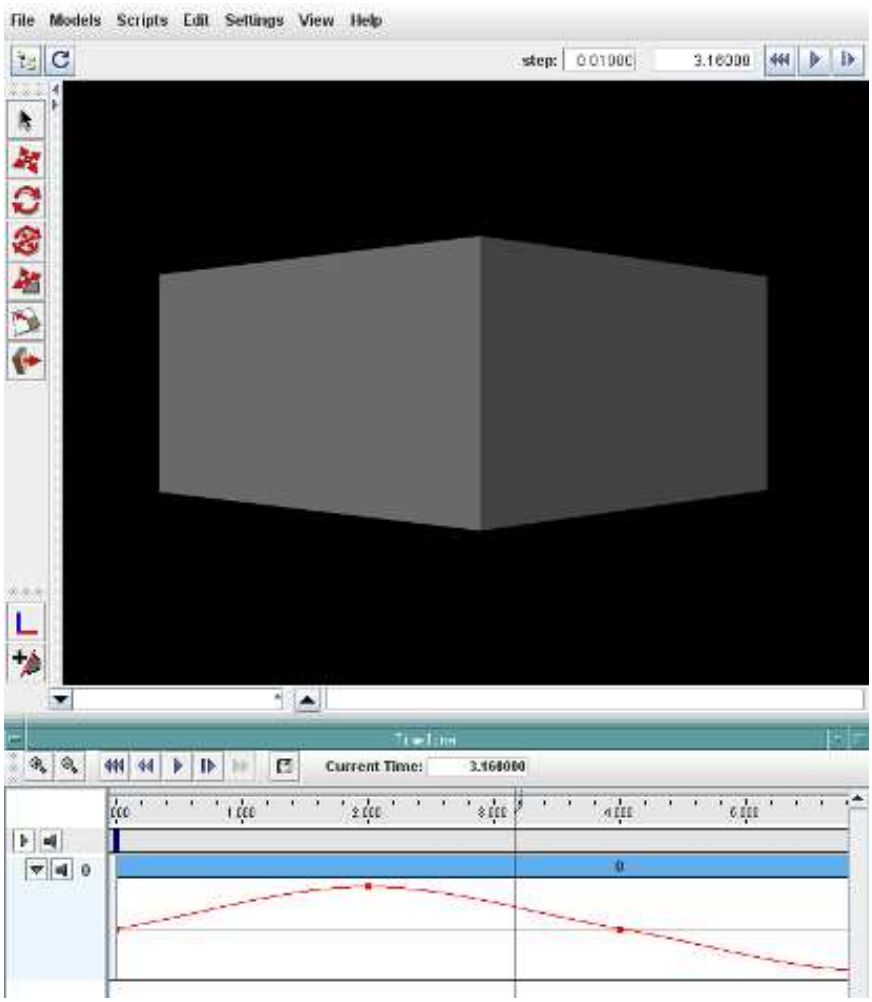



Figure 5.7: Screen shot of the `SpinControlDemo`, showing the numeric data in the timeline.

Alternatively, an application can subclass `NumericControlProbe` and override either its applyData(vec,t,trel) or apply(t) methods, as described for `NumericMonitorProbes` (Section 5.4.7).

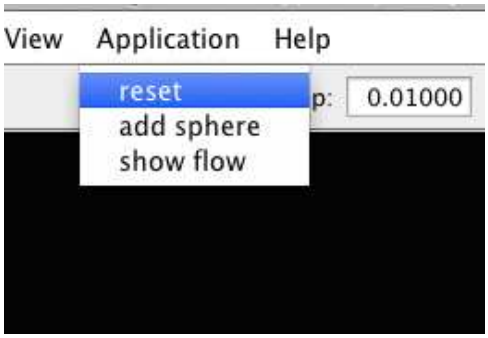



Figure 5.8: Application-defined menu items appearing under the ArtiSynth menu bar.

## 5.5 Application-Defined Menu Items

Application models can define custom *menu items* that appear under the `Application` menu in the main ArtiSynth menu bar.

This can be done by implementing the interface HasMenuItems in either the `RootModel` or any of its top-level components (e.g., models, controllers, probes, etc.). The interface contains a single method

```
public boolean getMenuItems(List<Object> items);
```

which, if the component has menu items to add, should append them to `items` and return `true`.

The `RootModel` and all models derived from ModelBase implement `HasMenuItems` by default, but with `getMenuItems()` returning `false`. Models wishing to add menu items should override this default declaration. Other component types, such as controllers, need to explicitly implement `HasMenuItems`.

> Note: the Application menu will only appear if `getMenuItems()` returns `true` for either the `RootModel` or one or more of its top-level components.

getMenuItems() will be called each time the Application menu is selected, so the menu itself is created on demand and can be varied to suite the current system state. In general, it should return items that are capable of being displayed inside a Swing `JMenu`; other items will be ignored. The most typical item is a Swing `JMenuItem`. The convenience method createMenuItem(listener,text,toolTip) can be used to quickly create menu items, as in the following code segment:

```
public boolean getMenuItems(List<Object> items) {
   items.add (GuiUtils.createMenuItem (this, "reset", ""));
   items.add (GuiUtils.createMenuItem (this, "add sphere", ""));
   items.add (GuiUtils.createMenuItem (this, "show flow", ""));
   return true;
}
```

This creates three menu items, each with `this` specified as an `ActionListener` and no tool-tip text, and appends them to `items`. They will then appear under the Application menu as shown in Figure 5.8.

To actually execute the menu commands, the items returned by `getMenuItems()` need to be associated with an `ActionListener` (defined in `java.awt.event`), which supplies the method `actionPerformed()` which is called when the menu item is selected. Typically the `ActionListener` is the component implementing `HasMenuItems`, as was assumed in the example declaration of `getMenuItems()` shown above. `RootModel` and other models derived from `ModelBase` implement `ActionListener` by default, with an empty declaration of `actionPerformed()` that should be overridden as required. A declaration of `actionPerformed()` capable of handling the menu example above might look like this:

```
public void actionPerformed (ActionEvent event) {
   String cmd = event.getActionCommand();
   if (cmd.equals ("reset")) {
```

```
            resetModel();
         }
         else if (cmd.equals ("add sphere")) {
            addSphere();
         }
         else if (cmd.equals ("show flow")) {
            showFlow();
         }
      }
```

# Chapter 6

# Finite Element Models

This chapter details how to construct three-dimensional finite element models, and how to couple them with the other simulation components described in previous sections (e.g. particles and rigid bodies). Finite element *muscles*, which have additional properties that allow them to contract given activation signals, are discussed in Section 6.9. An example FEM model of the masseter, coupled to a rigid jaw and maxilla, is shown in Figure 6.1.

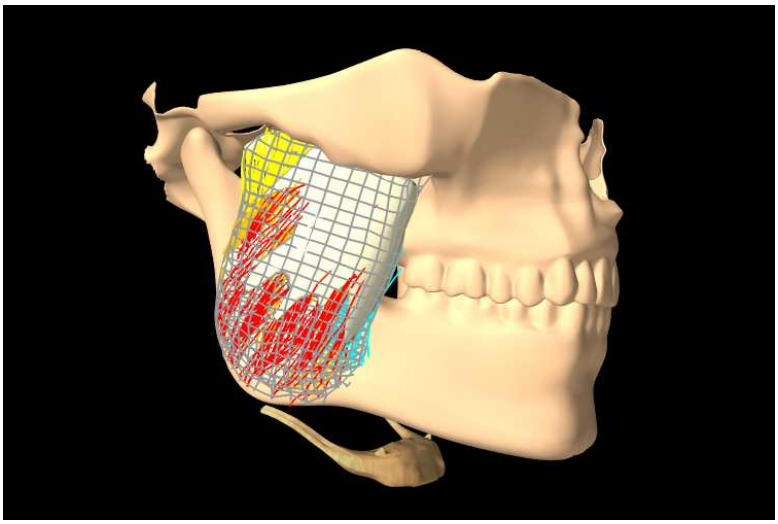


Figure 6.1: Finite element model of the masseter, coupled to the jaw and maxilla.

## 6.1 Overview

The finite element method (FEM) is a numerical technique used for solving a system of partial differential equations (PDEs) over some domain. The general approach is to divide the domain into a set of building blocks, referred to as *elements*. These partition the space, and form local domains over which the system of equations can be locally approximated. The corners of these elements, the *nodes*, become control points in a discretized system. The solution is then assumed to be smoothly interpolated across the elements based on values determined at the nodes. Using this discretization, the differential system is converted into an algebraic one, which is often linearized and solved iteratively.

In ArtiSynth, the PDEs considered are the governing equations of continuum mechanics: the conservation of mass, momentum, and energy. To complete the system, a *constitutive equation* is required that describes the stress-strain response of the material. This constitutive equation is what distinguishes between material types. The domain is the three-dimensional space that the model occupies. This must be divided into small elements which accurately represent the geometry. Within each element, the PDEs are sampled at a set of points, referred to as *integration points*, and terms are numerically integrated to form an algebraic system to solve.

The purpose of the rest of this chapter is to describe the construction and use of finite elements models within ArtiSynth. It does not further discuss the mathematical framework or theory. For an in-depth coverage of the nonlinear finite element method, as applied to continuum mechanics, the reader is referred to the textbook by Bonet and Wood [6].

### 6.1.1 FemModel3d

The basic type of finite element model is implemented in the class FemModel3d. This class controls some properties that are used by the model as a whole. The key ones that affect simulation dynamics are:

| Property | Description |
|---|---|
| `density` | The density of the model |
| `material` | An object that describes the material's *constitutive law* (i.e. its stress-strain relationship). |
| `particleDamping` | Proportional damping associated with the particle-like motion of the FEM nodes. |
| `stiffnessDamping` | Proportional damping associated with the system's stiffness term. |

These properties can be set and retrieved using the methods

| | |
|---|---|
| void setDensity (double density) | Sets the density. |
| double getDensity() | Gets the density. |
| void setMaterial (FemMaterial mat) | Sets the FEM's material. |
| FemMaterial getMaterial() | Gets the FEM's material. |
| void setParticleDamping (double d) | Sets the particle (mass) damping. |
| double getParticleDamping() | Gets the particle (mass) damping. |
| void setStiffnessDamping (double d) | Sets the stiffness damping. |
| double getStiffnessDamping() | Gets the stiffness damping. |

Keep in mind that ArtiSynth is essentially "unitless" (Section 4.2), so it is the responsibility of the developer to ensure that all properties are specified in a compatible way.

The density of the model is used to compute the mass distribution throughout the volume. Note that we use a *diagonally lumped mass matrix* (DLMM) formulation, so the mass is assumed to be concentrated at the location of the discretized FEM nodes. To allow for a spatially-varying density, densities can be explicitly set for individual elements, or masses can be explicitly set for individual nodes.

The FEM's material property is a delegate object used to compute stress and stiffness within individual elements. It handles the *constitutive* component of the model, as described in more detail in Sections 6.1.3 and 6.10. In addition to the main material defined for the model, it is also possible set a material on a per-element basis, and to define additional materials which augment the behavior of the main materials (Section 6.8).

The two damping parameters are related to *Rayleigh damping*, which is used to dissipate energy within finite element models. There are two proportional damping terms: one related to the system's mass, and one related to stiffness. The resulting damping force applied is

$$\mathbf{f}_d = -(d_M\mathbf{M} + d_K\mathbf{K})\mathbf{v}, \tag{6.1}$$

where $d_M$ is the value of `particleDamping`, $d_K$ is the value of `stiffnessDamping`, $\mathbf{M}$ is the FEM model's lumped mass matrix, $\mathbf{K}$ is the FEM's stiffness matrix, and $\mathbf{v}$ is the concatenated vector of FEM node velocities. Since the lumped mass matrix is diagonal, the mass-related component of damping can be applied separately to each FEM node. Thus, the mass component reduces to the same system as Equation (3.3), which is why it is referred to as "particle damping".

### 6.1.2 Component Structure

Each FemModel3d contains several lists of subcomponents:

`nodes`

The particle-like dynamic components of the model. These lie at the corners of the elements and carry all the mass (due to DLMM formulation).

`elements`

The volumetric model elements. These define the 3D sub-units over which the system is numerically integrated.

`shellElements`

The shell elements. These define additional 2D sub-units over which the system is numerically integrated.

`meshes`

The geometry in the model. This includes the surface mesh, and any other embedded geometries.

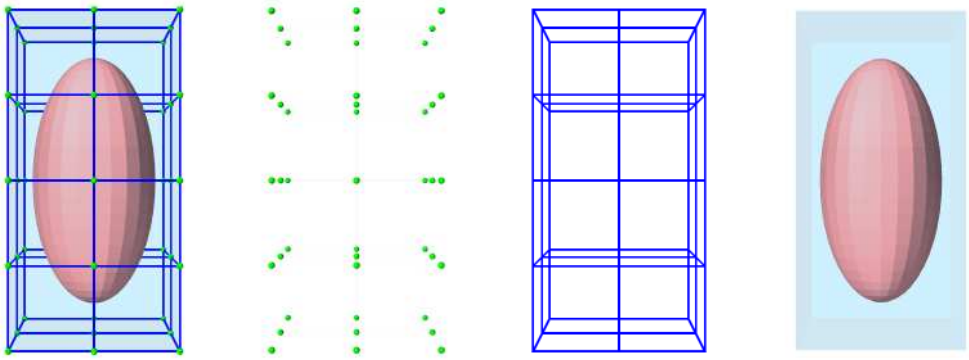
`materials`

Optional additional materials which can be added to the model to augment the behavior of the model's material property. This is described in more detail in Section 6.8.

`fields`

Optional *field* components which can be used to interpolate application-defined quantities over the FEM model's domain. Fields are described in detail in Chapter 7.

The `nodes`, `elements` and `meshes` components are illustrated in Figure 6.2.



(a) FEM model (b) Nodes (c) Elements (d) Geometry

Figure 6.2: Subcomponents of FemModel3d.

#### 6.1.2.1 Nodes

The set of nodes belong to a finite element model can be obtained by the method

| | |
|---|---|
| PointList<FemNode3d> getNodes() | Returns the list of FEM nodes. |

Nodes are implemented in the class FemNode3d, which is a subclass of Particle (Section 3.1). They are the main dynamic components of the finite element model. The key properties affecting simulation dynamics are:

| Property | Description |
|---|---|
| `restPosition` | The initial position of the node. |
| `position` | The current position of the node. |
| `velocity` | The current velocity of the node. |
| `mass` | The mass of the node. |
| `dynamic` | Whether the node is considered dynamic or parametric (e.g. boundary condition). |

Each of these properties has corresponding `getXxx()` and `setXxx(...)` functions to access and modify them.

The `restPosition` property defines the node's position in the FEM model's "natural" or "undeformed" state. Rest positions are used to compute an initial configuration for the model, from which strains are determined. A node's rest position can be updated in code using the method: FemNode3d.setRestPosition(Point3d).

> If any node's rest positions are changed, the current values for stress and stiffness will become invalid. They can be manually updated using the method FemModel3d.updateStressAndStiffness() for the parent model. Otherwise, stress and stiffness will be automatically updated at the beginning of the next time step.

The properties `position` and `velocity` give the node's current 3D state. These are common to all point-like particles, as is the `mass` property. Here, however, `mass` represents the lumped mass of the immediately surrounding material. Its

value is initialized by equally dividing mass contributions from each adjacent element, given their densities. For a finer control of spatially-varying density, node masses can be set manually after FEM creation.

The FEM node's `dynamic` property specifies whether or not the node is considered when computing the dynamics of the system. If not, it is treated as being parametrically controlled. This has implications when setting boundary conditions (Section 6.1.4).

#### 6.1.2.2 Elements

Elements are the 3D volumetric spatial building blocks of the domain. Within each element, the displacement (or strain) field is interpolated from displacements at nodes:

$$\mathbf{u}(\mathbf{x}) = \sum_{i=1}^{N} \phi_i(\mathbf{x})\mathbf{u}_i, \tag{6.2}$$

where $\mathbf{u}_i$ is the displacement of the $i$th node that is adjacent to the element, and $\phi_i(\cdot)$ is referred to as the *shape function* (or *basis function*) associated with that node. Elements are classified by their shape, number of nodes, and shape function order (Table 6.1). ArtiSynth supports the following element types, shown below with their node numberings:

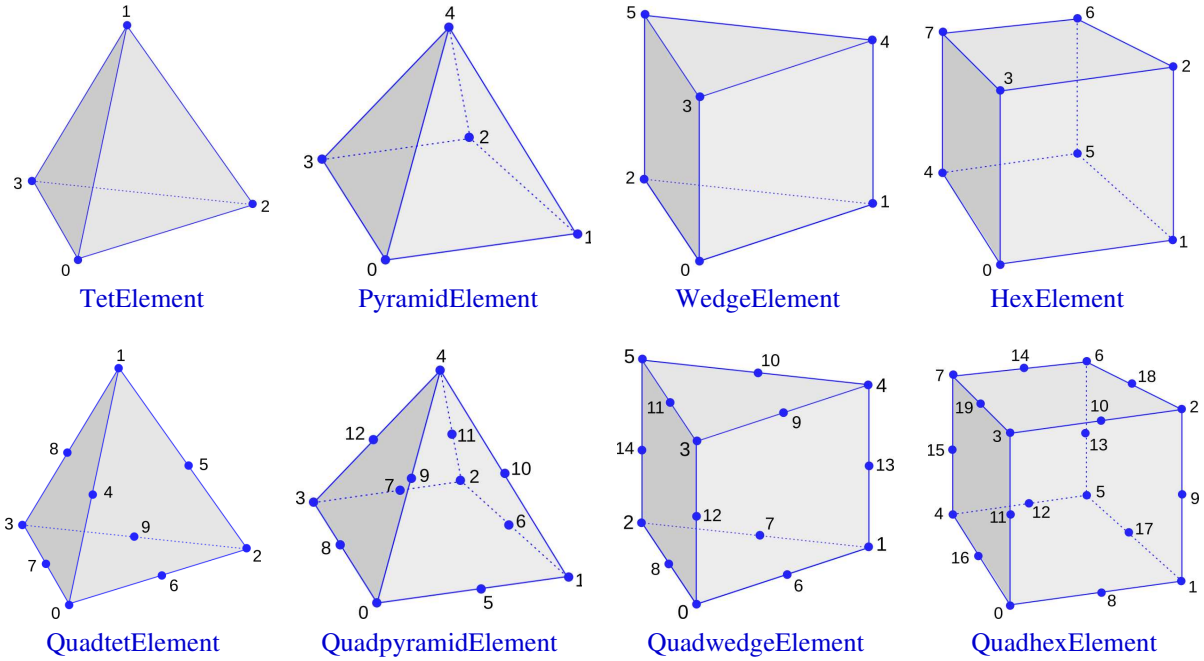



Table 6.1: Supported element types

| Element Type | # Nodes | Order | # Integration Points |
|---|---|---|---|
| TetElement | 4 | linear | 1 |
| PyramidElement | 5 | linear | 5 |
| WedgeElement | 6 | linear | 6 |
| HexElement | 8 | linear | 8 |
| QuadtetElement | 10 | quadratic | 4 |
| QuadpyramidElement | 13 | quadratic | 5 |
| QuadwedgeElement | 15 | quadratic | 9 |
| QuadhexElement | 20 | quadratic | 14 |

The base class for all of these is FemElement3d. A numerical integration is performed within each element to create the (tangent) stiffness matrix. This integration is performed by evaluating the stress and stiffness at a set of *integration points* within each element, and applying numerical quadrature. The list of elements in a model can be obtained with the method

| | |
|---|---|
| RenderableComponentList<FemElement3d> getElements() | Returns the list of volumetric elements. |

All objects of type FemElement3d have the following properties:

| Property | Description |
|---|---|
| `density` | Density of the element |
| `material` | An object that describes the *constitutive law* within the element (i.e. its stress-strain relationship). |

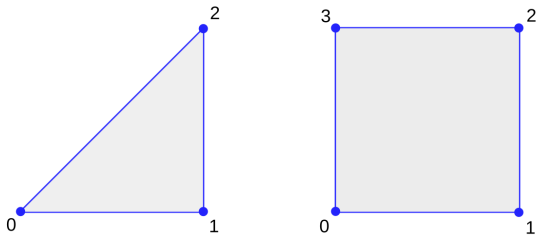
If left unspecified, the element's `density` is inherited from the containing `FemModel3d` object. When set, the mass of the element is computed and divided amongst all its nodes, updating the lumped mass matrix.

Each element's `material` property is also inherited by default from the containing `FemModel3d`. Specifying a material here allows for spatially-varying material properties across the model. Materials are discussed further in Sections 6.1.3 and 6.10.

#### 6.1.2.3 Shell elements

Shell elements are additional 2D spatial building blocks which can be added to a model. They are typically used to model structures which are too thin to be easily represented by 3D volumetric elements, or to provide additional internal stiffness within a set of volumetric elements.

ArtiSynth presently supports the following shell element types, with the number of nodes, shape function order, and integration point count described in Table 6.2:




ShellTriElement ShellQuadElement

Table 6.2: Supported shell element types

| Element Type | # Nodes | Order | # Integration Points |
|---|---|---|---|
| ShellTriElement | 3 | linear | 9 (3 if membrane) |
| ShellQuadElement | 4 | linear | 8 (4 if membrane) |

The base class for all shell elements is ShellElement3d, which contains the same density and material properties as FemElement3d, as well as the additional property defaultThickness, whose use will be described below.

The list of shell elements in a model can be obtained with the method

| | |
|---|---|
| RenderableComponentList<ShellElement3d> getShellElements() | Returns the list of shell elements. |

Both the volumetric elements (`FemElement3d`) and the shell elements (`ShellElement3d`) derive from the base class FemElement3dBase. To obtain *all* the elements in an FEM model, both shell and volumetric, one may use the method

| | |
|---|---|
| ArrayList<FemElement3dBase> getAllElements() | Returns a list of all elements. |

Each shell element can actually be instantiated in two forms:

- As a *regular* shell element, which has a bending stiffness;
- As a *membrane* element, which does not have bending stiffness.

Regular shell elements are implemented using the same *extensible director* formulation used by FEBio [13], and more specifically the front/back node formulation [12]. Each node associated with a (regular) shell element is assigned a *director*, which is a 3D vector providing a normal direction and virtual thickness at that node (Figure 6.3). This virtual thickness allows us to continue to use 3D materials to provide the constitutive laws that determine the shell's

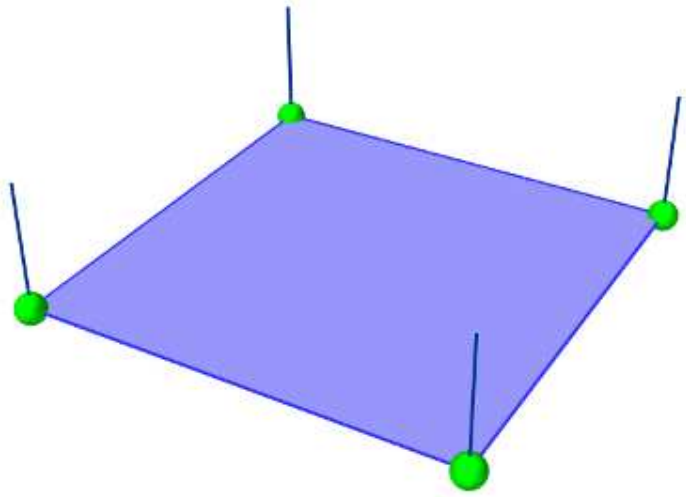


Figure 6.3: `ShellQuadElement` with the directors (dark blue lines) visible at the nodes.

stress/strain response, including its bending behavior. It also allows us to continue to use the element's density to determine its mass.

Director information is automatically assigned to a FemNode3d whenever one or more regular shell elements is connected to it. This information includes both the current value of the director, its *rest* value, and its velocity, with the difference between the first two determining the element's bending strain. These quantities can be queried using the methods

```
Vector3d getDirector ();         // return the current director value
void getDirector (Vector3d dir);

Vector3d getRestDirector ();     // return the rest director value
void getRestDirector (Vector3d dir);

Vector3d getDirectorVel ();      // return the director velocity

boolean hasDirector();           // does this node have a director?
```

For nodes which are not connected to regular shell elements, and therefore do not have director information assigned, these methods all return a zero-valued vector.

If not otherwise specified, the current and rest director values are computed automatically from the surrounding (regular) shell elements, with their value **d** being computed from

$$\mathbf{d} = \sum t_i \, \mathbf{n}_i$$

where $t_i$ is the value of the defaultThickness property and $\mathbf{n}_i$ is the surface normal of the $i$-th surrounding regular shell element. However, if necessary, it is also possible to explicitly assign these values, using the methods

```
setDirector (Vector3d dir);      // set the current director

setRestDirector (Vector3d dir); // set the rest director
```

> ArtiSynth FEM nodes can currently support only one director, which is shared by all regular shell elements associated with that node. This effectively means that all such elements must belong to the same "surface", and that two intersecting surfaces cannot share the same nodes.

As indicated above, shell elements can also be instantiated as membrane elements, which do not exhibit bending stiffness and therefore do not require director information. The regular/membrane distinction is specified in the element's constructor. For example, `ShellTriElement` and `ShellQuadElement` each have constructors with the signatures:

```
ShellTriElement (FemNode3d n0, FemNode3d n1, FemNode3d n2,
  double thickness, boolean membrane);

ShellQuadElement (FemNode3d n0, FemNode3d n1, FemNode3d n2, FemNode3d n3,
  double thickness, boolean membrane);
```

The `thickness` argument specifies the defaultThickness property, while `membrane` determines whether or not the element is a membrane element.

While membrane elements do not require explicit director information stored at the nodes, they do make use of an *inferred* director that is parallel to the element's surface normal, and has a constant length equal to the element's defaultThickness property. This gives the element a virtual volume, which (as with regular elements) is used to determine 3D strains and to compute the element's mass from it's density.

#### 6.1.2.4 Meshes

The geometry associated with a finite element model consists of a collection of meshes (e.g. PolygonalMesh, PolylineMesh, PointMesh) that move along with the model in a way that maintains the shape function interpolation equation (6.2) at each vertex location. These geometries can be used for visualizations, or for physical interactions like collisions. However, they have no physical properties themselves. FEM geometries will be discussed in more detail in Section 6.3. The list of meshes can be obtained with the method

```
MeshComponentList<FemMeshComp> getMeshComps();
```

### 6.1.3 Materials

The stress-strain relationship within each element is defined by a "material" delegate object, implemented by a subclass of FemMaterial. This material object is responsible for implementing the functions

```
void computeStressAndTangent (...)
```

which computes the stress tensor and (optionally) the tangent stiffness matrix at each integration point, based on the current local deformation at that point.

All supported materials are presented in detail in Section 6.10. The default material type is LinearMaterial, which linearly maps Cauchy strain $\varepsilon$ onto Cauchy stress $\sigma$ via

$$\sigma = \mathcal{D} : \varepsilon,$$

where $\mathcal{D}$ is the *elasticity tensor* determined from Young's modulus and Poisson's ratio. Anisotropic linear materials are provided by TransverseLinearMaterial and AnisotropicLinearMaterial. Linear materials in ArtiSynth implement corotation, which removes the rotation from the deformation gradient and allows Cauchy strain to be applied to large deformations [18, 16]. Nonlinear hyperelastic materials are also supported, including MooneyRivlinMaterial, OgdenMaterial, YeohMaterial, ArrudaBoyceMaterial, and VerondaWestmannMaterial.

### 6.1.4 Boundary conditions

Boundary conditions can be implemented in one of several ways:

1. Explicitly setting FEM node positions/velocities
2. Attaching FEM nodes to other dynamic components
3. Enabling collisions

To enforce an explicit (Dirichlet) boundary condition for a set of nodes, their `dynamic` property must be set to `false`. This notifies ArtiSynth that the state of these nodes (both position and velocity) will be controlled parametrically. By disabling dynamics, a fixed boundary condition is applied. For a moving boundary, positions and velocities of the boundary nodes must be explicitly set every timestep. This can be accomplished with either a Controller (see Section

5.3) or an InputProbe (see Section 5.4). Note that both the position *and* velocity of the nodes should be explicitly set for consistency.

Another type of supported boundary condition is to attach FEM nodes to other components, including particles, springs, rigid bodies, and locations within other FEM elements. Here, the node is still considered dynamic, but its motion is coupled to that of the attached component through a constraint mechanism. Attachments will be discussed further in Section 6.4.

Finally, the boundary of an FEM can be constrained by enabling collisions with other components. This will be covered in Chapter 8.

## 6.2 FEM model creation

Creating a finite element model in ArtiSynth typically follows the pattern:

```
// Create and add main MechModel
MechModel mech = new MechModel("mech");
addModel(mech);

// Create FEM
FemModel3d fem = new FemModel3d("fem");

/* ... Setup FEM structure and properties ... */

// Add FEM to model
mech.addModel(fem);
```

The main code block for the FEM setup should do the following:

- Build the node/element structure
- Set physical properties
  - density
  - damping
  - material
- Set boundary conditions
- Set render properties

Building the FEM structure can be done with the use of factory methods for simple shapes, by loading external files, or by writing code to manually assemble the nodes and elements.

### 6.2.1 Factory methods

For simple shapes such as beams and ellipsoids, there are factory methods to automatically build the node and element structure. These methods are found in the FemFactory class. Some common methods are

```
FemFactory.createGrid(...)          // basic beam
FemFactory.createCylinder(...)      // cylinder
FemFactory.createTube(...)          // hollowed cylinder
FemFactory.createEllipsoid(...)     // ellipsoid
FemFactory.createTorus(...)         // torus
```

The inputs specify the dimensions, resolution, and potentially the type of element to use. The following code creates a basic beam made up of hexahedral elements:

```
// Create FEM
FemModel3d beam = new FemModel3d("beam");

// Build FEM structure
double[] size = {1.0, 0.25, 0.25};  // widths
int[] res = {8, 2, 2};              // resolution (# elements)
```

```
FemFactory.createGrid(beam, FemElementType.Hex,
   size[0], size[1], size[2],
   res[0], res[1], res[2]);

/* ... Set FEM properties ... */

// Add FEM to model
mech.addModel(beam);
```

### 6.2.2 Loading external FEM meshes

For more complex geometries, volumetric meshes can be loaded from external files. A list of supported file types is provided in Table 6.3. To load a geometry, an appropriate file reader must be created. Readers capable of reading FEM models implement the interface FemReader, which has the method

```
readFem( FemModel3d fem )   // populates the FEM based on file contents
```

Additionally, many `FemReader` classes have static methods to handle the loading of files for convenience.

Table 6.3: Supported FEM geometry files

| Format | File extensions | Reader | Writer |
|---|---|---|---|
| ANSYS | .node, .elem | AnsysReader | AnsysWriter |
| TetGen | .node, .ele | TetGenReader | TetGenWriter |
| Abaqus | .inp | AbaqusReader | AbaqusWriter |
| VTK (ASCII) | .vtk | VtkAsciiReader | – |

The following code snippet demonstrates how to load a model using the AnsysReader.

```
// Create FEM
FemModel3d tongue = new FemModel3d("tongue");

// Read FEM from file
try {
   // Get files relative to THIS class
   String nodeFileName = PathFinder.getSourceRelativePath(this,
                            "data/tongue.node");
   String elemFileName = PathFinder.getSourceRelativePath(this,
                            "data/tongue.elem");

   AnsysReader.read(tongue, nodeFileName, elemFileName);

} catch (IOException ioe) {
   // Wrap error, fail to create model
   throw new RuntimeException("Failed to read model", ioe);
}

// Add to model
mech.addModel(tongue);
```

The method PathFinder.getSourceRelativePath() is used to find a path within the ArtiSynth source tree that is relative to the current model's source file (Section 2.6). Note the try-catch block. Most of these readers throw an `IOException` if the read fails.

### 6.2.3 Generating from surfaces

There are two ways an FEM model can be generated from a surface: by using a FEM mesh generator, and by extruding a surface along its normal direction.

ArtiSynth has the ability to interface directly with the TetGen library (http://tetgen.org) to create a tetrahedral volumetric mesh given a closed and manifold surface. The main Java class for calling TetGen directly is TetgenTessellator. The tessellator has several advanced options, allowing for the computation of convex hulls, and for adding points to a volumetric mesh. For simply creating an FEM from a surface, there is a convenience routine within FemFactory that handles both mesh generation and constructing a `FemModel3d`:

```
// Create an FEM from a manifold mesh with a given quality
FemFactory.createFromMesh( PolygonalMesh mesh, double quality );
```

If `quality` $> 0$, then points will be added in an attempt to bound the maximum radius-edge ratio (see the `-q` switch for TetGen). According to the TetGen documentation, the algorithm *usually* succeeds for a quality ratio of 1.2.

It's also possible to create thin layer of elements by extruding a surface along its normal direction.

```
// Create an FEM by extruding a surface
FemFactory.createExtrusion(
      FemModel3d model, int nLayers, double layerThickness, double zOffset,
      PolygonalMesh surface);
```

For example, to create a two-layer slice of elements centered about a surface of a tendon mesh, one might use

```
// Load the tendon surface mesh
PolygonalMesh tendonSurface = new PolygonalMesh("tendon.obj");

// Create the tendon
FemModel3d tendon = new FemModel3d("tendon");
int layers = 2;              // 2 layers
double thickness = 0.0005;   // 0.5 mm layer thickness
double offset = thickness;   // center the layers about the surface

// Create the extrusion
FemFactory.createExtrusion( tendon, layers, thickness, offset, tendonSurface );
```

For this type of extrusion, triangular faces become wedge elements, and quadrilateral faces become hexahedral elements.

> Note: for extrusions, no care is taken to ensure element quality; if the surface has a high curvature relative to the total extrusion thickness, then some elements will be inverted.

### 6.2.4 Building elements in code

A finite element model's structure can also be manually constructed in code. FemModel3d has the methods:

```
addNode ( FemNode3d );        // add a node to the model
addElement ( FemElement3d )   // add an element to the model
```

For an element to successfully be added, all its nodes must already have been added to the model. Nodes can be constructed from a 3D location, and elements from an array of nodes. A convenience routine is available in FemElement3d that automatically creates the appropriate element type given the number of nodes (Table 6.1):

```
// Creates an element using the supplied nodes
FemElement3d FemElement3d.createElement( FemNode3d[] nodes );
```

Be aware of node orderings when supplying nodes. For linear elements, ArtiSynth uses a clockwise convention with respect to the outward normal for the first face, followed by the opposite node(s). To determine the correct ordering for a particular element, check the coordinates returned by the function FemElement3dBase.getNodeCoords(). This returns the concatenated coordinate list for an "ideal" element of the given type.

### 6.2.5 Example: a simple beam model

A complete application model that implements a simple FEM beam is given below.

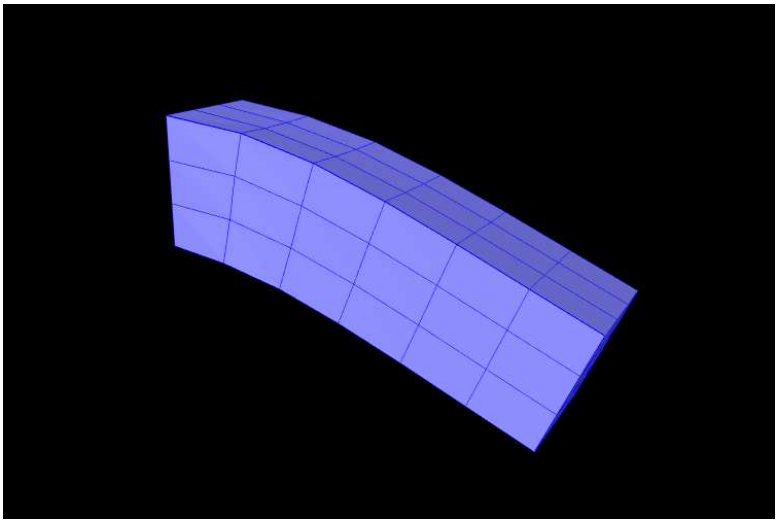


Figure 6.4: FemBeam model loaded into ArtiSynth.

```
package artisynth.demos.tutorial;

import java.awt.Color;
import java.io.IOException;

import maspack.render.RenderProps;
import artisynth.core.femmodels.FemFactory;
import artisynth.core.femmodels.FemModel.SurfaceRender;
import artisynth.core.femmodels.FemModel3d;
import artisynth.core.femmodels.FemNode3d;
import artisynth.core.materials.LinearMaterial;
import artisynth.core.mechmodels.MechModel;
import artisynth.core.workspace.RootModel;

public class FemBeam extends RootModel {

   // Models and dimensions
   FemModel3d fem;
   MechModel mech;
   double length = 1;
   double density = 10;
   double width = 0.3;
   double EPS = 1e-15;

   public void build (String[] args) throws IOException {

      // Create and add MechModel
      mech = new MechModel ("mech");
      addModel(mech);

      // Create and add FemModel
      fem = new FemModel3d ("fem");
      mech.add (fem);

      // Build hex beam using factory method
      FemFactory.createHexGrid (
         fem, length, width, width, /*nx=*/6, /*ny=*/3, /*nz=*/3);

      // Set FEM properties
      fem.setDensity (density);
      fem.setParticleDamping (0.1);
      fem.setMaterial (new LinearMaterial (4000, 0.33));
```

```
43
44        // Fix left-hand nodes for boundary condition
45        for (FemNode3d n : fem.getNodes()) {
46           if (n.getPosition().x <= -length/2+EPS) {
47              n.setDynamic (false);
48           }
49        }
50
51        // Set rendering properties
52        setRenderProps (fem);
53
54     }
55
56     // sets the FEM's render properties
57     protected void setRenderProps (FemModel3d fem) {
58        fem.setSurfaceRendering (SurfaceRender.Shaded);
59        RenderProps.setLineColor (fem, Color.BLUE);
60        RenderProps.setFaceColor (fem, new Color (0.5f, 0.5f, 1f));
61     }
62
63  }
```

This example can be found in `artisynth.demos.tutorial.FemBeam`. The `build()` method first creates a `MechModel` and `FemModel3d`. A FEM beam is created using a factory method on line 36. This beam is centered at the origin, so its length extends from `-length/2` to `length/2`. The density, damping and material properties are then assigned.

On lines 45–49, a fixed boundary condition is set to the left-hand side of the beam by setting the corresponding nodes to be non-dynamic. Due to numerical precision, a small `EPSILON` buffer is required to ensure all left-hand boundary nodes are identified (line 46).

Rendering properties are then assigned to the FEM model on line 52. These will be discussed further in Section 6.12.

## 6.3 FEM Geometry

Associated with each FEM model is a list of geometry with the heading `meshes`. This geometry can be used for either display purposes, or for interactions such as collisions (Section 8.3.2). A geometry itself has no physical properties; its motion is entirely governed by the FEM model that contains it.

All FEM geometries are of type FemMeshComp, which stores a reference to a mesh object (Section 2.5), as well as attachment information that links vertices of the mesh to points within the FEM. The attachments enforce the shape function interpolation in Equation (6.2) to hold at each mesh vertex, with constant shape function coefficients.

### 6.3.1 Surface meshes

By default, every `FemModel3d` has an auto-generated geometry representing the "surface mesh". The surface mesh consists of all un-shared element faces (i.e. the faces of individual elements that are exposed to the world), and its vertices correspond to the nodes that make up those faces. As the FEM nodes move, so do the mesh vertices due to the attachment framework.

The surface mesh can be obtained using one of the following functions in `FemModel3d`:

```
FemMeshComp getSurfaceMeshComp ();     // returns the FemMeshComp surface component
PolygonalMesh getSurfaceMesh ();  // returns the underlying polygonal surface mesh
```

The first returns the surface complete with attachment information. The latter method directly returns the `PolygonalMesh` that is controlled by the FEM.

It is possible to manually set the surface mesh:

```
setSurfaceMesh ( PolygonalMesh surface );  // manually set surface mesh
```

However, doing so is normally not necessary. It is always possible to add additional mesh geometries to a finite element model, and the visibility settings can be changed so that the default surface mesh is not rendered.

### 6.3.2 Embedding geometry within an FEM

Any geometry of type MeshBase can be added to a `FemModel3d`. To do so, first position the mesh so that its vertices are in the desired locations inside the FEM, then call one of the `FemModel3d` methods:

```
FemMeshComp addMesh ( MeshBase mesh );                // creates and returns ↩
    FemMeshComp
FemMeshComp addMesh ( String name, MeshBase mesh );
```

The latter is a convenience routine that also gives the newly embedded `FemMeshComp` a name.

### 6.3.3 Example: a beam with an embedded sphere

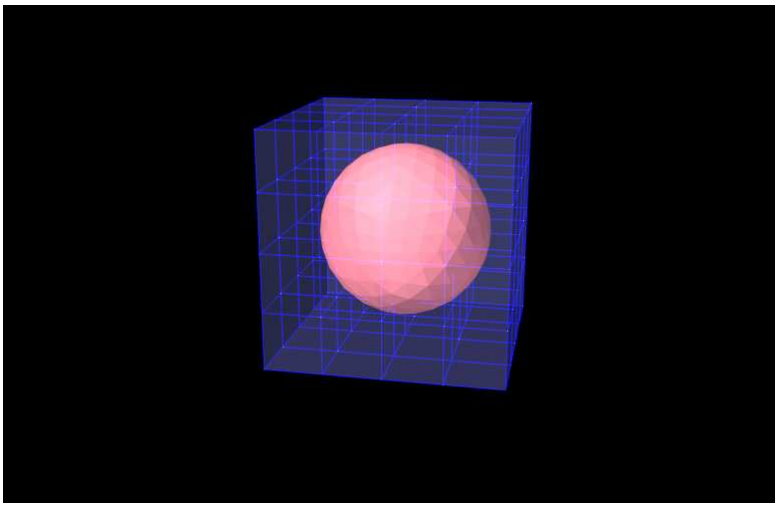


Figure 6.5: FemEmbeddedSphere model loaded into ArtiSynth.

A complete model demonstrating embedding a mesh is given below.

```
1  package artisynth.demos.tutorial;
2
3  import java.awt.Color;
4  import java.io.IOException;
5
6  import maspack.geometry.*;
7  import maspack.render.RenderProps;
8  import artisynth.core.mechmodels.Collidable.Collidability;
9  import artisynth.core.femmodels.*;
10 import artisynth.core.femmodels.FemModel.SurfaceRender;
11 import artisynth.core.materials.LinearMaterial;
12 import artisynth.core.mechmodels.MechModel;
13 import artisynth.core.workspace.RootModel;
14
15 public class FemEmbeddedSphere extends RootModel {
16
17    // Internal components
18    protected MechModel mech;
19    protected FemModel3d fem;
20    protected FemMeshComp sphere;
21
22    @Override
23    public void build(String[] args) throws IOException {
24       super.build(args);
25
26       mech = new MechModel("mech");
27       addModel(mech);
28
```

```
29        fem = new FemModel3d("fem");
30        mech.addModel(fem);
31
32        // Build hex beam and set properties
33        double[] size = {0.4, 0.4, 0.4};
34        int[] res = {4, 4, 4};
35        FemFactory.createHexGrid (fem,
36           size[0], size[1], size[2], res[0], res[1], res[2]);
37        fem.setParticleDamping(2);
38        fem.setDensity(10);
39        fem.setMaterial(new LinearMaterial(4000, 0.33));
40
41        // Add an embedded sphere mesh
42        PolygonalMesh sphereSurface = MeshFactory.createOctahedralSphere(0.15, 3);
43        sphere = fem.addMesh("sphere", sphereSurface);
44        sphere.setCollidable (Collidability.EXTERNAL);
45
46        // Boundary condition: fixed LHS
47        for (FemNode3d node : fem.getNodes()) {
48           if (node.getPosition().x == -0.2) {
49              node.setDynamic(false);
50           }
51        }
52
53        // Set rendering properties
54        setFemRenderProps (fem);
55        setMeshRenderProps (sphere);
56     }
57
58     // FEM render properties
59     protected void setFemRenderProps ( FemModel3d fem ) {
60        fem.setSurfaceRendering (SurfaceRender.Shaded);
61        RenderProps.setLineColor (fem, Color.BLUE);
62        RenderProps.setFaceColor (fem, new Color (0.5f, 0.5f, 1f));
63        RenderProps.setAlpha(fem, 0.2);    // transparent
64     }
65
66     // FemMeshComp render properties
67     protected void setMeshRenderProps ( FemMeshComp mesh ) {
68        mesh.setSurfaceRendering( SurfaceRender.Shaded );
69        RenderProps.setFaceColor (mesh, new Color (1f, 0.5f, 0.5f));
70        RenderProps.setAlpha (mesh, 1.0);    // opaque
71     }
72
73  }
```

This example can be found in `artisynth.demos.tutorial.FemEmbeddedSphere`. The model is very similar to `FemBeam`. A `MechModel` and `FemModel3d` are created and added. At line 41, a `PolygonalMesh` of a sphere is created using a factory method. The sphere is already centered inside the beam, so it does not need to be repositioned. At Line 42, the sphere is embedded inside model `fem`, creating a `FemMeshComp` with the name "sphere". The full model is shown in Figure 6.5.

## 6.4 Connecting FEM models to other components

To couple FEM models to other dynamic components, the "attachment" mechanism described in Section 1.2 is used. This involves creating and adding to the model attachment components, which are instances of DynamicAttachment, as described in Section 3.7. Common point-based attachment classes are listed in Table 6.4.

FEM models are connected to other model components by attaching their nodes to various components. This can be done by creating an attachment object of the appropriate type, and then adding it to the `MechModel` using

```
addAttachment (DynamicAttachment attach); // adds an attachment constraint
```

Table 6.4: Point-based attachments

| Attachment | Description |
|---|---|
| PointParticleAttachment | Attaches one "point" to one "particle" |
| PointFrameAttachment | Attaches one "point" to one "frame" |
| PointFem3dAttachment | Attaches one "point" to a linear combination of FEM nodes |

There are also convenience routines inside `MechModel` that will create the appropriate attachments automatically (see Section 3.7.1).

All attachments described in this section are based around FEM *nodes*. However, it is also possible to attach frame-based components (such as rigid bodies) directly to an FEM, as described in Section 6.6.

### 6.4.1 Connecting nodes to rigid bodies or particles

Since FemNode3d is a subclass of Particle, the same methods described in Section 3.7.1 for attaching particles to other particles and frames are available. For example, we can attach an FEM node to a rigid body using a either a statement of the form

```
mech.addAttachment (new PointFrameAttachment(body, node));
```

or the following equivalent statement which does the same thing:

```
mech.attachPoint (node, body);
```

Both of these create a `PointFrameAttachment` between a rigid body (called `body`) and an FEM node (called `node`) and then adds it to the `MechModel mech`.

One can also attach the nodes of one FEM model to the nodes of another using statements like

```
mech.addAttachment (new PointParticle (node1, node2));
```

or

```
mech.attachPoint (node2, node1);
```

which attaches `node2` to `node1`.

### 6.4.2 Example: connecting a beam to a block

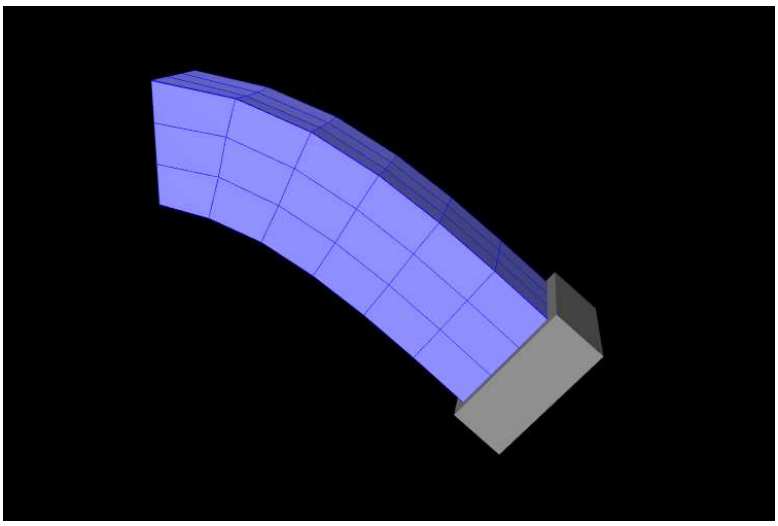


Figure 6.6: FemBeamWithBlock model loaded into artisynth.

The following model demonstrates attaching an FEM beam to a rigid block.

```
1  package artisynth.demos.tutorial;
2
3  import java.io.IOException;
4
5  import maspack.matrix.RigidTransform3d;
6  import artisynth.core.femmodels.FemNode3d;
7  import artisynth.core.mechmodels.PointFrameAttachment;
8  import artisynth.core.mechmodels.RigidBody;
9
10 public class FemBeamWithBlock extends FemBeam {
11
12    public void build (String[] args) throws IOException {
13
14       // Build simple FemBeam
15       super.build (args);
16
17       // Create a rigid block and move to the side of FEM
18       RigidBody block = RigidBody.createBox (
19          "block", width/2, 1.2*width, 1.2*width, 2*density);
20       mech.addRigidBody (block);
21       block.setPose (new RigidTransform3d (length/2+width/4, 0, 0));
22
23       // Attach right-side nodes to rigid block
24       for (FemNode3d node : fem.getNodes()) {
25          if (node.getPosition().x >= length/2-EPS) {
26             mech.addAttachment (new PointFrameAttachment (block, node));
27          }
28       }
29    }
30
31 }
```

This model extends the `FemBeam` example of Section 6.2.5. The `build()` method then creates and adds a `RigidBody` block (lines 18–20). On line 21, the block is repositioned to the side of the beam to prepare for the attachment. On lines 24–28, all right-most nodes of the beam are then set to be attached to the block using a `PointFrameAttachment`. In this case, the attachments are explicitly created. They could also have been attached using

```
mech.attachPoint (node, block);  // attach node to rigid block
```

### 6.4.3 Connecting nodes directly to elements

Typically, nodes do not align in a way that makes it possible to connect them to other FEM models and/or points based on simple point-to-node attachments. Instead, we use a different mechanism that allows us to attach a point to an arbitrary location within an FEM model. This is done using an attachment component of type PointFem3dAttachment, which implements an attachment where the position $\mathbf{p}$ and velocity $\mathbf{u}$ of the attached point is determined by a weighted sum of the positions $\mathbf{x}_k$ and velocities $\mathbf{u}_k$ of selected fem nodes:

$$\mathbf{p} = \sum \alpha_k \mathbf{x}_k \tag{6.3}$$

Any force $\mathbf{f}$ acting on the attached point is then propagated back to the nodes, according to the relation

$$\mathbf{f}_k = \alpha_k \mathbf{f} \tag{6.4}$$

where $\mathbf{f}_k$ is the force acting on node $k$ due to $\mathbf{f}$. This relation can be derived based on the conservation of energy. If $\mathbf{p}$ is embedded within a single element, then the $\mathbf{x}_k$ are simply the element nodes and the $\alpha_i$ are corresponding shape function values; this is known as an *element-based* attachment. On the other hand, as described below, it is sometimes desirable to form an attachment using a more general set of nodes that extends beyond a single element; this is known as a *nodal-based* attachment (Section 6.4.8).

An element-based attachment can be created using a code fragment of the form

```
PointFem3dAttachment ax = new PointFem3dAttachment(pnt);
ax.setFromElement (pnt.getPosition(), elem);
mech.addAttachment (ax);
```

First, a `PointFem3dAttachment` is created for the point `pnt`. Next, `setFromElement()` is used to determine the nodal weights within the element `elem` for the specified position (which in this case is simply the point's current position). To do this, it computes the "natural coordinates" coordinates of the position within the element. For this to be guaranteed to work, the position should be on or inside the element. If natural coordinates cannot be found, the method will return `false` and the nearest estimates coordinates will be used instead. However, it is sometimes possible to find natural coordinates outside a given element as long as the shape functions are well-defined. Finally, the attachment is added to the model.

More conveniently, the exact same functionality is provided by the `attachPoint()` method in `MechModel`:

```
mech.attachPoint (pnt, elem);
```

This creates an attachment identical to that created by the previous code fragment.

Often, one does not want to have to determine the element to which a point should be attached. In that case, one can call

```
PointFem3dAttachment ax = new PointFem3dAttachment(pnt);
ax.setFromFem (pnt.getPosition(), fem);
mech.addAttachment (ax);
```

or, equivalently,

```
mech.attachPoint (pnt, fem);
```

This will find the nearest element to the node in question and use that to create the attachment. If the node is outside the FEM model, then it will be attached to the nearest point on the FEM's surface.

### 6.4.4 Example: connecting two FEMs together

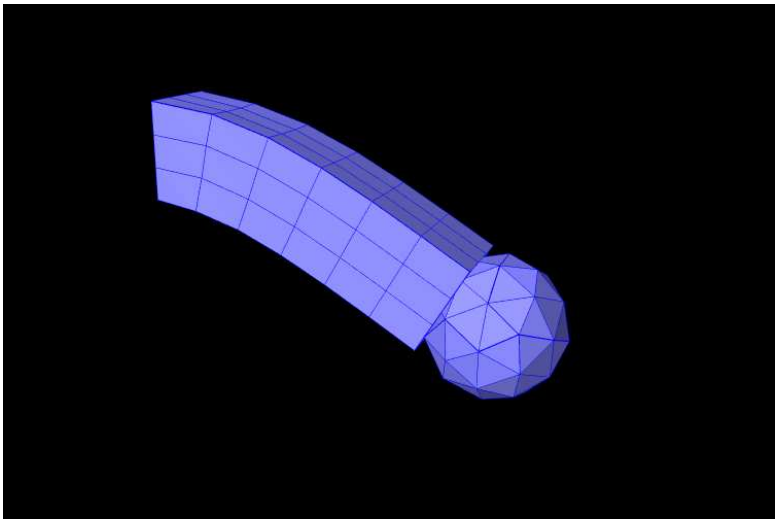


Figure 6.7: FemBeamWithFemSphere model loaded into ArtiSynth.

The following model demonstrates how to attach two FEM models together:

```
1 package artisynth.demos.tutorial;
2
3 import java.io.IOException;
4
5 import maspack.matrix.RigidTransform3d;
6 import artisynth.core.femmodels.*;
7 import artisynth.core.materials.LinearMaterial;
```

```
8  import maspack.util.PathFinder;
9
10 public class FemBeamWithFemSphere extends FemBeam {
11
12    public void build (String[] args) throws IOException {
13
14       // Build simple FemBeam
15       super.build (args);
16
17       // Create a FEM sphere
18       FemModel3d femSphere = new FemModel3d("sphere");
19       mech.addModel(femSphere);
20       // Read from TetGen file
21       TetGenReader.read(femSphere,
22          PathFinder.getSourceRelativePath(FemModel3d.class, "meshes/sphere2.1.node") ↩
     ,
23          PathFinder.getSourceRelativePath(FemModel3d.class, "meshes/sphere2.1.ele")) ↩
     ;
24       femSphere.scaleDistance(0.22);
25       // FEM properties
26       femSphere.setDensity(10);
27       femSphere.setParticleDamping(2);
28       femSphere.setMaterial(new LinearMaterial(4000, 0.33));
29
30       // Reposition FEM to side of beam
31       femSphere.transformGeometry( new RigidTransform3d(length/2+width/2, 0, 0) );
32
33       // Attach sphere nodes that are inside beam
34       for (FemNode3d node : femSphere.getNodes()) {
35          // Find element containing node (if exists)
36          FemElement3d elem = fem.findContainingElement(node.getPosition());
37          // Add attachment if node is inside "fem"
38          if (elem != null) {
39             mech.attachPoint(node, elem);
40          }
41       }
42
43       // Set render properties
44       setRenderProps(femSphere);
45
46    }
47
48 }
```

This example can be found in `artisynth.demos.tutorial.FemBeamWithFemSphere`. The model extends `FemBeam`, adding a finite element sphere and coupling them together. The sphere is created and added on lines 18–28. It is read from TetGen-generated files using the TetGenReader class. The model is then scaled to match the dimensions of the current model, and transformed to the right side of the beam. To create attachments, the code first checks for any nodes that belong to the sphere that fall inside the beam using the FemModel3d.findContainingElement(Point3d) method (line 36), which returns the containing element if the point is inside the model, or `null` if the point is outside. Internally, this spatial search uses a bounding volume hierarchy for efficiency (see BVTree and BVFeatureQuery). If the point is contained within the beam, then `mech.attachPoint()` is used to attach it to the nodes of the element (line 39).

### 6.4.5 Finding which nodes to attach

While it is straightforward to connect nodes to rigid bodies or other FEM nodes or elements, it is often necessary to determine *which* nodes to attach. This was evident in the example of Section 6.4.4, which attached nodes of an FEM sphere that were found to be *inside* an FEM beam.

As in that example, finding the nodes to attach can often be done using geometric queries. For example, we often select nodes based on how close they are to the other body we wish to attach to.

Various prolixity queries are available for this task. To find the distance of a point to a polygonal mesh, we can use the following PolygonalMesh methods,

| | |
|---|---|
| double distanceToPoint(Point3d pnt) | Returns the distance of `pnt` to the mesh. |
| int pointIsInside (Point3d pnt) | Returns `true` if `pnt` is inside the mesh. |

where the latter method returns 1 if the point is inside and 0 otherwise. For checking the distance of an FEM node, `pnt` can be obtained from `node.getPosition()` (or possibly `node.getRestPosition()`). For example, to find all nodes within a distance `tol` of the surface of a rigid body, we could use the code fragment:

```
RigidBody body;
FemModel3d fem;
...
double tol = 0.001;
PolygonalMesh surface = body.getSurfaceMesh();
ArrayList<FemNode3d> nearNodes = new ArrayList<FemNode3d>();
for (FemNode3d n : fem.getNodes()) {
   if (surface.distanceToPoint (n.getPosition()) < tol) {
      nearNodes.add (n);
   }
}
```

If we want to check only nodes that lie on the FEM surface, then we can filter them using the `FemModel3d` method isSurfaceNode():

```
for (FemNode3d n : fem.getNodes()) {
   if (fem.isSurfaceNode (n) &&
       surface.distanceToPoint (n.getPosition()) < tol) {
      nearNodes.add (n);
   }
}
```

Most of the mesh-based query methods work only for *triangular* meshes. The `PolygonalMesh` method isTriangular() can be used to determine if the mesh is triangular. If it is not, it can made triangular by calling triangulate(), although in general this should be done during model construction *before* the mesh-based component has been added to the model.

For connecting an FEM model to another FEM model, `FemModel3d` provides a number of query methods:

| Nearest element queries: | |
|---|---|
| FemElement3dBase findNearestElement( Point3d near, Point3d pnt) | Find nearest element (shell or volumetric) to `pnt`. |
| FemElement3dBase findNearestElement( Point3d near, Point3d pnt, ElementFilter filter) | Find nearest filtered element (shell or volumetric) to `pnt`. |
| FemElement3dBase findNearestSurfaceElement( Point3d near, Point3d pnt) | Find nearest surface element (shell or volumetric) to `pnt`. |
| FemElement3d findNearestVolumetricElement ( Point3d near, Point3d pnt) | Find nearest volumetric element to `pnt`. |
| ShellElement3d findNearestShellElement ( Point3d near, Point3d pnt) | Find nearest shell element to `pnt`. |
| FemElement3d findContainingElement(Point3d pnt) | Find volumetric element (if any) containing `pnt`. |

| Nearest node queries: | |
|---|---|
| FemNode3d findNearestNode ( Point3d pnt, double maxDist) | Find nearest node to pnt that is within `maxDist`. |
| ArrayList<FemNode3d> findNearestNodes ( Point3d pnt, double maxDist) | Find all nodes that are within maxDist of `pnt`. |

All the above queries are based on the FEM model's *current* nodal positions. The method findNearestElement(near,pnt,filter) allows the application to specify a FemModel.ElementFilter to restrict the elements that are searched.

The argument `near` that appears in some of the queries is an optional argument which, if not `null`, returns the location of the corresponding nearest point on the element. The distance from `pnt` to the element can then be found using

```
near.distance (pnt);
```

If the resulting distance is 0, then the point is on or inside the element. Otherwise, the point is outside the element, and if no element filters were used in the query, outside the FEM model itself.

> Typically, it is preferred attach a point to an element only if it lies on or inside an element. However, it is possible to attach points outside an element as long as the system is able to determine appropriate element "coordinates" for that point (which it may not be able to do if the point is far away). In addition, the motion behavior of an exterior attached point may sometimes appear counterintuitive.

The `FemModel3d` element and node queries can be used in a variety of ways.

findNearestNodes() can be used to find all nodes within a certain distance of a point, as part of the process of making nodal-based attachments (Section 6.4.8).

findNearestNode() is used in the FemMuscleBeam example (Section 6.9.5) to determine if a desired muscle point is near enough to a node to use that node directly, or if a marker should be created.

As another example, suppose we wish to connect the surface nodes of an FEM model `femA` to the surface elements of another model `femB` if they lie within a prescribed distance `tol` of the surface of `femB`. Then we could use the following code:

```
MechModel mech;
FemModel3d femA;
FemModel3d femB;
...
double tol = 0.001;
Point3d near = new Point3d();
for (FemNode3d n : femA.getNodes()) {
   if (femA.isSurfaceNode(n)) {
      FemElement3dBase elem =
         femB.findNearestSurfaceElement (near, n.getPosition());
      if (elem != null && near.distance(n.getPosition()) <= tol) {
         // attach if within distance
         mech.attachPoint (n, elem);
      }
   }
}
```

Finally, it is possible to identify nodes on the surface of an FEM model according to whether they belong to specific features, such as a smooth *patch* or a sharp *edge line*. Methods for doing this are provided as static methods in the class FemQuery, and include:

| Feature based node queries: | |
|---|---|
| ArrayList<FemNode3d> findPatchNodes ( FemModel3d fem, FemNode3d node0, double maxBendAng) | Find nodes in patch defined by a maximum bend angle. |
| ArrayList<FemNode3d> findPatchBoundaryNodes ( FemModel3d fem, FemNode3d node0, double maxBendAng) | Find the border nodes of a patch. |
| ArrayList<FemNode3d> findEdgeLineNodes ( FemModel3d fem, FemNode3d node0, double minBendAng, double maxEdgeAng, double allowBranching) | Find the nodes along an edge defined by a minimum bend angle. |

Details of how these methods work are given in their API documentation. They use the notion of a *bend angle*, which is the absolute value of the angle between two faces about their common edge. A patch is defined by a collection of faces

whose bend angles do not exceed a minimum value, while an edge line is a collection of edges with bend angles not below a maximum value. The feature methods start with an initial node (`node0`) and then grow the requested feature out from there. For example, suppose we have a regular hexahedral FEM grid, and we wish to find all the nodes on one of the faces. If we know *one* of the nodes on the face, then we can find *all* of the nodes using `findPatchNodes`:

```
FemModel3d fem =
   FemFactory.createHexGrid (null, 1.0, 0.5, 0.5, 40, 20, 20);

// find any point on the left face
FemNode3d node0 = fem.findNearestNode (new Point3d (-0.5, 0, 0), /*tol=*/1e-8);
// use this to find all nodes on the left face
ArrayList<FemNode3d> nodes =
   FemQuery.findPatchNodes (fem, node0, /*maxBendAngle=*/Math.toRadians(10));
```

Note that the feature query above uses a maximum bend angle of 10°. Because grid faces are flat, this choice is somewhat arbitrary; any angle larger than 0 (within machine precision) would also work.

### 6.4.6 Selecting nodes in the viewer

Often, it is most convenient to select nodes in the ArtiSynth viewer. For this, a node selection tool is available (Figure 6.8), as described in the section "Selecting FEM nodes" of the ArtiSynth User Interface Guide). It allows nodes to be selected in various ways, including the usual click, drag box and elliptical selection tools, as well as specialized operations that select nodes based on patches, edge lines, and distances to other bodies. Once nodes have been selected, their numbers can be saved to a *node number file* which can then be read by a model's `build()` method to determine which nodes to connect to some body.

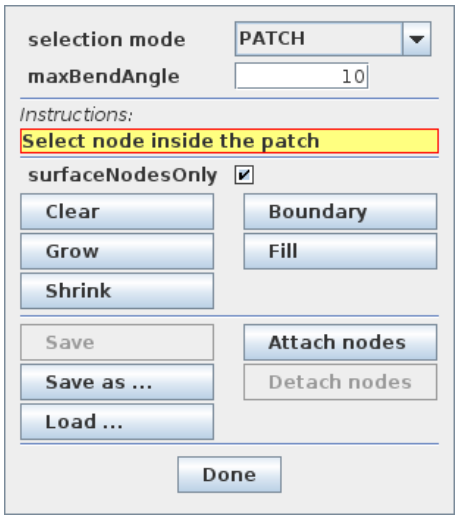



Figure 6.8: FEM node selection tool, described in detail in the User Interface Guide. It allows nodes to be selected, and the selected nodes to be saved to (or loaded from) a node number file.

Node number files are text files with a very simple format, consisting of integers (the node numbers), separated by white space. There is no limit to how many numbers can appear on a line but typically this is limited to ten or so to make the file more readable. Optionally, the numbers can be surrounded by square brackets (`[ ]`). The special character '`#`' is a comment character, commenting out all characters from itself to the end of the current line. For a file containing the node numbers 2, 12, 4, 8, 23 and 47, the following formats are all valid:

```
2 12 4 8 23 47
```

```
[ 2 12 4 8 23 47 ]
```

```
# this is a node number file
[ 2 12 4 8
  23 47
]
```

Once node numbers have been identified in the viewer and saved in a file, they can be read by the `build()` method using a NodeNumberReader. For convenience, this class supplies two static methods for extracting the FEM nodes specified by the numbers in a file:

| | |
|---|---|
| static ArrayList<FemNode3d> read( File file, FemModel3d fem) | Returns nodes in `fem` corresponding to file's numbers. |
| static ArrayList<FemNode3d> read( String filePath, FemModel3d fem) | Returns nodes in `fem` corresponding to file's numbers. |

Extracted nodes can be used to set boundary conditions or form connections with other bodies. For example, suppose we wish to connect a face of an FEM model `fem` to a rigid body `block`, using a set of nodes specified in a file called `faceNodes.txt`. A code fragment to accomplish this could be the following:

```
FemModel3d fem;   // FEM model to attach
RigidBody block; // block to attach fem to
MechModel mech;   // mech model containing fem and block
...

// Get the file path. Assume it is in a folder "geometry" beneath the
// model source folder:
String filePath = getSourceRelativePath ("geometry/faceNodes.txt");

// Read the nodes and attach them to block. Wrap the code in a try/catch
// block to handle I/O errors
try {
   ArrayList<FemNode3d> nodes = NodeNumberReader.read (filePath, fem);
   for (FemNode3d n : nodes) {
      mech.attachPoint (n, block);
   }
}
catch (IOException e) {
   System.out.println ("Couldn't read node file " + filePath);
}
```

The process of selecting nodes in the viewer, saving them in a file, and using them in a model can be done iteratively: if the selected nodes need to be adjusted, one can reopen the node selection tool, load the selection from file, adjust the selection, and resave the file, without needing to make any modifications to the model's `build()` code.

If desired, one can also determine a set of nodes in code, and then write their numbers to a file using the class NodeNumberWriter, which supplies static methods for writing number files:

| | |
|---|---|
| static void write( String filePath, Collection<FemNode3d> nodes) | Writes the numbers of `nodes` to the specified file. |
| static void write( File file, Collection<FemNode3d> nodes) | Writes the numbers of `nodes` to `file`. |
| static void write( File file, Collection<FemNode3d> nodes, int maxCols, int flags) | Writes the numbers of `nodes` to `file`, using the specified number of columns and format flags. |

For example:

```
FemModel3d fem;   // FEM model
...

// find the nodes to write:
ArrayList<FemNode3d> nodes = new ArrayList<>();
for (FemNode3d n : nodes) {
   if (n satisfies appropriate criteria) {
      nodes.add (n);
   }
}
// write the node numbers, using a try/catch block to handle I/O errors
```

```
   String filePath = getSourceRelativePath ("geometry/special.txt");
   try {
      NodeNumberWriter.write (filePath, nodes);
   }
   catch (IOException e) {
      System.out.println ("Couldn't write node file " + filePath);
   }
```

### 6.4.7 Example: two bodies connected by an FEM "spring"

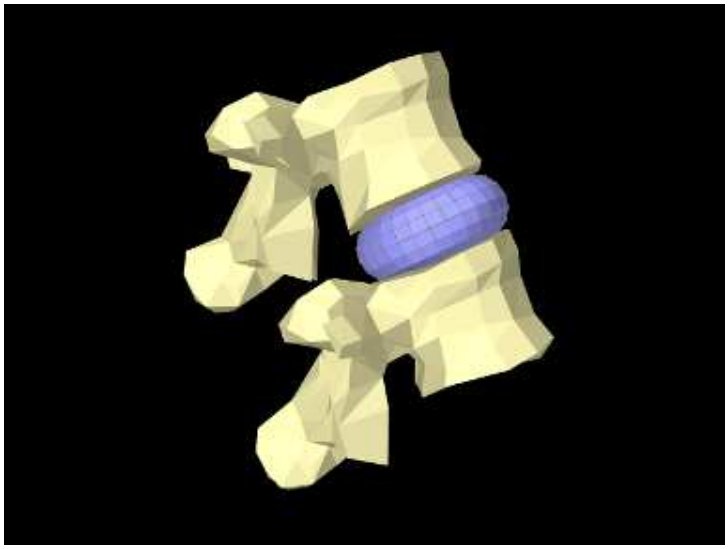


Figure 6.9: `LumbarFEMDisk` loaded into ArtiSynth, showing a simplified FEM model connecting two vertebrae.

The `LumbarFrameSpring` example in Section 3.5.4 uses a frame spring to connect two simplified lumbar vertebrae. However, it is also possible to use an FEM model in place of a frame spring, possibly providing a more realistic model of the intervertebral disc. A simple model which does this is defined in

`artisynth.demos.tutorial.LumbarFEMDisk`

The initial source code is similar to that for `LumbarFrameSpring`, but differs in the section where the FEM disk replaces the `FrameSpring`:

```
56       // create a torus shaped FEM model for the disk
57       FemModel3d fem = new FemModel3d();
58       fem.setDensity (1500);
59       fem.setMaterial (new LinearMaterial (20000, 0.4));
60       FemFactory.createHexTorus (fem, 0.011, 0.003, 0.008, 16, 30, 2);
61       mech.addModel (fem);
62
63       // position it betweem the disks
64       double DTOR = Math.PI/180.0;
65       fem.transformGeometry (
66          new RigidTransform3d (-0.012, 0.0, 0.040, 0, -DTOR*25, DTOR*90));
67
68       // find and attach nearest nodes to either the top or bottom mesh
69       double tol = 0.001;
70       for (FemNode3d n : fem.getNodes()) {
71          // top vertebra
72          double d = lumbar1.getSurfaceMesh().distanceToPoint(n.getPosition());
73          if (d < tol) {
74             mech.attachPoint (n, lumbar1);
75          }
```

```
76            // bottom vertebra
77            d = lumbar2.getSurfaceMesh().distanceToPoint(n.getPosition());
78            if (d < tol) {
79               mech.attachPoint (n, lumbar2);
80            }
81         }
82         // set render properties ...
83         fem.setSurfaceRendering (SurfaceRender.Shaded);
84         RenderProps.setFaceColor (fem, new Color (153/255f, 153/255f, 1f));
85         RenderProps.setFaceColor (mech, new Color (238, 232, 170)); // bone color
86      }
87   }
```

The simplified FEM model representing the "disk" is created at lines 57-61, using a torus-shaped model created by `FemFactory`. It is then repositioning using `transformGeometry()` ( Section 4.7) to place it between the vertebrae (line 64-66). After the FEM model is positioned, we find which nodes are within a distance `tol` of each vertebral surface and attach them to the appropriate body (lines 69-81).

To run this example in ArtiSynth, select All demos > tutorial > LumbarFEMDisk from the Models menu. The model should load and initially appear as in Figure 6.9. The behavior is best seem by running the model and using the pull controller to exert forces on the upper vertebra.

### 6.4.8 Nodal-based attachments

The example of Section 6.4.4 uses element-based attachments to connect the nodes of one FEM to elements of another. As mentioned above, element-based attachments assume that the attached point is associated with a specific FEM model element. While this often gives good results, there are situations where it may be desirable to distribute the connection more broadly among a larger set of nodes.

In particular, this is sometimes the case when connecting FEM models to point-to-point springs. The end-point of such a spring may end up exerting a large force on the FEM, and then if the number of nodes to which the end-point is attached are too small, the resulting forces on these nodes (Equation 6.4) may end up being too large. In other words, it may be desirable to distribute the spring's force more evenly throughout the FEM model.

To handle such situations, it is possible to create a *nodal-based* attachment in which the nodes and weights are explicitly specified. This involves explicitly creating a `PointFem3dAttachment` for the point or particle to be attached and the specifying the nodes and weights directly,

```
PointFem3dAttachment ax = new PointFem3dAttachment (part);
ax.setFromNodes (nodes, weights);
mech.addAttachment (ax);
```

where `nodes` and `weights` are arrays of `FemNode` and `double`, respectively. It is up to the application to determine these.

PointFem3dAttachment provides several methods for explicitly specifying nodes and weights. The signatures for these include:

```
void setFromNodes (FemNode[] nodes, double[] weights)
void setFromNodes (Collection<FemNode> nodes, VectorNd weights)
boolean setFromNodes (Point3d pos, FemNode[] nodes)
boolean setFromNodes (Point3d pos, Collection<FemNode> nodes)
```

The last two methods determine the weights automatically, using an inverse-distance-based calculation in which each weight $\alpha_k$ is initially computed as

$$\alpha_k = \frac{d_{\max}}{d_k + d_{max}} \tag{6.5}$$

where $d_k$ is the distance from node $k$ to `pos` and $d_{\max}$ is the maximum distance. The weights are then adjusted to ensure that they sum to one and that the weighted sum of the nodes equals `pos`. In some cases, the specified nodes may not provide enough support for the last condition to be met, in which case the methods return `false`.

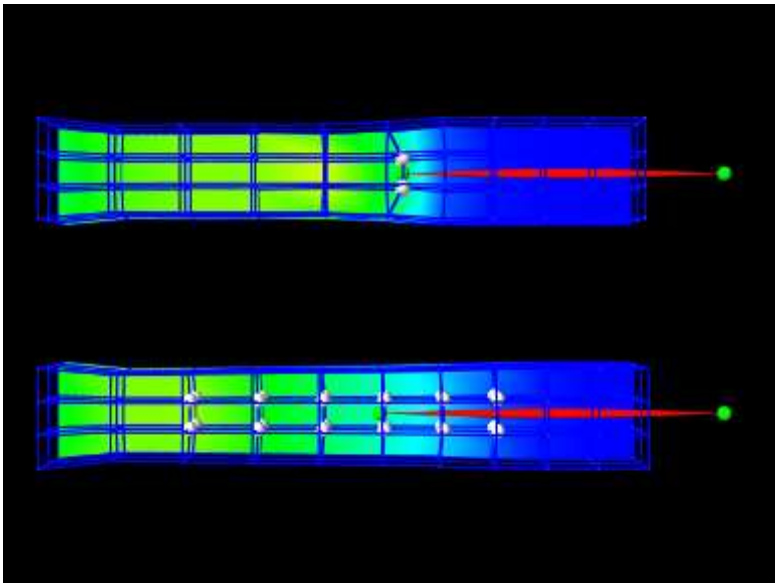


Figure 6.10: PointFemAttachment loaded into ArtiSynth and run until stable. The top and bottom models are connected to their springs using element and nodal-based attachments, respectively. The nodes associated with each attachment are rendered as white spheres.

### 6.4.9 Example: element vs. nodal-based attachments

The model demonstrating the difference between element and nodal-based attachments is defined in

```
artisynth.demos.tutorial.PointFemAttachment
```

It creates two FEM models, each with a single point-to-point spring attached to a particle at their center. The model at the top (`fem1` in the code below) is connected to the particle using an element-based attachment, while the lower model (`fem2` in the code) is connected using a nodal-based attachment with a larger number of nodes. Figure 6.10 shows the result after the model is run until stable. The element-based attachment results in significantly higher deformation in the immediate vicinity around the attachment, while for the nodal-based attachment, the deformation is distributed much more evenly through the model.

The build method and some of the auxiliary code for this model is shown below. Code for the other auxiliary methods, including `addFem()`, `addParticle()`, `addSpring()`, and `setAttachedNodesWhite()`, can be found in the actual source file.

```
1     // Filter to select only elements for which the nodes are entirely on the
2     // positive side of the x-z plane.
3     private class MyFilter extends ElementFilter {
4        public boolean elementIsValid (FemElement e) {
5           for (FemNode n : e.getNodes()) {
6              if (n.getPosition().y < 0) {
7                 return false;
8              }
9           }
10          return true;
11       }
12    }
13
14    // Collect and return all the nodes of an FEM model associated with a
15    // set of elements specified by an array of element numbers
16    private HashSet<FemNode3d> collectNodes (FemModel3d fem, int[] elemNums) {
17       HashSet<FemNode3d> nodes = new LinkedHashSet<FemNode3d>();
18       for (int i=0; i<elemNums.length; i++) {
19          FemElement3d e = fem.getElements().getByNumber (elemNums[i]);
20          for (FemNode3d n : e.getNodes()) {
21             nodes.add (n);
```

```
22            }
23         }
24         return nodes;
25      }
26
27      public void build (String[] args) {
28         MechModel mech = new MechModel ("mech");
29         addModel (mech);
30         mech.setGravity (0, 0, 0); // turn off gravity
31
32         // create and add two FEM beam models centered at the specified locations
33         FemModel3d fem1 = addFem (mech,  0.0, 0.0,  0.25);
34         FemModel3d fem2 = addFem (mech,  0.0, 0.0, -0.25);
35
36         // reconstruct the FEM surface meshes so that they show only elements on
37         // the positive side of the x-y plane. Also, set surface rendering to
38         // show strain values.
39         fem1.createSurfaceMesh (new MyFilter());
40         fem1.setSurfaceRendering (SurfaceRender.Strain);
41         fem2.createSurfaceMesh (new MyFilter());
42         fem2.setSurfaceRendering (SurfaceRender.Strain);
43
44         // create and add the particles for the point-to-point springs
45         // that will apply forces to each FEM.
46         Particle p1 = addParticle (mech,  0.9, 0.0,  0.25);
47         Particle p2 = addParticle (mech,  0.9, 0.0, -0.25);
48         Particle m1 = addParticle (mech,  0.0, 0.0,  0.25);
49         Particle m2 = addParticle (mech,  0.0, 0.0, -0.25);
50
51         // attach spring end-point to fem1 using an element-based marker
52         mech.attachPoint (m1, fem1);
53
54         // attach spring end-point to fem2 using a larger number of nodes, formed
55         // from the node set for elements 22, 31, 40, 49, and 58. This is done by
56         // explicitly creating the attachment and then setting it to use the
57         // specified nodes
58         HashSet<FemNode3d> nodes =
59            collectNodes (fem2, new int[] { 22, 31, 40, 49, 58 });
60
61         PointFem3dAttachment ax = new PointFem3dAttachment (m2);
62         ax.setFromNodes (m2.getPosition(), nodes);
63         mech.addAttachment (ax);
64
65         // finally, create the springs
66         addSpring (mech, /*stiffness=*/10000, p1, m1);
67         addSpring (mech, /*stiffness=*/10000, p2, m2);
68
69         // set the attachments nodes for m1 and m2 to render as white spheres
70         setAttachedNodesWhite (m1);
71         setAttachedNodesWhite (m2);
72         // set render properties for m1
73         RenderProps.setSphericalPoints (m1, 0.015, Color.GREEN);
74      }
```

The `build()` method begins by creating a `MechModel` and then adding to it two FEM beams (created using the auxiliary method `addFem()`. Rendering of each FEM model's surface is then set up to show strain values (`setSurfaceRendering()`, lines 41 and 43). The surface meshes themselves are also redefined to exclude the frontmost elements, allowing the strain values to be displayed closer model centers. This redefinition is done using calls to `createSurfaceMesh()` (lines 40, 41) with a custom `ElementFilter` defined at lines 3-12.

Next, the end-point particles for the axial springs are created (using the auxiliary method `addParticle()`, lines 46-49), and particle `m1` is attached to `fem1` using `mech.attachPoint()` (line 52), which creates an element-based attachment at the point's current location. Point `m2` is then attached to `fem2` using a nodal-based attachment. The nodes for these are collected as the union of all nodes for a specified set of elements (lines 58-59, and the method `collectNodes()` defined

at lines 16-25). These are then used to create a nodal-based attachment (lines 61-63), where the weights are determined automatically using the method associated with equation (6.5).

Finally, the springs are created (auxiliary method `addSpring()`, lines 66-67), the nodes associated for each attachment are set to render as white spheres (`setAttachedNodesWhites()`, lines 70-71), and the particles are set to render as green spheres.

To run this example in ArtiSynth, select All demos > tutorial > PointFemAttachment from the Models menu. Running the model will cause it to settle into the state shown in Figure 6.10. Selecting and dragging one of the spring anchor points at the right will cause the spring tension to vary and further illustrate the difference between the element and nodal-based attachments.

## 6.5 FEM markers

Just as there are `FrameMarkers` to act as anchor points on a frame or rigid body (Section 3.2.1), there are also `FemMarkers` that can mark a point inside a finite element. They are frequently used to provide anchor points for attaching springs and forces to a point inside an element, but can also be used for graphical purposes.

FEM markers are implemented by the class FemMarker, which is a subclass of Point. They are essentially massless points that contain their own attachment component, so when creating and adding a marker there is no need to create a separate attachment component.

Within the component hierarchy, FEM markers are typically stored in the `markers` list of their associated FEM model. They can be created and added using a code fragment of the form

```
FemMarker mkr = new FemMarker (1, 0, 0);
mkr.setFromFem (fem); // attach to the nearest fem element
fem.addMarker (mkr);  // add to fem
```

This creates a marker at the location $(1,0,0)$ (in world coordinates), calls `setFromFem()` to attach it to the nearest element in the FEM model ( which is either the containing element or the nearest element on the model's surface), and then adds it to the `markers` list.

If the marker's attachment has not already been set when `addMarker()` is called, then `addMarker()` will call `setFromFem()` automatically. Therefore the above code fragment is equivalent to the following:

```
FemMarker mkr = new FemMarker (1, 0, 0);
fem.addMarker (mkr);
```

Alternatively, one may want to explicitly specify the nodes associated with the attachment, as described in Section 6.4.8:

```
FemMarker mkr = new FemMarker (1, 0, 0);
mkr.setFromNodes (nodes, weights);
fem.addMarker (mkr);
```

There are a variety of methods available to set the attachment, mirroring those available in the underlying base class `PointFem3dAttachment`:

```
void setFromFem (FemModel3d fem)
boolean setFromElement (FemElement3d elem)
void setFromNodes (FemNode[] nodes, double[] weights)
void setFromNodes (Collection<FemNode> nodes, VectorNd weights)
boolean setFromNodes (FemNode[] nodes)
boolean setFromNodes (Collection<FemNode> nodes)
```

The last two methods compute nodal weights automatically, as described in Section 6.4.8, based on the marker's currently assigned position. If the supplied nodes do not provide sufficient support, then the methods return `false`.

Another set of convenience methods are supplied by `FemModel3d`, which combine these with the `addMarker()` call:

```
void addMarker (FemMarker mkr, FemElement3d elem)
void addMarker (FemMarker mkr, FemNode[] nodes, double[] weights)
void addMarker (FemMarker mkr, Collection<FemNode> nodes, VectorNd weights)
boolean addMarker (FemMarker mkr, FemNode[] nodes)
boolean addMarker (FemMarker mkr, Collection<FemNode> nodes)
```

For example, one can do

```
FemMarker mkr = new FemMarker (1, 0, 0);
fem.addMarker (mkr, nodes, weights);
```

Markers are often used to track movement within an FEM model. For that, one can examine their positions and velocities, as with any other particles, using the methods

```
Point3d getPosition();     // returns the current position
Vector3d getVelocity();     // returns the current velocity
```

The return values from these methods should not be modified. Alternatively, when a 3D force **f** is applied to the marker, it is distributed to the attached nodes according to the nodal weights, as described in Equation (6.4).

### 6.5.1 Example: attaching an FEM beam to a muscle

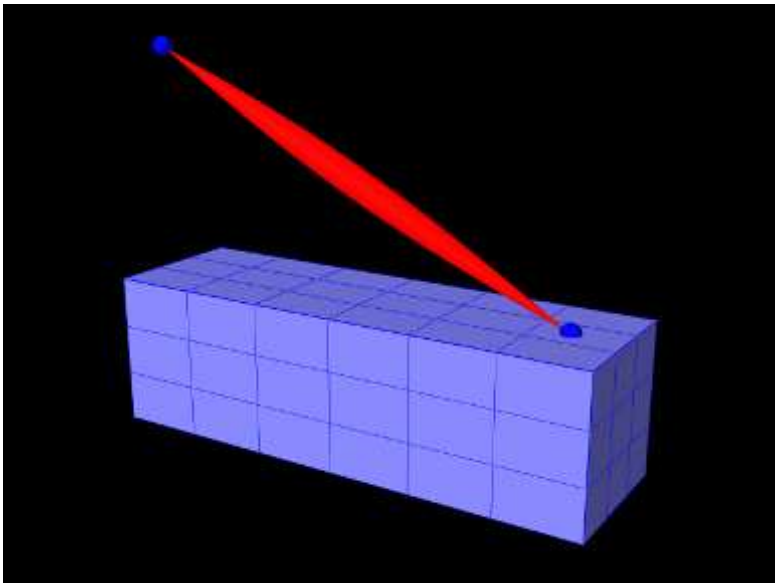


Figure 6.11: FemBeamWithMuscle model loaded into ArtiSynth.

A complete application model that employs a fem marker as an anchor for a spring is given below.

```
1  package artisynth.demos.tutorial;
2
3  import java.awt.Color;
4  import java.io.IOException;
5
6  import maspack.render.RenderProps;
7  import maspack.render.Renderer;
8  import artisynth.core.femmodels.FemMarker;
9  import artisynth.core.femmodels.FemModel3d;
10 import artisynth.core.materials.SimpleAxialMuscle;
11 import artisynth.core.mechmodels.Muscle;
12 import artisynth.core.mechmodels.Particle;
13 import artisynth.core.mechmodels.Point;
14
15 public class FemBeamWithMuscle extends FemBeam {
16
17    // Creates a point-to-point muscle
18    protected Muscle createMuscle () {
19       Muscle mus = new Muscle (/*name=*/null, /*restLength=*/0);
20       mus.setMaterial (
21          new SimpleAxialMuscle (/*stiffness=*/20, /*damping=*/10, /*maxf=*/10));
22       RenderProps.setLineStyle (mus, Renderer.LineStyle.SPINDLE);
```

```
23        RenderProps.setLineColor (mus, Color.RED);
24        RenderProps.setLineRadius (mus, 0.03);
25        return mus;
26     }
27
28     // Creates a FEM Marker
29     protected FemMarker createMarker (
30        FemModel3d fem, double x, double y, double z) {
31        FemMarker mkr = new FemMarker (/*name=*/null, x, y, z);
32        RenderProps.setSphericalPoints (mkr, 0.02, Color.BLUE);
33        fem.addMarker (mkr);
34        return mkr;
35     }
36
37     public void build (String[] args) throws IOException {
38
39        // Create simple FEM beam
40        super.build (args);
41
42        // Add a particle fixed in space
43        Particle p1 = new Particle (/*mass=*/0, -length/2, 0, 2*width);
44        mech.addParticle (p1);
45        p1.setDynamic (false);
46        RenderProps.setSphericalPoints (p1, 0.02, Color.BLUE);
47
48        // Add a marker at the end of the model
49        FemMarker mkr = createMarker (fem, length/2-0.1, 0, width/2);
50
51        // Create a muscle between the point an marker
52        Muscle muscle = createMuscle();
53        muscle.setPoints (p1, mkr);
54        mech.addAxialSpring (muscle);
55     }
56
57  }
```

This example can be found in `artisynth.demos.tutorial.FemBeamWithMuscle`. This model extends the `FemBeam` example, adding a `FemMarker` for the spring to attach to. The method `createMarker(...)` on lines 29–35 is used to create and add a marker to the FEM. Since the element is initially set to `null`, when it is added to the FEM, the model searches for the containing or nearest element. The loaded model is shown in Figure 6.11.

## 6.6 Frame attachments

It is also possible to attach frame components, including rigid bodies, directly to FEM models, using the attachment component FrameFem3dAttachment. Analogously to PointFem3dAttachment, the attachment is implemented by connecting the frame to a set of FEM nodes, and attachments can be either element-based or nodal-based. The frame's origin is computed in the same way as for point attachments, using a weighted sum of node positions (Equation 6.3), while the orientation is computed using a polar decomposition on a deformation gradient determined from either element shape functions (for element-based attachments) or a Procrustes type analysis using nodal rest positions (for nodal-based attachments).

An element-based attachment can be created using either a code fragment of the form

```
FrameFem3dAttachment ax = new FrameFem3dAttachment(frame);
ax.setFromElement (frame.getPose(), elem);
mech.addAttachment (ax);
```

or, equivalently, the `attachFrame()` method in `MechModel`:

```
mech.attachFrame (frame, elem);
```

This attaches the frame `frame` to the nodes of the FEM element `elem`. As with `PointFem3dAttachment`, if the frame's origin is not inside the element, it may not be possible to accurately compute the internal nodal weights, in which case `setFromElement()` will return `false`.

In order to have the appropriate element located automatically, one can instead use

```
FrameFem3dAttachment ax = new FrameFem3dAttachment(frame);
ax.setFromFem (frame.getPose(), fem);
mech.addAttachment (ax);
```

or, equivalently,

```
mech.attachFrame (frame, fem);
```

As with point-to-FEM attachments, it may be desirable to create a nodal-based attachment in which the nodes and weights are not tied to a specific element. The reasons for this are generally the same as with nodal-based point attachments (Section 6.4.8): the need to distribute the forces and moments acting on the frame across a broader set of element nodes. Also, element-based frame attachments use element shape functions to determine the frame's orientation, which may produce slightly asymmetric results if the frame's origin is located particularly close to a specific node.

FrameFem3dAttachment provides several methods for explicitly specifying nodes and weights. The signatures for these include:

```
void setFromNodes (RigidTransform3d TFW, FemNode[] nodes, double[] weights)
void setFromNodes (RigidTransform3d TFW, Collection<FemNode> nodes,
                   VectorNd weights)
boolean setFromNodes (RigidTransform3d TFW, FemNode[] nodes)
boolean setFromNodes (RigidTransform3d TFW, Collection<FemNode> nodes)
```

Unlike their counterparts in `PointFem3dAttachment`, the first two methods also require the current desired pose of the frame `TFW` (in world coordinates). This is because while nodes and weights will unambiguously specify the frame's origin, they do not specify the desired orientation.

### 6.6.1 Example: attaching frames to an FEM beam

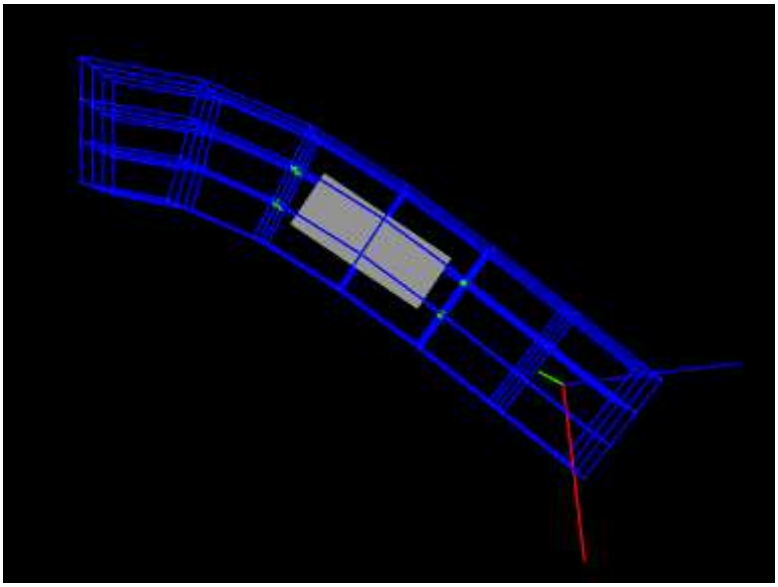


Figure 6.12: FrameFemAttachment loaded into ArtiSynth and run until stable.

A model illustrating how to connect frames to an FEM model is defined in

```
artisynth.demos.tutorial.FrameFemAttachment
```

It creates an FEM beam, along with a rigid body block and a massless coordinate frame, that are then attached to the beam using nodal and element-based attachments. The build method is shown below:

```
1     public void build (String[] args) {
2
3        MechModel mech = new MechModel ("mech");
4        addModel (mech);
5
6        // create and add FEM beam
7        FemModel3d fem = FemFactory.createHexGrid (null, 1.0, 0.2, 0.2, 6, 3, 3);
8        fem.setMaterial (new LinearMaterial (500000, 0.33));
9        RenderProps.setLineColor (fem, Color.BLUE);
10       RenderProps.setLineWidth (mech, 2);
11       mech.addModel (fem);
12       // fix leftmost nodes of the FEM
13       for (FemNode3d n : fem.getNodes()) {
14          if ((n.getPosition().x-(-0.5)) < 1e-8) {
15             n.setDynamic (false);
16          }
17       }
18
19       // create and add rigid body box
20       RigidBody box = RigidBody.createBox (
21          "box", 0.25, 0.1, 0.1, /*density=*/1000);
22       mech.add (box);
23
24       // create a basic frame and set its pose and axis length
25       Frame frame = new Frame();
26       frame.setPose (new RigidTransform3d (0.4, 0, 0, 0, Math.PI/4, 0));
27       frame.setAxisLength (0.3);
28       mech.addFrame (frame);
29
30       mech.attachFrame (frame, fem); // attach using element-based attachment
31
32       // attach the box to the FEM, using all the nodes of elements 31 and 32
33       HashSet<FemNode3d> nodes = collectNodes (fem, new int[] { 22, 31 });
34       FrameFem3dAttachment attachment = new FrameFem3dAttachment(box);
35       attachment.setFromNodes (box.getPose(), nodes);
36       mech.addAttachment (attachment);
37
38       // render the attachment nodes for the box as spheres
39       for (FemNode n : attachment.getNodes()) {
40          RenderProps.setSphericalPoints (n, 0.007, Color.GREEN);
41       }
42    }
```

Lines 3-22 create a `MechModel` and populate it with an FEM beam and a rigid body box. Next, a basic `Frame` is created, with a specified pose and an axis length of 0.3 (to allow it to be seen), and added to the `MechModel` (lines 25-28). It is then attached to the FEM beam using an element-based attachment (line 30). Meanwhile, the box is attached to using a nodal-based attachment, created from all the nodes associated with elements 22 and 31 (lines 33-36). Finally, all attachment nodes are set to be rendered as green spheres (lines 39-41).

To run this example in ArtiSynth, select All demos > tutorial > FrameFemAttachment from the Models menu. Running the model will cause it to settle into the state shown in Figure 6.12. Forces can interactively be applied to the attached block and frame using the pull tool, causing the FEM model to deform (see the section "Pull Manipulation" in the ArtiSynth User Interface Guide).

### 6.6.2 Adding joints to FEM models

The ability to connect frames to FEM models, as described in Section 6.6, makes it possible to interconnect different FEM models directly using joints, as described in Section 3.3. This is done internally by using `FrameFem3dAttachments` to connect frames C and D of the joint (Figure 3.8) to their respective FEM models.

As indicated in Section 3.3.3, most joints have a constructor of the form

```
JointType (bodyA, bodyB, TDW);
```

that creates a joint connecting `bodyA` to `bodyB`, with the initial pose of the D frame given (in world coordinates) by `TDW`. The same body and transform settings can be made on an existing joint using the method setBodies(bodyA, bodyB, TDW). For these constructors and methods, it is possible to specify FEM models for `bodyA` and/or `bodyB`. Internally, the joint then creates a `FrameFem3dAttachment` to connect frame C and/or D of the joint (See Figure 3.8) to the corresponding FEM model.

However, unlike joints involving rigid bodies or frames, there are no associated $\mathbf{T}_{CA}$ or $\mathbf{T}_{DB}$ transforms (since there is no fixed frame within an FEM to define such transforms). Methods or constructors which utilize $\mathbf{T}_{CA}$ or $\mathbf{T}_{DB}$ can therefore not be used with FEM models.

### 6.6.3 Example: two FEM beams connected by a joint

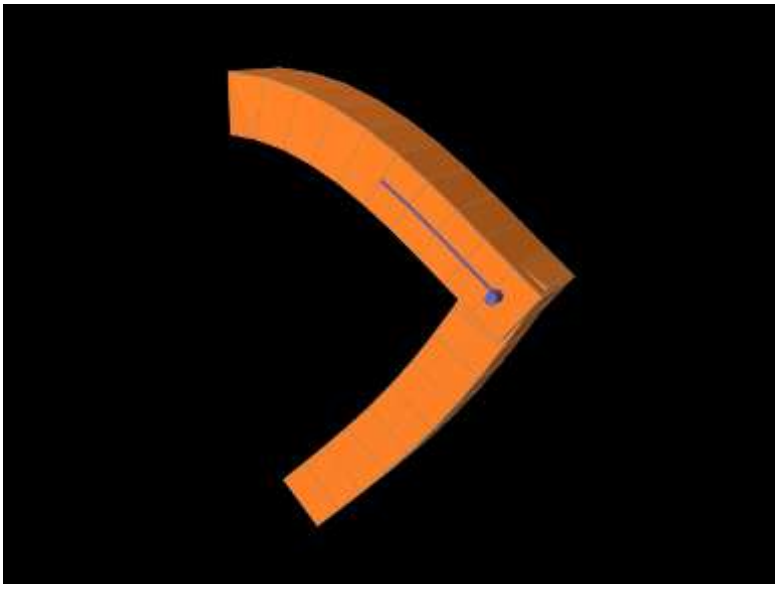


Figure 6.13: JointedFemBeams loaded into ArtiSynth and run until stable.

A model connecting two FEM beams by a joint is defined in

```
artisynth.demos.tutorial.JointedFemBeams
```

It creates two FEM beams and connects them via a special slotted-revolute joint. The build method is shown below:

```
1    public void build (String[] args) {
2
3       MechModel mech = new MechModel ("mechMod");
4       addModel (mech);
5
6       double stiffness = 5000;
7       // create first fem beam and fix the leftmost nodes
8       FemModel3d fem1 = addFem (mech, 2.4, 0.6, 0.4, stiffness);
9       for (FemNode3d n : fem1.getNodes()) {
10         if (n.getPosition().x <= -1.2) {
11            n.setDynamic(false);
12         }
13      }
14      // create the second fem beam and shift it 1.5 to the right
15      FemModel3d fem2 = addFem (mech, 2.4, 0.4, 0.4, 0.1*stiffness);
16      fem2.transformGeometry (new RigidTransform3d (1.5, 0, 0));
17
18      // create a slotted revolute joint that connects the two fem beams
19      RigidTransform3d TDW = new RigidTransform3d(0.5, 0, 0, 0, 0, Math.PI/2);
```

```
20        SlottedRevoluteJoint joint = new SlottedRevoluteJoint (fem2, fem1, TDW);
21        mech.addBodyConnector (joint);
22
23        // set ranges and rendering properties for the joint
24        joint.setShaftLength (0.8);
25        joint.setMinX (-0.5);
26        joint.setMaxX (0.5);
27        joint.setSlotDepth (0.63);
28        joint.setSlotWidth (0.08);
29        RenderProps.setFaceColor (joint, myJointColor);
30     }
```

Lines 3-16 create a `MechModel` and populates it with two FEM beams, `fem1` and `fem2`, using an auxiliary method `addFem()` defined in the model source file. The leftmost nodes of `fem1` are set fixed. A `SlottedRevoluteJoint` is then created to interconnect `fem1` and `fem2` at a location specified by `TDW` (lines 19-21). Lines 24-29 set some parameters for the joint, along with various render properties.

To run this example in ArtiSynth, select All demos > tutorial > JointedFemBeams from the Models menu. Running the model will cause it drop and flex under gravity, as shown in 6.13. Forces can interactively be applied to the beams using the pull tool (see the section "Pull Manipulation" in the ArtiSynth User Interface Guide).

## 6.7 Incompressibility

FEM incompressibility within ArtiSynth is enforced by trying to ensure that the volume of an FEM remains locally constant. This, in turn, is accomplished by constraining nodal velocities so that the local volume change, or *divergence*, is zero (or close to zero). There are generally two ways to do this:

- *Hard incompressibility*, which sets up explicit constraints on the nodal velocities;
- *Soft incompressibility*, which uses a restoring pressure based on a potential field to try to keep the volume constant.

Both of these methods operate independently, and both can be used either separately or together. Generally speaking, hard incompressibility will result in incompressibility being more rigorously enforced, but at the cost of increased computation time and (sometimes) less stability. Soft incompressibility allows the application to control the restoring force used to enforce incompressibility, usually by adjusting the value of the *bulk modulus* material property. As the bulk modulus is increased, soft incompressibility starts to act more like 'hard' incompressibility, with an infinite bulk modulus corresponding to perfect incompressibility. However, very large bulk modulus values will generally produce stability problems.

> Incompressibility is not currently implemented for shell elements. Applying hard incompressibility to a shell element will have no effect on its behavior. If soft incompressibility is applied, by supplying the element with an incompressible material, then only the deviatoric component of that material will have any effect; the dilational component will generate no stress.

### 6.7.1 Volume regions and locking

Both hard and soft incompressibility can be applied to different regions of local volume. From larger to smaller, these regions are:

- *Nodal* - the local volume surrounding each node;
- *Element* - the volume of each element;
- *Full* - the volume at each integration point.

Element-based incompressibility is the standard method generally seen in the literature. However, it tends not to work well for tetrahedral meshes, because constraining the volume of each tet in a tetrahedral mesh tends to over constrain the system. This is because the number of tets in a large tetrahedral mesh is often $O(5n)$, where $n$ is the number of nodes, and so putting a volume constraint on each element may result in $O(5n)$ constraints, which exceeds the $3n$ degrees of freedom (DOF) in the FEM. This overconstraining results in an artificially increased stiffness known as *locking*. Because of locking, for tetrahedrally based meshes it may be better to use nodal-based incompressibility, which creates a single volume constraint around each node, resulting in only $n$ constraints, leaving $2n$ DOF to handle the remaining deformation. However, nodal-based incompressibility is computationally more costly than element-based and may not be as stable.

Generally, the best solution for incompressible problems is to use element-based incompressibility with a mesh consisting of hexahedra, or primarily hexahedra and a mix of other elements (the latter commonly being known as a *hex dominant* mesh). For hex-based meshes, the number of elements is roughly equal to the number of nodes, and so adding a volume constraint for each element imposes $n$ constraints on the model, which (like nodal incompressibility) leaves $2n$ DOF to handle the remaining deformation.

Full incompressibility tries to control the volume at each integration point within each element, which almost always results in a large number of volumetric constraints and hence locking. It is therefore not commonly used and is provided mostly for debugging and diagnostic purposes.

### 6.7.2 Hard incompressibility

Hard incompressibility is controlled by the incompressible property of the FEM, which can be set to one of the following values of the enumerated type `FemModel.IncompMethod`:

**OFF** No hard incompressibility enforced.

**ELEMENT** Element-based hard incompressibility enforced (Section 6.7.1).

**NODAL** Nodal-based hard incompressibility enforced (Section 6.7.1).

**AUTO** Selects either `ELEMENT` or `NODAL`, with the former selected if the number of elements is less than or equal to the number of nodes.

**ON** Same as `AUTO`.

Hard incompressibility uses explicit constraints on the nodal velocities to enforce the incompressibility, which increases computational cost. Also, if the number of constraints is too large, *perturbed pivot* errors may be encountered by the solver. However, hard incompressibility can in principle handle situations where complete incompressibility is required. It is equivalent to the mixed u-P formulation used in commercial FEM codes (such as ANSYS), and the Lagrange multipliers computed for the constraints are pressure impulses.

Hard incompressibility can be applied in addition to soft incompressibility, in which case it will provide additional incompressibility enforcement on top of that provided by the latter. It can also be applied to linear materials, which are not themselves able to emulate true incompressible behavior (Section 6.7.4).

### 6.7.3 Soft incompressibility

Soft incompressibility enforces incompressibility using a restoring pressure that is controlled by a volume-based energy potential. It is only available for FEM materials that are subclasses of IncompressibleMaterial. The energy potential $U(J)$ is a function of the determinant $J$ of the deformation gradient, and is scaled by the material's *bulk modulus* $\kappa$. The restoring pressure $p$ is given by

$$p = \frac{\partial U}{\partial J}. \tag{6.6}$$

Different potentials can be selected by setting the bulkPotential property of the incompressible material, whose value is an instance of `IncompressibleMaterial.BulkPotential`. Currently there are two different potentials:

**QUADRATIC** The potential and associated pressure are given by

$$U(J) = \frac{1}{2}\kappa(J-1)^2, \quad p = \kappa(J-1). \tag{6.7}$$

**LOGARITHMIC** The potential and associated pressure are given by

$$U(J) = \frac{1}{2}\kappa(\ln J)^2, \quad p = \kappa\frac{\ln J}{J} \tag{6.8}$$

The default potential is `QUADRATIC`, which may provide slightly improved stability characteristics. However, we have not noticed significant differences between the two potentials in practice.

How soft incompressibility is applied within an FEM model is controlled by the FEM's softIncompMethod property, which can be set to one of the following values of the enumerated type `FemModel.IncompMethod`:

**ELEMENT** Element-based soft incompressibility enforced (Section 6.7.1).

**NODAL** Nodal-based soft incompressibility enforced (Section 6.7.1).

**AUTO** Selects either `ELEMENT` or `NODAL`, with the former selected if the number of elements is less than or equal to the number of nodes.

**FULL** Incompressibility enforced at each integration point (Section 6.7.1).

### 6.7.4 Incompressibility and linear materials

Within a linear material, incompressibility is controlled by Poisson's ratio $\nu$, which for isotropic materials can assume a value in the range $[-1, 0.5]$. This specifies the amount of transverse contraction (or expansion) exhibited by the material as it compressed or extended along a particular direction. A value of 0 allows the material to be compressed or extended without any transverse contraction or expansion, while a value of 0.5 in theory indicates a perfectly incompressible material. However, setting $\nu = 0.5$ in practice causes a division by zero, so only values close to 0.5 (such as 0.49) can be used.

Moreover, the incompressibility only applies to small displacements, so that even with $\nu = 0.49$ it is still possible to squash a linear FEM completely flat if enough force is applied. If true incompressible behavior is desired with a linear material, then one must also use hard incompressibility (Section 6.7.2).

### 6.7.5 Using incompressibility in practice

As mentioned above, when modeling incompressible models, we have found that the best practice is to use, if possible, either a hex or hex-dominant mesh, along with element-based incompressibility.

Hard incompressibility allows the handling of full incompressibility but at the expense of greater computational cost and often less stability. When modeling biomechanical materials, it is often permissible to use only soft incompressibility, partly since biomechanical materials are rarely completely incompressible. When implementing soft incompressibility, it is common practice to set the bulk modulus to something like 100 times the other (deviatoric) stiffnesses of the material.

We have found stability behavior to be complex, and while hard incompressibility often results in less stable behavior, this is not always the case: in some situations the stronger enforcement afforded by hard incompressibility actually improves stability.

## 6.8 Varying and augmenting material behaviors

The default material used by all elements of an FEM model is supplied by the model's material property. However, it is often the case that one wishes to specify different material behaviors for different sets of elements within an FEM model. This may be particularly true when combining volumetric and shell elements.

There are several ways to vary material behavior within a model. These include:

- **Setting an explicit material for specific elements**, using their material property. An element's material is `null` by default, but if set to a material, it will override the default material supplied by the FEM model. While this method is quite straightforward, it does have one disadvantage: because material settings are *copied* by each element, subsequent interactive or online changes to the material require resetting the material in *all* the affected elements.

- **Binding one or more material parameters to a field**. Sometimes certain material parameters, such as stiffness quantities or direction information, may need to vary across the FEM domain. While sometimes this can be handled by setting material properties for specific elements, it may be more convenient to bind the varying properties to a *field*, which can specify varying values over a domain composed of either a regular grid or an FEM mesh. Only one material needs to be used, and any properties which are not bound can be adjusted interactively or online. Fields and their bindings are described in detail in Chapter 7.
- **Adding augmenting material behaviors using MaterialBundles**. A material bundle may be specified either for all elements, or for a subset of them, and each provides one material (via its own material property) whose behavior is *added* to that of the indicated elements. This also provides an easy way to *combine* the behaviors of two of more materials in the same element. One also has the option of setting the material property of the certain elements to NullMaterial, so that *only* the augmenting material(s) are applied.
- **Adding muscle behaviors using MuscleBundles**. This is analogous to using `MaterialBundles`, except that `MuscleBundles` are restricted to using an instance of a MuscleMaterial, and include support for handling the excitation value, as well as the activation directions (which usually vary across the FEM domain). `MuscleBundles` are only present in the `FemMuscleModels` subclass of `FemModel3d`, and are described in detail in Section 6.9.

The remainder of this section will discuss `MaterialBundles`.

Adding a MaterialBundle to an FEM model is illustrated by the following code fragment:

```
FemMaterial extraMat;   // material to be added
FemModel3d fem;         // FEM model
...

MaterialBundle bun = new MaterialBundle ("mybundle", extraMat);
// add volumetric elements to the bundle
for (FemElement3d e : fem.getElements()) {
   if (/* e should be added to the bundle */) {
      bun.addElement (e);
   }
}
fem.addMaterialBundle (bun);
```

Once added, the stress computed by the bundle's material will be *added* to the stress computed by any other materials which are active for the bundle's elements.

When deciding what elements to add to a bundle, one is free to choose any means necessary. The example above inspects all the volumetric elements in the FEM. To instead inspect all the shell elements, or all volumetric and shell elements, one could use the code fragments such as the following:

```
// add shell elements to the bundle
for (ShellElement3d e : fem.getShellElements()) {
   if (/* e should be added to the bundle */) {
      bun.addElement (e);
   }
}

// add volumetric or shell elements to the bundle
for (FemElement3dBase e : fem.getAllElements()) {
   if (/* e should be added to the bundle */) {
      bun.addElement (e);
   }
}
```

Of course, if the elements are known through other means, then they can be added directly.

The element composition of a bundle can be controlled by the methods

```
void addElement (FemElement3dBase e)        // adds an element
boolean removeElement (FemElement3dBase e)  // removes an element
void clearElements()                        // removes all elements

int numElements()                           // gets the number of elements
FemElment3dBase getElement (int idx)        // gets the idx-th element
```

It is also possible to create a `MaterialBundle` whose material is added to *all* the FEM elements. This can done either by using one of the following constructors

```
MaterialBundle (String name, boolean useAllElements)
MaterialBundle (String name, FemMaterial mat, boolean useAllElements)
```

with `useAllElements` set to `true`, or by calling the method

```
void setUseAllElements (boolean enable)
```

When a bundle is set to use all the FEM elements, it clears its own element list, and one is not permitted to add elements using `addElement(elem)`.

After a bundle has been created, it is possible to get or set its material property using the methods

```
FemMateral getMaterial()                  // get the material
FemMateral setMaterial (FemMaterial mat)  // set the material
```

Again, because materials are copied internally, any modification to a material *after* it has been used as an input to `setMaterial()` will *not* be noticed by the bundle. Instead, one should modify the material *before* calling `setMaterial()`, or modify the *copied* material which can be obtained by calling `getMaterial()` or by storing the value returned by `setMaterial()`.

Finally, a `MuscleBundle` is a renderable component. Setting its elementWidgetSize property to a value greater than zero will cause the rendering of all its elements, using a solid widget representation of each at a scale controlled by elementWidgetSize: 0.5 for half size, 1.0 for full size, etc. The color of the widgets is controlled by the faceColor property of the bundle's renderProps. Being able to render the elements makes it easy to select the bundle and visualize which elements it contains.

### 6.8.1 Example: FEM sheet with a stiff spine

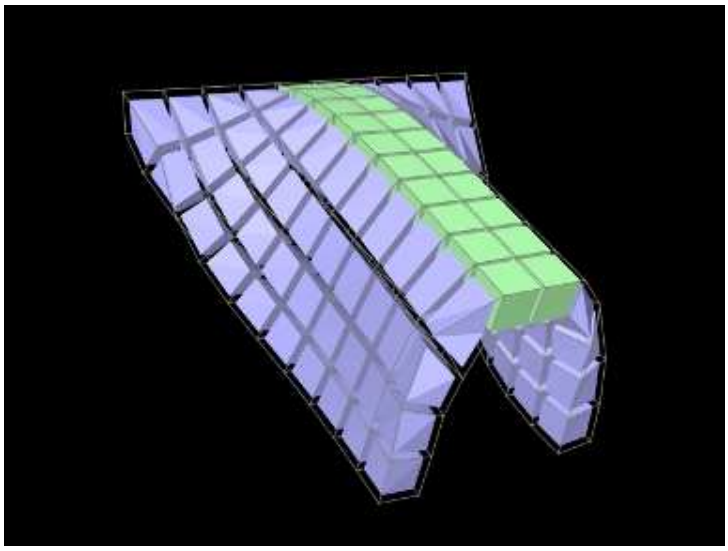


Figure 6.14: MaterialBundleDemo model after being run in ArtiSynth.

A simple model demonstrating the use of material bundles is defined in

```
artisynth.demos.tutorial.MaterialBundleDemo
```

It consists of a simple thin hexahedral sheet in which a material bundle is used to stiffen elements close to the x-axis, creating a stiff "spine". While the same effect could be achieved by simply setting a different material property for the "spine" elements, it does provide a good example of muscle bundle usage. The model's `build` method is given below:

```
1     public void build (String[] args) {
2        MechModel mech = new MechModel ("mech");
3        addModel (mech);
4
5        // create a fem model consisting of a thin sheet of hexes
6        FemModel3d fem = FemFactory.createHexGrid (null, 1.0, 1.0, 0.1, 10, 10, 1);
7        fem.setDensity (1000);
8        fem.setMaterial (new LinearMaterial (10000, 0.45));
9        mech.add (fem);
10       // fix the left-most nodes:
11       double EPS = 1e-8;
12       for (FemNode3d n : fem.getNodes()) {
13          if (n.getPosition().x <= -0.5+EPS) {
14             n.setDynamic (false);
15          }
16       }
17       // create a "spine" of stiffer elements using a MaterialBundle with a
18       // stiffer material
19       MaterialBundle bun =
20          new MaterialBundle ("spine", new NeoHookeanMaterial (5e6, 0.45), false);
21       for (FemElement3d e : fem.getElements()) {
22          // use element centroid to determine which elements are on the "spine"
23          Point3d pos = new Point3d();
24          e.computeCentroid (pos);
25          if (Math.abs(pos.y) <= 0.1+EPS) {
26             bun.addElement (e);
27          }
28       }
29       fem.addMaterialBundle (bun);
30
31       // add a control panel to control both the fem and bundle materials,
32       // as well as the fem and bundle widget sizes
33       ControlPanel panel = new ControlPanel();
34       panel.addWidget ("fem material", fem, "material");
35       panel.addWidget ("fem widget size", fem, "elementWidgetSize");
36       panel.addWidget ("bundle material", bun, "material");
37       panel.addWidget ("bundle widget size", bun, "elementWidgetSize");
38       addControlPanel (panel);
39
40       // set rendering properties, using element widgets
41       RenderProps.setFaceColor (fem, new Color (0.7f, 0.7f, 1.0f));
42       RenderProps.setFaceColor (bun, new Color (0.7f, 1.0f, 0.7f));
43       bun.setElementWidgetSize (0.9);
44       fem.setElementWidgetSize (0.8);
45    }
```

Lines 6-9 create a thin FEM hex sheet, centered on the origin, with size $1 \times 1 \times 0.1$ and $10 \times 10 \times 1$ elements along each axis. Its material is set to a linear material with a Young's modulus of 10000. The leftmost nodes are then fixed (lines 11-16).

Next, a `MuscleBundle` is added and used to apply an additional material to the elements near the x axis (lines 19-29). It is given a neo-hookean material with a much higher stiffness, and the "spine" elements for it are selected by finding those whose centroids have a $y$ value within 0.1 of the x axis.

After the bundle is added, a control panel is created allowing interactive control of the materials for both the FEM model and the bundle, along with their elementWidgetSize properties.

Finally, some render properties are set (lines 41-44). The idea is to render the elements of both the FEM model and the bundle using element widgets (both of which will be visible since there is no surface rendering and the element widget sizes for both are greater than 0). The widget size for the bundle is made larger than that of the FEM model to ensure its widgets will cover those of the latter. Widget colors are controlled by the FEM and bundle face colors, set in lines 41-42.

The example can be run in ArtiSynth by selecting All demos > tutorial > MaterialBundleDemo from the Models menu. When it is run, the sheet will fall under gravity but be much stiffer along the spine (Figure 6.14). The control panel can be used to interactively adjust the material parameters for both the FEM and the bundle. This is one advantage of using

bundles: the same material can be used to control multiple elements. The panel also allows interactive adjustment of the widget sizes, to illustrate what they look like and how they are rendered.

## 6.9 Muscle activated FEM models

Finite element muscle models are an extension to regular FEM models. As such, everything previously discussed for regular FEM models also applies to FEM muscles. Muscles have additional properties that allow them to contract when activated. There are two types of muscles supported:

**Fibre-based:** Point-to-point muscle fibres are embedded in the model.

**Material-based:** An auxiliary material is added to the constitutive law to embed muscle properties.

In this section, both types will be described.

### 6.9.1 FemMuscleModel

The main class for FEM-based muscles is FemMuscleModel, a subclass of FemModel3d. It differs from a basic FEM model in that it has the new property

| Property | Description |
|---|---|
| `muscleMaterial` | An object that adds an activation-dependent 'muscle' term to the *constitutive law*. |

This is a delegate object of type MuscleMaterial, described in detail in Section 6.10.3, that computes activation-dependent stress and stiffness in the muscle. In addition to this property, `FemMuscleModel` adds two new lists of subcomponents:

`bundles`
: Groupings of muscle sub-units (fibres or elements) that can be activated.

`exciters`
: Components that control the activation of a set of bundles or other exciters.

#### 6.9.1.1 Bundles

Muscle bundles allow for a muscle to be partitioned into separate groupings of fibres/elements, where each bundle can be activated independently. They are implemented in the class MuscleBundle. Bundles have three key properties:

| Property | Description |
|---|---|
| `excitation` | Activation level of the muscle, $a \in [0,1]$. |
| `fibresActive` | Enable/disable "fibre-based" muscle components. |
| `muscleMaterial` | An object that adds an activation-dependent 'muscle' term to the *constitutive law* (Section 6.10.3). |

The `excitation` property controls the level of muscle activation, with zero being no muscle action, and one being fully activated. The `fibresActive` property is a boolean variable that controls whether or not to treat any contained fibres as point-to-point-like muscles ("fibre-based"). If false, the fibres are ignored. The third property, `muscleMaterial`, allows for a `MuscleMaterial` to be specified per bundle. By default, its value is inherited from `FemMuscleModel`.

Once a muscle bundle is created, muscle sub-units must be assigned to it. These are either point-to-point fibres, or material-based muscle element descriptors. The two types will be covered in Sections 6.9.2 and 6.9.3, respectively.

#### 6.9.1.2 Exciters

A MuscleExciter component enables you to simultaneously activate a group of "excitation components". This includes: point-to-point muscles, muscle bundles, muscle fibres, material-based muscle elements, and other muscle exciters. Components that can be excited all implement the ExcitationComponent interface. To add or remove a component to the exciter, use

```
    addTarget (ExcitationComponent ex);     // adds a component to the exciter
    addTarget (ExcitationComponent ex,      // adds a component with a gain factor
        double gain);
    removeTarget (ExcitationComponent ex); // removes a component
```

If a gain factor is specified, the activation is scaled by the gain for that component.

### 6.9.2 Fibre-based muscles

In fibre-based muscles, a set of point-to-point muscle fibres are added between FEM nodes or markers. Each fibre is assigned an AxialMuscleMaterial, just like for regular point-to-point muscles (Section 4.5.1). Note that these muscle materials typically have a "rest length" property, that will likely need to be adjusted for each fibre. Once the set of fibres are added to a `MuscleBundle`, they need to be enabled. This is done by setting the `fibresActive` property of the bundle to `true`.

Fibres are added to a `MuscleBundle` using one of the functions:

```
  addFibre( Muscle muscle );              // adds a point-to-point fibre
  Muscle addFibre( Point p0, Point p1,    // creates and adds a fibre
      AxialMuscleMaterial mat);
```

The latter returns the newly created `Muscle` fibre. The following code snippet demonstrates how to create a fibre-based `MuscleBundle` and add it to an FEM muscle.

```
1  // Create a muscle bundle
2  MuscleBundle bundle = new MuscleBundle("fibres");
3  Point3d[] fibrePoints = ...   //create a sequential list of points
4
5  // Add fibres
6  Point pPrev = fem.addMarker(fibrePoints[0]);     // create an FEM marker
7  for (int i=1; i<=fibrePoints.length; i++) {
8     Point pNext = fem.addMarker(fibrePoint[i]);
9
10    // Create fibre material
11    double l0 = pNext.distance(pPrev);           // rest length
12    AxialMuscleMaterial fibreMat =
13          new BlemkerAxialMuscle(
14                1.4*l0, l0, 3000, 0, 0);
15
16    // Add a fibre between pPrev and pNext
17    bundle.addFibre(pPrev, pNext, fibreMat);     // add fibre to bundle
18    pPrev = pNext;
19 }
20
21 // Enable use of fibres (default is disabled)
22 bundle.setFibresActive(true);
23 fem.addMuscleBundle(bundle);                    // add the bundle to fem
```

In these fibre-based muscles, force is only exerted between the anchor points of the fibres; it is a discrete approximation. These models are typically more stable than material-based ones.

### 6.9.3 Material-based muscles

In material-based muscles, the constitutive law is augmented with additional terms to account for muscle-specific properties. This is a continuous representation within the model.

The basic building block for a material-based muscle bundle is a MuscleElementDesc. This object contains a reference to a FemElement3d, a MuscleMaterial, and either a single direction or set of directions that specify the direction of contraction. If a single direction is specified, then it is assumed the entire element contracts in the same direction. Otherwise, a direction can be specified for each *integration point* within the element. A null direction signals that there is no muscle at the corresponding point. This allows for a sub-element resolution for muscle definitions. The positions of integration points for a given element can be obtained with:

```
// loop through all integration points for a given element
for ( IntegrationPoint3d ipnt : elem.getIntegrationPoints() ) {
   Point3d curPos = new Point3d();
   Point3d restPos = new Point3d();
   ipnt.computePosition (curPos, elem);          // computes current position
   ipnt.computeRestPosition (restPos, elem);     // computes rest position
}
```

By default, the MuscleMaterial is inherited from the bundle's material property. Muscle materials are described in detail in Section 6.10.3 and include GenericMuscle, BlemkerMuscle, and FullBlemkerMuscle. The Blemker-type materials are based on [5].

Elements can be added to a muscle bundle using one of the methods:

```
// Adds a muscle element
addElement (MuscleElementDesc elem);
// Creates and adds a muscle element
MuscleElementDesc addElement (FemElement3d elem, Vector3d dir);
// Sets a direction per integration point
MuscleElementDesc addElement (FemElement3d elem, Vector3d[] dirs);
```

The following snippet demonstrates how to create and add a material-based muscle bundle:

```
1  // Create muscle bundle
2  MuscleBundle bundle = new MuscleBundle("embedded");
3
4  // Muscle material
5  MuscleMaterial muscleMat = new BlemkerMuscle(
6        1.4, 1.0, 3000, 0, 0);
7  bundle.setMuscleMaterial(muscleMat);
8
9  // Muscle direction
10 Vector3d dir = Vector3d.X_UNIT;
11
12 // Add elements to bundle
13 for (FemElement3d elem : beam.getElements()) {
14    bundle.addElement(elem, dir);
15 }
16
17 // Add bundle to model
18 beam.addMuscleBundle(bundle);
```

### 6.9.4 Example: toy FEM muscle model

A simple example showing an FEM with material-based muscles is given by artisynth.demos.tutorial.ToyMuscleFem. It consists of a hex-element beam, with 12 muscles added in groups along its length, allowing it to flex in the x-z plane. A frame object is also attached to the right end. The code for the model, without the package and include directives, is listed below:

```
1  public class ToyMuscleFem extends RootModel {
2
3     protected MechModel myMech;      // overall mechanical model
4     protected FemMuscleModel myFem;  // FEM muscle model
5     protected Frame myFrame;         // frame attached to the FEM
6
7     public void build (String[] args) throws IOException {
```

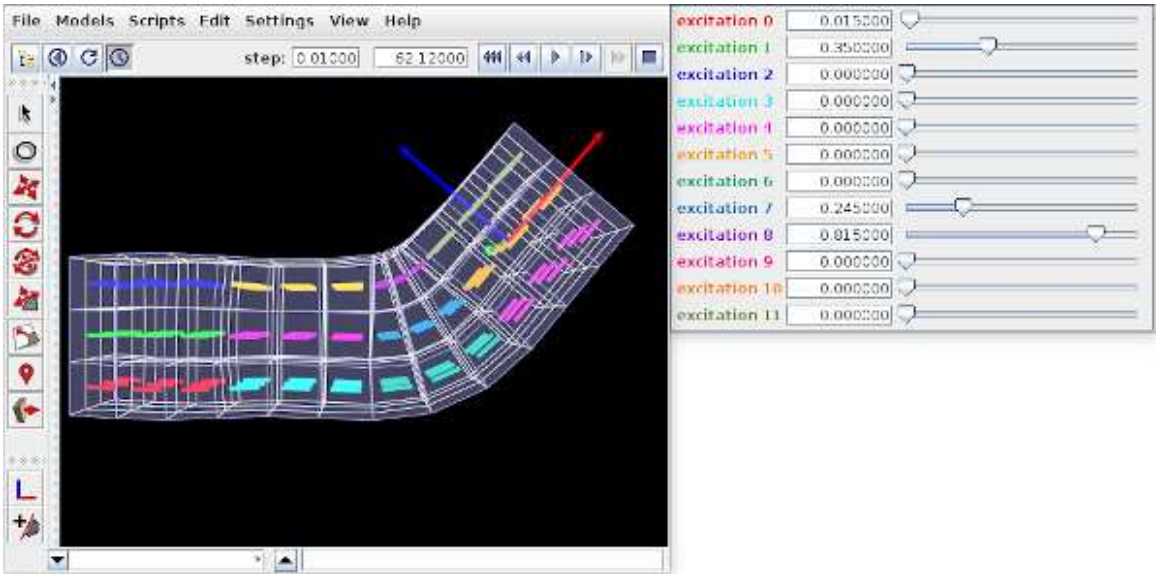



Figure 6.15: `ToyMuscleFem`, showing the deformation from setting the muscle excitations in the control panel.

```
8         myMech = new MechModel ("mech");
9         myMech.setGravity (0, 0, 0);
10        addModel (myMech);
11
12        // create a FemMuscleModel and then create a hex grid with it.
13        myFem = new FemMuscleModel ("fem");
14        FemFactory.createHexGrid (
15           myFem, /*wx*/1.2, /*wy*/0.3, /*wz*/0.3, /*nx*/12, /*ny*/3, /*nz*/3);
16        // give it a material with Poisson's ratio 0 to allow it to compress
17        myFem.setMaterial (new LinearMaterial (100000, 0.0));
18        // fem muscles will be material-based, using a SimpleForceMuscle
19        myFem.setMuscleMaterial (new SimpleForceMuscle(/*maxstress=*/100000));
20        // set density + particle and stiffness damping for optimal stability
21        myFem.setDensity (100.0);
22        myFem.setParticleDamping (1.0);
23        myFem.setStiffnessDamping (1.0);
24        myMech.addModel (myFem);
25
26        // fix the nodes on the left side
27        for (FemNode3d n : myFem.getNodes()) {
28           Point3d pos = n.getPosition();
29           if (pos.x == -0.6) {
30              n.setDynamic (false);
31           }
32        }
33        // Create twelve muscle bundles, bottom to top and left to right. Muscles
34        // are material-based, each with a set of 9 elements in the x-y plane and
35        // a rest direction parallel to the x axis.
36        Point3d pe = new Point3d(); // element center point
37        for (int sec=0; sec<4; sec++) {
38           for (int k=0; k<3; k++) {
39              MuscleBundle bundle = new MuscleBundle();
40              // locate elements based on their center positions
41              pe.set (-0.55+sec*0.3, -0.1, -0.1+k*0.1);
42              for (int i=0; i<3; i++) {
43                 pe.y = -0.1;
44                 for (int j=0; j<3; j++) {
45                    FemElement3d e = myFem.findNearestVolumetricElement(null, pe);
46                    bundle.addElement (e, Vector3d.X_UNIT);
47                    pe.y += 0.1;
```

```
48                  }
49                  pe.x += 0.1;
50               }
51               // set the line color for each bundle using a color from the probe
52               // display palette.
53               RenderProps.setLineColor (
54                  bundle, PlotTraceInfo.getPaletteColors()[sec*3+k]);
55               myFem.addMuscleBundle (bundle);
56            }
57         }
58         // create a panel for controlling all 12 muscle excitations
59         ControlPanel panel = new ControlPanel ();
60         for (MuscleBundle b : myFem.getMuscleBundles()) {
61            LabeledComponentBase c = panel.addWidget (
62               "excitation "+b.getNumber(), b, "excitation");
63            // color the exciter labels using the muscle bundle color
64            c.setLabelFontColor (b.getRenderProps().getLineColor());
65         }
66         addControlPanel (panel);
67
68         // create and attach a frame to the right end of the FEM
69         myFrame = new Frame();
70         myFrame.setPose (new RigidTransform3d (0.45, 0, 0));
71         myMech.addFrame (myFrame);
72         myMech.attachFrame (myFrame, myFem); // attach to the FEM
73
74         // render properties:
75         // show muscle bundles by rendering activation directions within elements
76         RenderProps.setLineWidth (myFem.getMuscleBundles(), 4);
77         myFem.setDirectionRenderLen (0.6); //
78         // draw frame by showing its coordinate axis
79         myFrame.setAxisLength (0.3);
80         myFrame.setAxisDrawStyle (AxisDrawStyle.ARROW);
81         // set FEM line and surface colors to blue-gray
82         RenderProps.setLineColor (myFem, new Color (0.7f, 0.7f, 1f));
83         RenderProps.setFaceColor (myFem, new Color (0.7f, 0.7f, 1f));
84         // render FEM surface transparent
85         myFem.setSurfaceRendering (SurfaceRender.Shaded);
86         RenderProps.setAlpha (myFem.getSurfaceMeshComp(), 0.4);
87      }
88   }
```

Within the `build()` method, the mech model is created in the usual way (lines 8-10). Gravity is turned off to give the muscles greater control over the FEM model's configuration. The `MechModel` reference is stored in the class field `myMech` to enable any subclasses to have easy access to it; the same will be done for the FEM model and the attached frame.

The FEM model itself is created at lines (12-24). An instance of `FemMuscleModel` is created and then passed to `FemFactory.createHexGrid()`; this allows the model to be an instance of `FemMuscleModel` instead of `FemModel3d`. The model is assigned a linear material with Poisson's ratio of 0 to allow it to be easily compressed. Muscle bundles will be material-based and will all use the same `SimpleForceMuscle` material that is assigned to the model's `muscleMaterial` property. Density and damping parameters are defined so as to improve model stability when muscles are excited. Once the model is created, its left-hand nodes are fixed to provide anchoring (lines 27-32).

Muscle bundles are created at lines 36-58. There are 12 of them, arranged in three vertical layers and four horizontal sections. Each bundle is populated with 9 adjacent elements in the x-y plane. To find these elements, their center positions are calculated and then passed to the FEM method `findNearestVolumetricElement()`. The element is then added to the bundle with a resting activation direction parallel to the x axis. For visualization, each bundle's line color is set to a distinct color from the numeric probe display palette, based on the bundle's number (lines 53-54).

After the bundles have been added to the FEM model, a control is created to allow interactive control of their excitations (lines 88-95). Each widget is labeled `excitation N`, where `N` is the bundle number, and the label is colored to match the bundle's line color. Finally, a frame is created and attached to the FEM model (lines 59-62), and render properties are set at lines 106-115: Muscle bundles are drawn by showing the activation direction in each element; the frame is

displayed as a solid arrow with axis lengths of 0.3; the FEM line and face colors are set to blue-gray, and the surface is rendered and made transparent.

To run this example in ArtiSynth, select All demos > tutorial > ToyMuscleFem from the Models menu. Running the model and adjusting the exciters in the control panel will cause the FEM model to deform in various ways (Figure 6.15).

### 6.9.5 Example: comparison with two beam examples

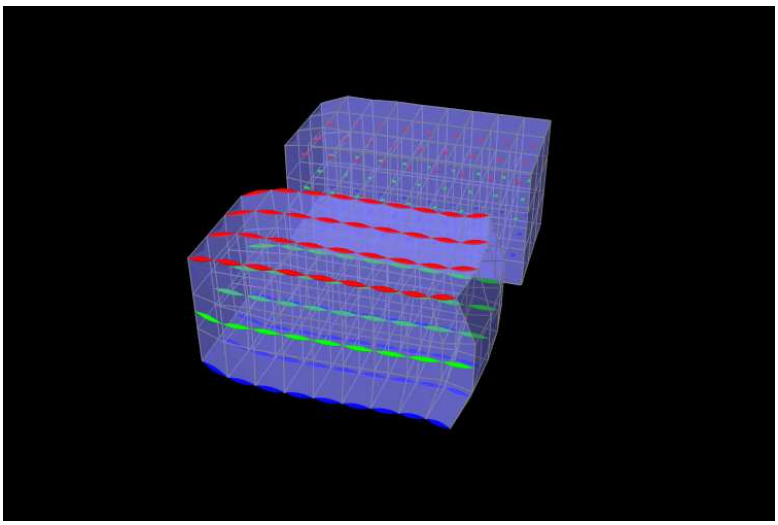


Figure 6.16: FemMuscleBeams model loaded into ArtiSynth.

An example comparing a fibre-based and a material-based muscle is shown in Figure 6.16. The code can be found in `artisynth.demos.tutorial.FemMuscleBeams`. There are two `FemMuscleModel` beams in the model: one fibre-based, and one material-based. Each has three muscle bundles: one at the top (red), one in the middle (green), and one at the bottom (blue). In the figure, both muscles are fully activated. Note the deformed shape of the beams. In the fibre-based one, since forces only act between point on the fibres, the muscle seems to bulge. In the material-based muscle, the entire continuous volume contracts, leading to a uniform deformation.

Material-based muscles are more realistic. However, this often comes at the cost of stability. The added terms to the constitutive law are highly nonlinear, which may cause numerical issues as elements become highly contracted or highly deformed. Fibre-based muscles are, in general, more stable. However, they can lead to bulging and other deformation artifacts due to their discrete nature.

## 6.10 Material types

ArtiSynth FEM models support a variety of material types, including linear, hyperelastic, and activated muscle materials, described in the sections below. These can be used to supply the primary material for either an entire FEM model or for specific elements (using the `setMaterial()` methods for either). They can also be used to supply auxiliary materials, whose behavior is superimposed on the underlying material, via either material bundles (Section 6.8) or, for FemMuscleModels, muscle bundles (Section 6.9). Many of the properties for a given material can be bound to a scalar or vector field (Section 7.2) to allow their values to vary across an FEM model. In the descriptions below, properties for which this is true will have a ✓ indicated under the "Field" entry in the material's property table.

All materials are defined in the package `artisynth.core.materials`.

### 6.10.1 Linear

Linear materials determine Cauchy stress $\sigma$ as a linear mapping from the small deformation Cauchy strain $\varepsilon$,

$$\sigma = \mathcal{D} : \varepsilon, \tag{6.9}$$

where $\mathcal{D}$ is the *elasticity tensor*. Both the stress and strain are symmetric 3 matrices,

$$\sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{pmatrix}, \quad \varepsilon = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{xy} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz} \end{pmatrix}, \tag{6.10}$$

and can be expressed as 6-vectors using Voigt notation:

$$\hat{\sigma} \equiv \begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{pmatrix}, \quad \hat{\varepsilon} \equiv \begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ 2\varepsilon_{xy} \\ 2\varepsilon_{yz} \\ 2\varepsilon_{xz} \end{pmatrix}. \tag{6.11}$$

This allows the constitutive mapping to be expressed in matrix form as

$$\hat{\sigma} = \mathbf{D}\,\hat{\varepsilon}, \tag{6.12}$$

where $\mathbf{D}$ is the $6 \times 6$ *elasticity matrix*.

> Different Voigt notation mappings appear in the literature with regard to off-diagonal matrix entries. We use the one employed by FEBio [12]. Another common mapping is
>
> $$\hat{\sigma} \equiv \begin{pmatrix} \sigma_{xx} & \sigma_{yy} & \sigma_{zz} & \sigma_{yz} & \sigma_{xz} & \sigma_{xy} \end{pmatrix}^T.$$

Traditionally, Cauchy strain is computed from the symmetric part of the deformation gradient $\mathbf{F}$ using the formula

$$\varepsilon = \frac{\mathbf{F}^T + \mathbf{F}}{2} - \mathbf{I}, \tag{6.13}$$

where $\mathbf{I}$ is the $3 \times 3$ identity matrix. However, ArtiSynth materials support *corotation*, in which rotations are first removed from $\mathbf{F}$ using a polar decomposition

$$\mathbf{F} = \mathbf{R}\mathbf{P}, \tag{6.14}$$

where $\mathbf{R}$ is a (right-handed) rotation matrix and $\mathbf{P}$ is symmetric matrix. $\mathbf{P}$ is then used to compute the Cauchy strain:

$$\varepsilon = \mathbf{P} - \mathbf{I}. \tag{6.15}$$

Corotation is the default behavior for linear materials in ArtiSynth and allows models to handle large scale rotational deformations [18, 16].

For linear materials, the stress/strain response is computed at a *single* integration point in the center of each element. This is done to improve computational efficiency, as it allows the precomputation of stiffness matrices that map nodal displacements onto nodal forces for each element. This precomputed matrix can then be adjusted during each simulation step to account for the rotation $\mathbf{R}$ computed at each element's integration point [18, 16].

Specific linear material types are listed in the subsections below. All are subclasses of LinearMaterialBase, and in addition to their individual properties, all export a corotation property (default value `true`) that controls whether corotation is applied.

#### 6.10.1.1 LinearMaterial

LinearMaterial is a standard isotropic linear material, which is also the default material for FEM models. Its elasticity matrix is given by

$$\mathbf{D} = \begin{bmatrix} \lambda+2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda+2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda+2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix}, \tag{6.16}$$

where the Lamé parameters $\lambda$ and $\mu$ are derived from Young's modulus $E$ and Poisson's ratio $\nu$ via

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}, \quad \mu = \frac{E}{2(1+\nu)}. \tag{6.17}$$

The material behavior is controlled by the following properties:

| | Property | Description | Default | Field |
|---|---|---|---|---|
| $E$ | YoungsModulus | Young's modulus | 500000 | ✓ |
| $\nu$ | PoissonsRatio | Poisson's ratio | 0.33 | |
| | corotated | applies corotation if `true` | `true` | |

`LinearMaterials` can be created with the following constructors:

| | |
|---|---|
| LinearMaterial() | Create with default properties. |
| LinearMaterial (double E, double nu) | Create with specified $E$ and $\nu$. |
| LinearMaterial (double E, double nu, boolean corotated) | Create with specified $E$, $\nu$ and corotation. |

#### 6.10.1.2 TransverseLinearMaterial

TransverseLinearMaterial is a transversely isotropic linear material whose behavior is symmetric about a prescribed *polar axis*. If the polar axis is parallel to the $z$ axis, then the elasticity matrix is specified most easily in terms of its inverse, or *compliance* matrix, according to

$$\mathbf{D}^{-1} = \begin{bmatrix} \frac{1}{E_{xy}} & -\frac{\nu_{yx}}{E_{xy}} & -\frac{\nu_z}{E_z} & 0 & 0 & 0 \\ -\frac{\nu_{yx}}{E_{xy}} & \frac{1}{E_{xy}} & -\frac{\nu_z}{E_z} & 0 & 0 & 0 \\ -\frac{\nu_z}{E_z} & -\frac{\nu_z}{E_z} & \frac{1}{E_z} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2(1+\nu_{yx})}{E_{xy}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G} \end{bmatrix},$$

where $E_{xy}$ and $E_z$ are Young's moduli transverse and parallel to the axis, respectively, $\nu_{xy}$ and $\nu_z$ are Poisson's ratios transverse and parallel to the axis, and $G$ is the shear modulus.

The material behavior is controlled by the following properties:

| | Property | Description | Default | Field |
|---|---|---|---|---|
| $(E_{xy}, E_z)^T$ | youngsModulus | Young's moduli transverse and parallel to the axis | $(500000, 500000)^T$ | ✓ |
| $(\nu_{xy}, \nu_z)^T$ | poissonsRatio | Poisson's ratios transverse and parallel to the axis | $(0.33, 0.33)^T$ | |
| $G$ | shearModulus | shear modulus | 187970 | ✓ |
| | direction | direction of the polar axis | $(0,0,1)^T$ | ✓ |
| | corotated | applies corotation if `true` | `true` | |

The youngsModulus and poissonsRatio properties are both described by Vector2d objects, while direction is described by a Vector3d. The direction property (as well as youngsModulus and shearModulus) can be bound to a field to allow its value to vary over an FEM model (Section 7.2).

`TransverseLinearMaterials` can be created with the following constructors:

| | |
|---|---|
| TransverseLinearMaterial() | Create with default properties. |
| TransverseLinearMaterial (Vector2d E, double G, Vector2d nu, boolean corotated) | Create with specified $E$, $G$, $\nu$ and corotation. |

#### 6.10.1.3 AnisotropicLinearMaterial

AnisotropicLinearMaterial is a general anisotropic linear material whose behavior is specified by an arbitrary (symmetric) elasticity matrix **D**. The material behavior is controlled by the following properties:

| | Property | Description | Default | Field |
|---|---|---|---|---|
| **D** | stiffnessTensor | symmetric $6 \times 6$ elasticity matrix | | |
| | corotated | applies corotation if `true` | `true` | |

The default value for stiffnessTensor is an isotropic elasticity matrix corresponding to $E = 500000$ and $\nu = 0.33$.

`AnisotropicLinearMaterials` can be created with the following constructors:

| | |
|---|---|
| AnisotropicLinearMaterial() | Create with default properties. |
| AnisotropicLinearMaterial (Matrix6dBase D) | Create with specified elasticity **D**. |
| AnisotropicLinearMaterial (Matrix6dBase D, boolean corotated) | Create with specified **D** and corotation. |

### 6.10.2 Hyperelastic materials

A *hyperelastic* material is defined by a strain energy density function $W()$ that is in general a function of the deformation gradient **F**, and more specifically a quantity derived from **F** that is rotationally invariant, such as the right Cauchy Green tensor $\mathbf{C} \equiv \mathbf{F}^T\mathbf{F}$, the left Cauchy Green tensor $\mathbf{B} \equiv \mathbf{F}\mathbf{F}^T$, or Green strain

$$\mathbf{E} = \frac{1}{2}\left(\mathbf{F}^T\mathbf{F} - \mathbf{I}\right). \tag{6.18}$$

$W()$ is often described with respect to the first or second invariants of these quantities, denoted by $I_1$ and $I_2$ and defined with respect to a given matrix **A** by

$$I_1 \equiv \mathrm{tr}(\mathbf{A}), \quad I_2 \equiv \frac{1}{2}\left(\mathrm{tr}(\mathbf{A})^2 - \mathrm{tr}(\mathbf{A}^2)\right). \tag{6.19}$$

Alternatively, $W()$ is sometimes defined with respect to the three *principal stretches* of the deformation, $\lambda_i, i \in 1,2,3$, which are the eigenvalues of the symmetric component **P** of the polar decomposition of **F** in (6.14).

If $W()$ is expressed with respect to **E**, then the second Piola-Kirchhoff stress **S** is given by

$$\mathbf{S} = \frac{\partial W}{\partial \mathbf{E}} \tag{6.20}$$

from which the Cauchy stress can be obtained via

$$\sigma = \frac{1}{J}\mathbf{F}\mathbf{S}\mathbf{F}^T, \quad J \equiv \det\mathbf{F}. \tag{6.21}$$

Many of the hyperelastic materials described below are *incompressible*, in the sense that $W$ is partitioned into a *deviatoric* component that is volume invariant and a *dilational* component that depends solely on volume changes. Volume change is characterized by $J = \det\mathbf{F}$, with the change in volume given by $dV = J^{1/3}$ and $J = 1$ indicating no volume change. Deviatoric changes are characterized by $\bar{\mathbf{F}}$, with

$$\bar{\mathbf{F}} = J^{-1/3}\mathbf{F}, \tag{6.22}$$

and so the partitioned strain energy density function assumes the form

$$W(\mathbf{F}) = \bar{W}(\bar{\mathbf{F}}) + U(J), \tag{6.23}$$

where the $\bar{W}()$ term is the deviatoric contribution and $U(J)$ is the volumetric potential that enforces incompressibility. $\bar{W}()$ may also be expressed with respect to the right deviatoric Cauchy Green tensor $\bar{\mathbf{C}}$ or the left deviatoric Cauchy Green tensor $\bar{\mathbf{B}}$, respectively defined by

$$\bar{\mathbf{C}} = J^{-2/3}\mathbf{C}, \quad \bar{\mathbf{B}} = J^{-2/3}\mathbf{B}. \tag{6.24}$$

ArtiSynth supplies different forms of $U(J)$, as specified by a material's bulkPotential property and detailed in Section 6.7.3. All of the available potentials depend on a bulkModulus property $\kappa$, and so $U(J)$ is often expressed as $U(\kappa, J)$. A larger bulk modulus will make the material more incompressible, with effective incompressibility typically achieved by setting $\kappa$ to a value that exceeds the other elastic moduli in the material by a factor of 100 or more. All incompressible materials are subclasses of IncompressibleMaterialBase.

#### 6.10.2.1 St Venant-Kirchoff material

StVenantKirchoffMaterial is a compressible isotropic material that extends isotropic linear elasticity to large deformations. The strain energy density is most easily expressed as a function of the Green strain $\mathbf{E}$:

$$W(\mathbf{E}) = \frac{\lambda}{2}\operatorname{tr}(\mathbf{E})^2 + \mu \operatorname{tr}(\mathbf{E}^2), \tag{6.25}$$

where the Lamé parameters $\lambda$ and $\mu$ are derived from Young's modulus $E$ and Poisson's ratio $\nu$ according to (6.17). From this definition it follows that the second Piola-Kirchoff stress tensor is given by

$$\mathbf{S} = \lambda \operatorname{tr}(\mathbf{E})\mathbf{I} + 2\mu\mathbf{E}. \tag{6.26}$$

The material behavior is controlled by the following properties:

| | Property | Description | Default | Field |
|---|---|---|---|---|
| $E$ | YoungsModulus | Young's modulus | 500000 | ✓ |
| $\nu$ | PoissonsRatio | Poisson's ratio | 0.33 | |

`StVenantKirchoffMaterials` can be created with the following constructors:

| | |
|---|---|
| StVenantKirchoffMaterial() | Create with default properties. |
| StVenantKirchoffMaterial (double E, double nu) | Create with specified elasticity $E$ and $\nu$. |

#### 6.10.2.2 Neo-Hookean material

NeoHookeanMaterial is a compressible isotropic material with a strain energy density expressed as a function of the first invariant $I_1$ of the right Cauchy-Green tensor $\mathbf{C}$ and $J = \det \mathbf{F}$:

$$W(\mathbf{C}, J) = \frac{\mu}{2}(I_1 - 3) - \mu \ln(J) + \frac{\lambda}{2}\ln(J)^2, \tag{6.27}$$

where the Lamé parameters $\lambda$ and $\mu$ are derived from Young's modulus $E$ and Poisson's ratio $\nu$ via (6.17). The Cauchy stress can be expressed in terms of the left Cauchy-Green tensor $\mathbf{B}$ as

$$\sigma = \frac{\mu}{J}\mathbf{B} + \frac{\lambda \ln(J) - \mu}{J}\mathbf{I}. \tag{6.28}$$

The material behavior is controlled by the following properties:

| | Property | Description | Default | Field |
|---|---|---|---|---|
| $E$ | YoungsModulus | Young's modulus | 500000 | ✓ |
| $\nu$ | PoissonsRatio | Poisson's ratio | 0.33 | |

`NeoHookeanMaterials` can be created with the following constructors:

| | |
|---|---|
| NeoHookeanMaterial() | Create with default properties. |
| NeoHookeanMaterial (double E, double nu) | Create with specified elasticity $E$ and $\nu$. |

#### 6.10.2.3 Incompressible neo-Hookean material

IncompNeoHookeanMaterial is an incompressible version of the neo-Hookean material, with a strain energy density expressed in terms of the first invariant $\bar{I}_1$ of the *deviatoric* right Cauchy-Green tensor $\bar{\mathbf{C}}$, plus a potential function $U(\kappa, J)$ to enforce incompressibility:

$$W(\bar{\mathbf{C}}, J) = \frac{G}{2}(\bar{I}_1 - 3) + U(\kappa, J). \tag{6.29}$$

The material behavior is controlled by the following properties:

| | Property | Description | Default | Field |
|---|---|---|---|---|
| $G$ | shearModulus | shear modulus | 150000 | ✓ |
| $\kappa$ | bulkModulus | bulk modulus for incompressibility | 100000 | ✓ |
| | bulkPotential | incompressibility potential function $U(\kappa, J)$ | `QUADRATIC` | |

`IncompNeoHookeanMaterials` can be created with the following constructors:

| | |
|---|---|
| IncompNeoHookeanMaterial() | Create with default properties. |
| IncompNeoHookeanMaterial (double G, double kappa) | Create with specified elasticity $G$ and $\kappa$. |

#### 6.10.2.4 Mooney-Rivlin material

MooneyRivlinMaterial is an incompressible isotropic material with a strain energy density expressed as a polynomial of the first and second invariants $\bar{I}_1$ and $\bar{I}_2$ of the right deviatoric Cauchy-Green tensor $\bar{\mathbf{C}}$. ArtiSynth supplies a five parameter version of this model, with a strain energy density given by

$$W(\bar{\mathbf{C}},J) = C_{10}(\bar{I}_1-3)+C_{01}(\bar{I}_2-3)+C_{11}(\bar{I}_1-3)(\bar{I}_2-3)+C_{20}(\bar{I}_1-3)^2+C_{02}(\bar{I}_2-3)^2+U(\kappa,J). \tag{6.30}$$

A two-parameter version ($C_1$, $C_2$) is often found in the literature, consisting of only the first two terms:

$$W(\bar{\mathbf{C}},J) = C_1(\bar{I}_1-3)+C_2(\bar{I}_2-3)+U(\kappa,J), \qquad C_1 \equiv C_{10}, \quad C_2 \equiv C_{01}. \tag{6.31}$$

The material behavior is controlled by the following properties:

| | Property | Description | Default | Field |
|---|---|---|---|---|
| $C_{10}$ | C10 | first coefficient | 150000 | ✓ |
| $C_{01}$ | C01 | second coefficient | 0 | ✓ |
| $C_{11}$ | C11 | third coefficient | 0 | ✓ |
| $C_{20}$ | C20 | fourth coefficient | 0 | ✓ |
| $C_{02}$ | C02 | fifth coefficient | 0 | ✓ |
| $\kappa$ | bulkModulus | bulk modulus for incompressibility | 100000 | ✓ |
| | bulkPotential | incompressibility potential function $U(\kappa,J)$ | QUADRATIC | |

MooneyRivlinMaterials can be created with the following constructors:

| | |
|---|---|
| MooneyRivlinMaterial() | Create with default properties. |
| MooneyRivlinMaterial (double C10, double C01, double C11, double C20, double C02, double kappa) | Create with specified coefficients. |

#### 6.10.2.5 Ogden material

OgdenMaterial is an incompressible material with a strain energy density expressed as a function of the deviatoric principal stretches $\bar{\lambda}_i, i \in 1,2,3$:

$$W(\bar{\lambda}_1,\bar{\lambda}_2,\bar{\lambda}_3,J) = \sum_{k=1}^{N} \frac{\mu_k}{\alpha_k^2}\left(\bar{\lambda}_1^{\alpha_k}+\bar{\lambda}_2^{\alpha_k}+\bar{\lambda}_3^{\alpha_k}-3\right)+U(\kappa,J). \tag{6.32}$$

ArtiSynth allows a maximum of six terms, corresponding to $N = 6$, and the material behavior is controlled by the following properties:

| | Property | Description | Default | Field |
|---|---|---|---|---|
| $\mu_1$ | Mu1 | first multiplier | 300000 | ✓ |
| $\mu_2$ | Mu2 | second multiplier | 0 | ✓ |
| ... | ... | ... | 0 | ✓ |
| $\mu_6$ | Mu6 | final multiplier | 0 | ✓ |
| $\alpha_1$ | Alpha1 | first multiplier | 2 | ✓ |
| $\alpha_2$ | Alpha2 | second multiplier | 2 | ✓ |
| ... | ... | ... | 2 | ✓ |
| $\alpha_6$ | Alpha6 | final multiplier | 2 | ✓ |
| $\kappa$ | bulkModulus | bulk modulus for incompressibility | 100000 | ✓ |
| | bulkPotential | incompressibility potential function $U(\kappa,J)$ | QUADRATIC | |

OgdenMaterials can be created with the following constructors:

| | |
|---|---|
| OgdenMaterial() | Create with default properties. |
| OgdenMaterial (double[] mu, double[] alpha, double kappa) | Create with specified $\mu$, $\alpha$ and $\kappa$ values. |

#### 6.10.2.6 Fung orthotropic material

FungOrthotropicMaterial is an incompressible orthotropic material defined with respect to an $x$-$y$-$z$ coordinate system expressed by a $3 \times 3$ rotation matrix $\mathbf{R}$. The strain energy density is expressed as a function of the deviatoric Green strain

$$\bar{\mathbf{E}} \equiv \frac{1}{2}\left(\bar{\mathbf{C}} - \mathbf{I}\right) \tag{6.33}$$

and the three unit direction vectors $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$ representing the columns of $\mathbf{R}$. Letting $\mathbf{A}_i \equiv \mathbf{a}_i \mathbf{a}_i^T$ and defining

$$\alpha_i \equiv \mathbf{A}_i : \bar{\mathbf{E}} = \mathrm{tr}(\mathbf{a}_i^T \bar{\mathbf{E}}\, \mathbf{a}_i), \quad \beta_i \equiv \mathbf{A}_i : \bar{\mathbf{E}}^2 = \mathrm{tr}(\mathbf{a}_i^T \bar{\mathbf{E}}^2 \mathbf{a}_i), \tag{6.34}$$

the energy density is given by

$$W(\bar{\mathbf{E}}, J) = \frac{C}{2}(e^q - 1) + U(\kappa, J), \quad q = \sum_{i=1}^{3}\left(2\mu_i\beta_i + \sum_{j=1}^{3}\lambda_{ij}\alpha_i\alpha_j\right), \tag{6.35}$$

where $C$ is a material coefficient and $\mu_i$ and $\lambda_{ij}$ are the orthotropic Lamé parameters.

At present, the coordinate frame $\mathbf{R}$ is not defined in the material, but can be specified on a per-element basis using the element method setFrame(Matrix3dBase). For example:

```
RotationMatrix3d R;
FemModel3d fem;
...
for (FemElement3d elem : fem.getElements()) {
   ... computer R as required for the element ...
   elem.setFrame (R);
}
```

> One should be careful to ensure that the argument to `setFrame()` is in fact an orthogonal rotation matrix as this will not be otherwise enforced.

The material behavior is controlled by the following properties:

| | Property | Description | Default | Field |
|---|---|---|---|---|
| $\mu_1$ | mu1 | second Lamé parameter along $x$ | 1000 | ✓ |
| $\mu_2$ | mu2 | second Lamé parameter along $y$ | 1000 | ✓ |
| $\mu_3$ | mu3 | second Lamé parameter along $z$ | 1000 | ✓ |
| $\lambda_{11}$ | lam11 | first Lamé parameter along $x$ | 2000 | ✓ |
| $\lambda_{22}$ | lam22 | first Lamé parameter along $y$ | 2000 | ✓ |
| $\lambda_{33}$ | lam33 | first Lamé parameter along $z$ | 2000 | ✓ |
| $\lambda_{12}$ | lam12 | first Lamé parameter for $x, y$ | 2000 | ✓ |
| $\lambda_{23}$ | lam23 | first Lamé parameter for $y, z$ | 2000 | ✓ |
| $\lambda_{31}$ | lam31 | first Lamé parameter for $z, x$ | 2000 | ✓ |
| $C$ | C | C coefficient | 1500 | ✓ |
| $\kappa$ | bulkModulus | bulk modulus for incompressibility | 100000 | ✓ |
| | bulkPotential | incompressibility potential function $U(\kappa, J)$ | QUADRATIC | |

FungOrthotropicMaterials can be created with the following constructors:

| | |
|---|---|
| FungOrthotropicMaterial() | Create with default properties. |
| FungOrthotropicMaterial (double mu1, double mu2, double mu3, double lam11, double lam22, double lam33, double lam12, double lam23, double lam31, double C, double kappa) | Create with specified properties. |

#### 6.10.2.7 Yeoh material

YeohMaterial is an incompressible isotropic material that implements a Yeoh model with a strain energy density containing up to five terms:

$$W(\bar{\mathbf{C}}, J) = \sum_{i=1}^{5} C_i(\bar{I}_1 - 3)^i, \tag{6.36}$$

where $\bar{I}_1$ is the first invariant of the right deviatoric Cauchy Green tensor and $C_i$ are the material coefficients. The material behavior is controlled by the following properties:

| | Property | Description | Default | Field |
|---|---|---|---|---|
| $C_1$ | C1 | first coefficient (shear modulus) | 150000 | ✓ |
| $C_2$ | C2 | second coefficient | 0 | ✓ |
| $C_3$ | C3 | third coefficient | 0 | ✓ |
| $C_4$ | C4 | fourth coefficient | 0 | ✓ |
| $C_5$ | C5 | fifth coefficient | 0 | ✓ |
| $\kappa$ | bulkModulus | bulk modulus for incompressibility | 100000 | ✓ |
| | bulkPotential | incompressibility potential function $U(\kappa,J)$ | QUADRATIC | |

YeohMaterials can be created with the following constructors:

| | |
|---|---|
| YeohMaterial() | Create with default properties. |
| YeohMaterial (double C1, double C2, double C3, double kappa) | Create three term material with $C_1$, $C_2$, $C_3$, $\kappa$. |
| YeohMaterial (double C1, double C2, double C3, double C4, double C5, double kappa) | Create five term material with $C_1$, ..., $C_5$, $\kappa$. |

#### 6.10.2.8 Arruda-Boyce material

ArrudaBoyceMaterial is an incompressible isotropic material that implements the Arruda-Boyce model [2]. Its strain energy density is given by

$$W(\bar{\mathbf{C}},J) = \mu \sum_{i=1}^{5} \frac{C_i}{\lambda_L^{2(i-1)}}(\bar{I}_1^i - 3^i) + U(\kappa,J), \tag{6.37}$$

where $\mu$ is the *initial modulus*, $\lambda_L$ the *locking stretch*, $\bar{I}_1$ the first invariant of the right deviatoric Cauchy Green tensor, and

$$C = \left\{ \frac{1}{2}, \frac{1}{20}, \frac{11}{1050}, \frac{19}{7000}, \frac{519}{673750} \right\}.$$

The material behavior is controlled by the following properties:

| | Property | Description | Default | Field |
|---|---|---|---|---|
| $\mu$ | mu | initial modulus | 1000 | ✓ |
| $\lambda_L$ | lambdaMax | locking stretch | 2.0 | ✓ |
| $\kappa$ | bulkModulus | bulk modulus for incompressibility | 100000 | ✓ |
| | bulkPotential | incompressibility potential function $U(\kappa,J)$ | QUADRATIC | |

ArrudaBoyceMaterials can be created with the following constructors:

| | |
|---|---|
| ArrudaBoyceMaterial() | Create with default properties. |
| ArrudaBoyceMaterial (double mu, double lmax, double kappa) | Create with specified $\mu$, $\lambda_L$, $\kappa$. |

#### 6.10.2.9 Veronda-Westmann material

VerondaWestmannMaterial is an incompressible isotropic material that implements the Veronda-Westmann model [27]. Its strain energy density is given by

$$\mathbf{W}(\bar{\mathbf{C}},J) = C_1 \left( e^{C_2(\bar{I}_1 - 3)} - 1 \right) - \frac{C_1 C_2}{2}(\bar{I}_2 - 3) + U(\kappa,J), \tag{6.38}$$

where $C_1$ and $C_2$ are material coefficients and $\bar{I}_1$ and $\bar{I}_2$ the first and second invariants of the right deviatoric Cauchy Green tensor.

The material behavior is controlled by the following properties:

| | Property | Description | Default | Field |
|---|---|---|---|---|
| $C_1$ | C1 | first coefficient | 1000 | ✓ |
| $C_2$ | C2 | second coefficient | 10 | ✓ |
| $\kappa$ | bulkModulus | bulk modulus for incompressibility | 100000 | ✓ |
| | bulkPotential | incompressibility potential function $U(\kappa,J)$ | QUADRATIC | |

VerondaWestmannMaterials can be created with the following constructors:

| | |
|---|---|
| VerondaWestmannMaterial() | Create with default properties. |
| VerondaWestmannMaterial (double C1, double C2, double kappa) | Create with specified $\mathbf{C}_1$, $C_2$, and $\kappa$. |

#### 6.10.2.10 Incompressible material

IncompressibleMaterial is an incompressible isotropic material that implements pure incompressibility, with an energy density function given by

$$W(J) = U(\kappa, J). \tag{6.39}$$

Because it responds only to dilational strains, it must be used in conjunction with another material to resist deviatoric strains. In this context, it can be used to provide dilational support to deviatoric-only materials such as the FullBlemker-Muscle (Section 6.10.3.3).

The material behavior is controlled by the following properties:

| | Property | Description | Default | Field |
|---|---|---|---|---|
| $\kappa$ | bulkModulus | bulk modulus for incompressibility | 100000 | ✓ |
| | bulkPotential | incompressibility potential function $U(\kappa, J)$ | QUADRATIC | |

IncompressibleMaterials can be created with the following constructors:

| | |
|---|---|
| IncompressibleMaterial() | Create with default values. |
| IncompressibleMaterial (double kappa) | Create with specified $\kappa$. |

### 6.10.3 Muscle materials

Muscle materials are used to exert stress along a particular direction within a material. This stress may contain both an active component, which depends on the muscle excitation, and a passive component, which depends solely on the stretch along the prescribed direction. Because most muscle materials act only along one direction, they are typically deployed within an FEM as *additional* materials that act in addition to an underlying material. This can be done using either *muscle bundles* within a FemMuscleModel (Section 6.9.1.1) or material bundles within any FEM model (Sections 6.8 and 7.5.2).

All of the muscle materials described below assume (near) incompressibility, and so the directional stress is a function of the deviatoric stretch $\bar{\lambda}$ along the muscle direction instead of the overall stretch $\lambda$. The former can be determined from the latter via

$$\bar{\lambda} = \lambda J^{-1/3}, \tag{6.40}$$

where $J$ is the determinant of the deformation gradient. The directional Cauchy stress $\sigma_d$ resulting from the material is computed from

$$\sigma_d = \frac{\bar{\lambda} f_d(\bar{\lambda})}{J}\left(\mathbf{a}\mathbf{a}^T - \frac{1}{3}\mathbf{I}\right), \tag{6.41}$$

where $f_d(\bar{\lambda})$ is a force term, $\mathbf{a}$ is a unit vector indicating the current muscle direction in spatial coordinates, and $\mathbf{I}$ is the $3 \times 3$ identity matrix. In a purely passive case when the force arises from a stress energy density function $W(\bar{\lambda})$, we have

$$f_d(\bar{\lambda}) = \frac{\partial W(\bar{\lambda})}{\partial \bar{\lambda}}. \tag{6.42}$$

The muscle direction is specified in one of two ways, depending on how the muscle material is deployed. All muscle materials export a property restDir which specifies the direction in material (rest) coordinates. It has a default value of $(1,0,0)^T$, and can be either set explicitly or bound to a field to allow it to vary over the entire model (Section 7.5.2). However, if a muscle material is deployed within a muscle bundle (Section 6.9.1), then the restDir property is ignored and the direction is instead specified by the MuscleElementDesc components contained within the bundle (Section 6.9.3).

Likewise, muscle excitation is specified using the material's excitation property, unless the material is deployed within a muscle bundle, in which case it is controlled by the excitation property of the bundle.

#### 6.10.3.1 Generic muscle

GenericMuscle is a muscle material whose force term $f_d(\bar{\lambda})$ is given by

$$f_d(\bar{\lambda}) = a\sigma_{max} + f_p(\bar{\lambda}), \tag{6.43}$$

where $a$ is the excitation signal, $\sigma_{max}$ is a maximum stress term, and $f_p(\bar{\lambda})$ is a passive force given by

$$f_p(\bar{\lambda}) = \begin{cases} 0, & \bar{\lambda} < 1 \\ P_1(e^{P_2(\bar{\lambda}-1)} - 1)/\bar{\lambda}, & \bar{\lambda} < \bar{\lambda}_{max} \\ (P_3\bar{\lambda} + P_4)/\bar{\lambda}, & \text{otherwise.} \end{cases} \tag{6.44}$$

In the equation above, $P_1$ is the exponential stress coefficient, $P_2$ is the uncrimping factor, and $P_3$ and $P_4$ are computed to provide $C(1)$ continuity at $\bar{\lambda} = \bar{\lambda}_{max}$.

The material behavior is controlled by the following properties:

| | Property | Description | Default | Field |
|---|---|---|---|---|
| $\sigma_{max}$ | maxStress | maximum stress | 30000 | ✓ |
| $\bar{\lambda}_{max}$ | maxLambda | $\bar{\lambda}$ at upper limit of exponential section of $f_p(\bar{\lambda})$ | 1.4 | ✓ |
| $P_1$ | expStressCoeff | exponential stress coefficient | 0.05 | ✓ |
| $P_2$ | uncrimpingFactor | uncrimping factor | 6.6 | ✓ |
| $\mathbf{a}_0$ | restDir | direction in material coordinates | $(1,0,0)^T$ | ✓ |
| $a$ | excitation | activation signal | 0 | |

`GenericMuscle`s can be created with the following constructors:

| | |
|---|---|
| GenericMuscle() | Create with default values. |
| GenericMuscle (double maxLambda, double maxStress, double expStressCoef, double uncrimpFactor) | Create with specified $\bar{\lambda}_{max}$, $\sigma_{max}$, $P_1$, $P_2$. |

The constructors do not specify restDir. If the material is not being deployed within a muscle bundle, then restDir should also be set appropriately, either directly or via a field (Section 7.5.2).

#### 6.10.3.2 Blemker muscle

BlemkerMuscle is a muscle material implementing the directional component of the model proposed by Sylvia Blemker [5]. Its force term $f_d(\bar{\lambda})$ is given by

$$f_d(\bar{\lambda}) = \sigma_{max}(af_a(\bar{\lambda}) + f_p(\bar{\lambda}))/\bar{\lambda}_{opt}, \tag{6.45}$$

where $\sigma_{max}$ is a maximum stress term, $a$ is the excitation signal, $f_a(\bar{\lambda})$ is an active force-length curve, and $f_p(\bar{\lambda})$ is the passive force. $f_a(\bar{\lambda})$ and $f_p(\bar{\lambda})$ are described in terms of $\hat{\lambda} \equiv \bar{\lambda}/\bar{\lambda}_{opt}$:

$$f_a(\hat{\lambda}) = \begin{cases} 9(\hat{\lambda} - 0.4)^2, & \hat{\lambda} \le 0.6 \\ 1 - 4(\hat{\lambda} - 1)^2, & 0.6 < \hat{\lambda} < 1.4 \\ 9(\hat{\lambda} - 1.6)^2, & \text{otherwise,} \end{cases} \tag{6.46}$$

and

$$f_p(\hat{\lambda}) = \begin{cases} 0, & \hat{\lambda} \le 1 \\ P_1(e^{P_2(\hat{\lambda}-1)} - 1), & 1 < \hat{\lambda} < \bar{\lambda}_{max}/\bar{\lambda}_{opt} \\ (P_3\hat{\lambda} + P_4), & \text{otherwise.} \end{cases} \tag{6.47}$$

In the equation for $f_p(\bar{\lambda})$, $P_1$ is the exponential stress coefficient, $P_2$ is the uncrimping factor, and $P_3$ and $P_4$ are computed to provide $C(1)$ continuity at $\hat{\lambda} = \bar{\lambda}_{max}/\bar{\lambda}_{opt}$.

The material behavior is controlled by the following properties:

| | Property | Description | Default | Field |
|---|---|---|---|---|
| $\sigma_{max}$ | maxStress | maximum stress | 300000 | ✓ |
| $\bar{\lambda}_{opt}$ | optLambda | $\bar{\lambda}$ at peak of $f_a(\bar{\lambda})$ | 1 | ✓ |
| $\bar{\lambda}_{max}$ | maxLambda | $\bar{\lambda}$ at upper limit of exponential section of $f_p(\bar{\lambda})$ | 1.4 | ✓ |
| $P_1$ | expStressCoeff | exponential stress coefficient | 0.05 | ✓ |
| $P_2$ | uncrimpingFactor | uncrimping factor | 6.6 | ✓ |
| $\mathbf{a}_0$ | restDir | direction in material coordinates | $(1,0,0)^T$ | ✓ |
| $a$ | excitation | activation signal | 0 | |

`BlemkerMuscles` can be created with the following constructors:

| | |
|---|---|
| BlemkerMuscle() | Create with default values. |
| BlemkerMuscle (double maxLam, double optLam, double maxStress, double expStressCoef, double uncrimpFactor) | Create with specified $\bar{\lambda}_{max}$, $\bar{\lambda}_{opt}$, $\sigma_{max}$, $P_1$, $P_2$. |

The constructors do not specify restDir. If the material is not being deployed within a muscle bundle, then restDir should also be set appropriately, either directly or via a field (Section 7.5.2).

#### 6.10.3.3 Full Blemker muscle

FullBlemkerMuscle is a muscle material implementing the entire model proposed by Sylvia Blemker [5]. This includes the directional stress described in Section 6.10.3.2, plus additional passive terms to model the tissue material matrix. The latter is based on a strain energy density function $W_m$ given by

$$W_m(\bar{\mathbf{B}}, \lambda) = G_1 \bar{B}_1^2 + G_2 \bar{B}_2^2 \tag{6.48}$$

where $G_1$ and $G_2$ are the *along-fibre* and *cross-fibre* shear moduli, respectively, and $\bar{B}_1$ and $\bar{B}_2$ are Criscione invariants. The latter are defined as follows: Let $\bar{I}_1$ and $\bar{I}_2$ be the invariants of the deviatoric left Cauchy Green tensor $\bar{\mathbf{B}}$, $\mathbf{a}$ the unit vector giving the current muscle direction in spatial coordinates, and $\bar{I}_4$ and $\bar{I}_5$ additional invariants defined by

$$\bar{I}_4 \equiv \bar{\lambda}^2, \quad \bar{I}_5 \equiv \bar{I}_4 \mathbf{a}^T \bar{\mathbf{B}} \mathbf{a}. \tag{6.49}$$

Then, defining

$$g \equiv \frac{\bar{I}_5}{\bar{I}_4^2} - 1, \quad y \equiv \frac{\bar{I}_1 \bar{I}_4 - \bar{I}_5}{2\bar{\lambda}},$$

$\bar{B}_1$ and $\bar{B}_2$ are given by

$$\bar{B}_1 = \begin{cases} \sqrt{g}, & g > 0 \\ 0, & \text{otherwise,} \end{cases} \tag{6.50}$$

$$\bar{B}_2 = \begin{cases} \operatorname{acosh}(y), & y > 1 \\ 0, & \text{otherwise.} \end{cases} \tag{6.51}$$

> At present, `FullBlemkerMuscle` does not implement a volumetric potential function $U(J)$. That means it responds solely to deviatoric strains, and so if it is being used as a model's primary material, it should be augmented with an additional material to provide resistance to compression. A simple choice for this is the purely incompressible material IncompressibleMaterial, described in Section 6.10.2.10.

The material behavior is controlled by the following properties:

| | Property | Description | Default | Field |
|---|---|---|---|---|
| $\sigma_{max}$ | maxStress | maximum stress | 300000 | ✓ |
| $\bar{\lambda}_{opt}$ | optLambda | $\bar{\lambda}$ at peak of $f_a(\bar{\lambda})$ | 1 | ✓ |
| $\bar{\lambda}_{max}$ | maxLambda | $\bar{\lambda}$ at upper limit of exponential section of $f_p(\bar{\lambda})$ | 1.4 | ✓ |
| $P_1$ | expStressCoeff | exponential stress coefficient | 0.05 | ✓ |
| $P_2$ | uncrimpingFactor | uncrimping factor | 6.6 | ✓ |
| $G_1$ | G1 | along-fibre shear modulus | 0 | ✓ |
| $G_2$ | G2 | cross-fibre shear modulus | 0 | ✓ |
| $\mathbf{a}_0$ | restDir | direction in material coordinates | $(1,0,0)^T$ | ✓ |
| $a$ | excitation | activation signal | 0 | |

`FullBlemkerMuscles` can be created with the following constructors:

| | |
|---|---|
| FullBlemkerMuscle() | Create with default values. |
| BlemkerMuscle (double maxLam, double optLam, double maxStress, double expStressCoef, double uncrimpFactor, double G1, double G2) | Create with $\bar{\lambda}_{max}$, $\bar{\lambda}_{opt}$, $\sigma_{max}$, $P_1$, $P_2$, $G_1$, $G_2$. |

The constructors do not specify restDir. If the material is not being deployed within a muscle bundle, then restDir should also be set appropriately, either directly or via a field (Section 7.5.2).

#### 6.10.3.4 Simple force muscle

SimpleForceMuscle is a very simple muscle material whose force term $f_d(\bar{\lambda})$ is given by

$$f_d(\bar{\lambda}) = a\sigma_{max}, \tag{6.52}$$

where $a$ is the excitation signal and $\sigma_{max}$ is a maximum stress term. It can be useful as a debugging tool.

The material behavior is controlled by the following properties:

| | Property | Description | Default | Field |
|---|---|---|---|---|
| $\sigma_{max}$ | maxStress | maximum stress | 30000 | ✓ |
| $\mathbf{a}_0$ | restDir | direction in material coordinates | $(1,0,0)^T$ | ✓ |
| $a$ | excitation | activation signal | 0 | |

`SimpleForceMuscles` can be created with the following constructors:

| | |
|---|---|
| SimpleForceMuscle() | Create with default values. |
| SimpleForceMuscle (double maxStress) | Create with specified $\sigma_{max}$. |

The constructors do not specify restDir. If the material is not being deployed within a muscle bundle, then restDir should also be set appropriately, either directly or via a field (Section 7.5.2).

## 6.11 Stress, strain and strain energy

An ArtiSynth FEM model has the capability to compute a variety of stress and strain measures at each of its nodes, as well as strain energy for each element and for the entire model.

### 6.11.1 Computing nodal values

Stress, strain, and strain energy density can be computed at each node of an FEM model. By default, these quantities are *not* computed, in order to conserve computing power. However, their computation can be enabled with the `FemModel3d` properties computeNodalStress, computeNodalStrain and computeNodalEnergyDensity, which can be set either in the GUI, or using the following accessor methods:

| | |
|---|---|
| void setComputeNodalStress(boolean) | Enable stress to be computed at each node. |
| boolean getComputeNodalStress() | Queries if nodal stress computation enabled. |
| void setComputeNodalStrain(boolean) | Enable strain to be computed at each node. |
| boolean getComputeNodalStrain() | Queries if nodal strain computation enabled. |
| void setComputeNodalEnergyDensity(boolean) | Enable strain energy density to be computed at each node. |
| boolean getComputeNodalEnergyDensity() | Queries if nodal strain energy density computation enabled. |

Setting these properties to `true` will cause their respective values to be computed at the nodes during each subsequent simulation step. This is done by computing values at the element integration points, extrapolating these values to the nodes, and then averaging them across the contributions from all surrounding elements.

Once computed, the values may be obtained at each node using the following `FemNode3d` methods:

| | |
|---|---|
| SymmetricMatrix3d getStress() | Returns the current Cauchy stress at the node. |
| SymmetricMatrix3d getStrain() | Returns the current strain at the node. |
| double getEnergyStrain() | Returns the current strain energy density at the node. |

The stress value is the Cauchy stress. The strain values depend on the primary material within each element: if the material is linear, strain will be the small deformation Cauchy strain (co-rotated if the material's corotated property is `true`), while otherwise it will be Green strain $\mathbf{E}$:

$$\mathbf{E} = \frac{1}{2}(\mathbf{C} - \mathbf{I}), \tag{6.53}$$

where $\mathbf{C}$ is the right Cauchy-Green tensor.

### 6.11.2 Scalar stress/strain measures

Stress or strain values are tensor quantities represented by symmetric $3 \times 3$ matrices. If they are being computed at the nodes, a variety of scalar quantities can be extracted from them, using the `FemNode3d` method

```
double getStressStrainMea-
sure(StressStrainMeasure m)
```

where StressStrainMeasure is an enumerated type with the following entries:

**VonMisesStress**

von Mises stress, equal to

$$\sqrt{3\mathbf{J}_2},$$

where $\mathbf{J}_2$ the second invariant of the average deviatoric stress.

**MAPStress**

Absolute value of the maximum principle Cauchy stress.

**MaxShearStress**

Maximum shear stress, given by

$$\max(|s_0 - s_1|/2, |s_1 - s_2|/2, |s_2 - s_0|/2),$$

where $s_0$, $s_1$ and $s_2$ are the eigenvalues of the stress tensor.

**VonMisesStrain**

von Mises strain, equal to

$$\sqrt{4/3\mathbf{J}_2},$$

where $\mathbf{J}_2$ the second invariant of the average deviatoric strain.

**MAPStrain**

Absolute value of the maximum principle strain.

**MaxShearStrain**

Maximum shear strain, given by

$$\max(|s_0 - s_1|/2, |s_1 - s_2|/2, |s_2 - s_0|/2),$$

where $s_0$, $s_1$ and $s_2$ are the eigenvalues of the strain tensor.

**EnergyDensity**

Strain energy density.

These quantities can also be queried with the following convenience methods:

| | |
|---|---|
| double getVonMisesStress() | Return the von Mises stress. |
| double getMAPStress() | Return the maximum absolute principle stress. |
| double getMaxShearStress() | Return the maximum shear stress. |
| double getVonMisesStrain() | Return the von Mises strain. |
| double getMAPStrain() | Return the maximum absolute principle strain. |
| double getMaxShearStrain() | Return the maximum shear strain. |
| double getEnergyDensity() | Return the strain energy density. |

> For muscle-activated FEM materials, strain energy density is currently computed only for the *passive* portions of the material; no energy density is computed for the stress component that depends on muscle activation.

As indicated above, extracting these measures requires that the appropriate underling quantity (stress, strain or strain energy density) is being computed at the nodes; otherwise, 0 will be returned. To ensure that the appropriate underlying quantity is being computed for a specific measure, one may use the `FemModel3d` method

```
void setComputeNodalStressStrain (StressStrainMeasure m)
```

These measures can also be used to produce color maps on FEM mesh components or cut planes, as described in Sections 6.12.3 or 6.12.7. When color maps are being produced, the system automatically enables the computation of the required stress, strain or energy density.

### 6.11.3 Strain energy

Strain energy density can be integrated over an FEM model's volume to compute a total strain energy. Again, to save computational resources, this must be explicitly enabled using the `FemModel3d` property computeStrainEnergy, which can be set either in the GU or using the accessor methods:

| | |
|---|---|
| void setComputeStrainEnergy(boolean) | Enable strain energy computation. |
| boolean getComputeStrainEnergy() | Queries strain energy computation. |

Once computation is enabled, strain energy is exported for the entire model using the read-only strainEnergy property, which can be accessed using

```
double getStrainEnergy()
```

The `FemElement` method getStrainEnergy() returns the strain energy for individual elements.

## 6.12 Rendering and Visualizations

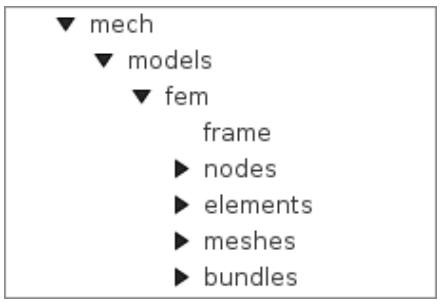



Figure 6.17: Component lists of an FEM muscle model displayed in the navigation panel.

An ArtiSynth FEM model can be rendered in a wide variety of ways, by adjusting the rendering of its nodes, elements, meshes (including the surface and other embedded meshes), and, for `FemMuscleModel`, muscle bundles. Properties for controlling the rendering include both the standard `RenderProps` (Section 4.3) as well as more specific properties. These can be set either in code, as described below, or by selecting the desired components (or their lists) and then choosing either Edit render props ... (for standard render properties) or Edit properties ... (for component-specific render properties) from the context menu. As mentioned in Section 4.3, standard render properties are *inherited*, and so if not explicitly specified within a given component, will assume whatever value has been set in the nearest ancestor component, or their default value. Some component-specific render properties are inherited as well.

One very direct way to control the rendering is to control the visibility of the various component lists. This can be done by selecting them in the navigation panel (Figure 6.17) and then choosing "Set invisible" or "Set visible", as appropriate. It can also be done in code, as shown in the fragment below making the elements and nodes of an FEM model invisible:

```
FemModel3d fem;

// ... initialize the model ...

RenderProps.setVisible (fem.getNodes(), false);    // make nodes invisible
RenderProps.setVisible (fem.getElements(), false); // make elements invisible
```

### 6.12.1 Nodes

Node rendering is controlled by the *point* render properties of the standard `RenderProps`. By default, nodes are set to render as gray points one pixel wide, and so are not highly visible. To increase their visibility, and make them easier to select in the viewer, one can make them appear as spheres of a specified radius and color. In code, this can be done using the RenderProps.setSphericalPoints() method, as in this example,

```
import java.awt.Color;
import maspack.render.RenderProps;
   ...

   FemModel3d fem;
   ...

   RenderProps.setSphericalPoints (fem, 0.01, Color.GREEN);
```

which sets the default rendering for all points within the FEM model (which includes the nodes) to be green spheres of radius 0.01. To restrict this to the node list specifically, one could supply `fem.getNodes()` instead of `fem` to `setSphericalPoints()`.

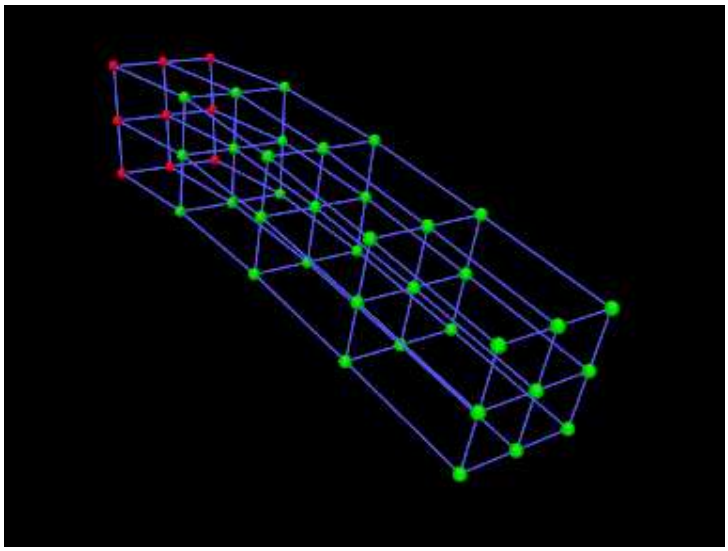


Figure 6.18: FEM model with nodes rendered as spheres.

It is also possible to set render properties for individual nodes. For instance, one may wish to mark nodes that are non-dynamic using a different color:

```
import java.awt.Color;
  ...

  for (FemNode3d node : fem.getNodes()) {
     if (!node.isDynamic()) {
        RenderProps.setPointColor (node, Color.RED);
     }
  }
```

Figure 6.18 shows an FEM model with nodes rendered as spheres, with the dynamic nodes colored green and the non-dynamic ones red.

### 6.12.2 Elements

By default, elements are rendered as wireframe outlines, using the lineColor and lineWidth properties of the standard render properties, with defaults of “gray” and 1. To improve visibility, one may wish the change the line color or size, as illustrated by the following fragment:

```
import java.awt.Color;
import maspack.render.RenderProps;
   ...

   FemModel3d fem;
   ...

   RenderProps.setLineColor (fem, Color.CYAN);
   RenderProps.setLineWidth (fem, 2);
```

This will set the default line color and width for all components within the FEM model. To restrict this to the elements only, one can specify `fem.getElements()` (and/or `fem.getShellElements()`) to the `set` methods. The fragment above is illustrated in Figure 6.19 (left) for a model created with a call to FemFactory.createHexTorus().

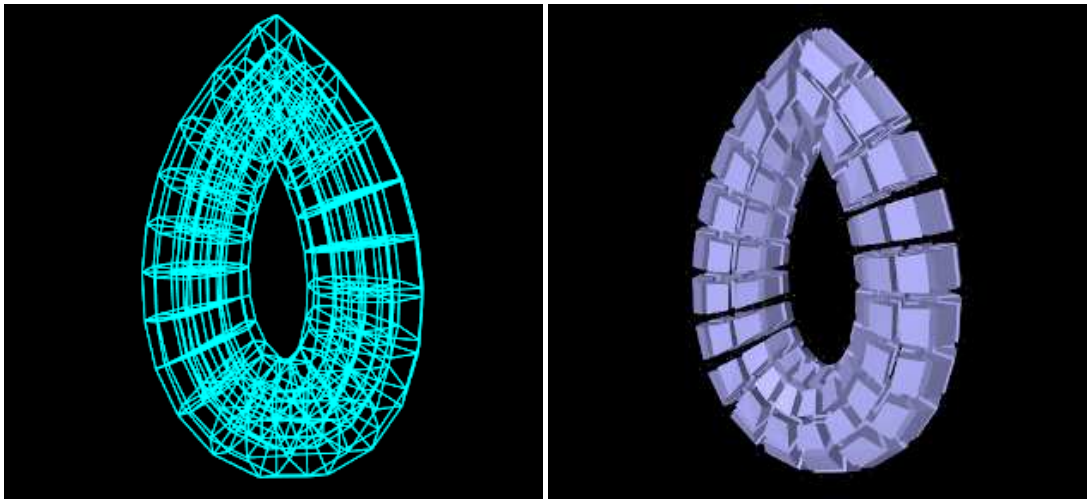


Figure 6.19: Elements of a torus shaped FEM model, rendered as wireframe (left), and using element widgets with an elementWidgetSize of 0.7 (right).

Elements can also be rendered as widgets that depict an approximation of their shape displayed at some fraction of the element's size (this shape will be 3D for volumetric elements and 2D for shell elements). This is controlled using the `FemModel3d` property elementWidgetSize, which is restricted to the range $[0, 1]$ and describes the size of the widgets relative to the element size. If elementWidgetSize is 0 then no widgets are displayed. The widget color is controlled using the faceColor and alpha properties of the standard render properties. The following code fragment enables element widget rendering for an FEM model, as illustrated in Figure 6.19 (right):

```
import java.awt.Color;
import maspack.render.RenderProps;
   ...

   FemModel3d fem;
   ...

   fem.setElementWidgetSize (0.7);
   RenderProps.setFaceColor (fem, new Color(0.7f, 0.7f, 1f));
   RenderProps.setLineWidth (fem, 0); // turn off element wire frame
```

Since setting faceColor for the whole FEM model will also set the default face color for its meshes, one may wish to restrict setting this to the elements only, by specifying `fem.getElements()` and/or `fem.getShellElements()` to `setFaceColor()`.

Element widgets can provide a easy way to select specific elements. The elementWidgetSize property is also present as an inherited property in the volumetric and shell element lists (`elements` and `shellElements`), as well as individual elements, and so widget rendering can be controlled on a per-element basis.

### 6.12.3 Surface and other meshes

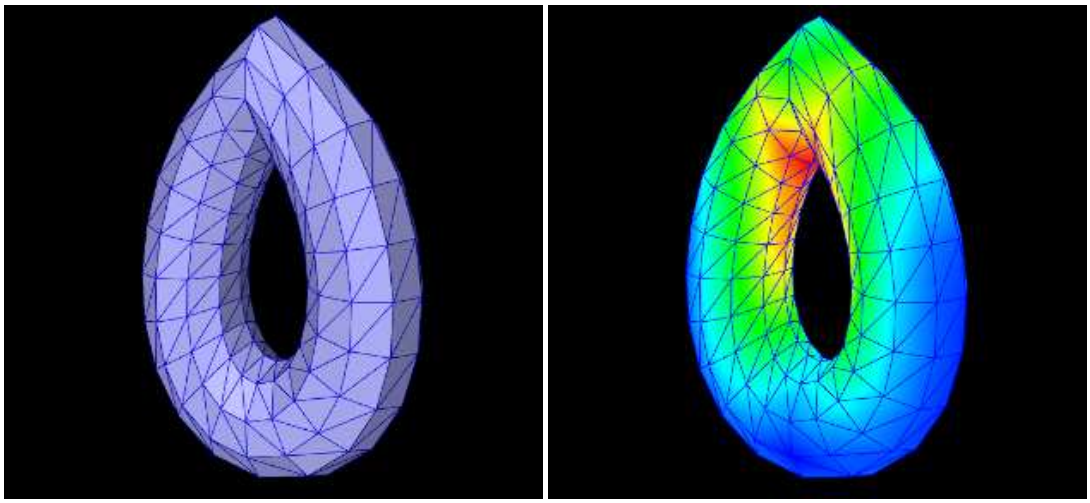


Figure 6.20: Surface mesh of a torus rendered as a regular mesh (left, with surfaceRendering set to `Shaded`), and as a color map of the von Mises stress (right, with surfaceRendering set to `Stress`).

A FEM model can also be rendered using its surface mesh and/or other mesh geometry (Section 6.19). How meshes are rendered is controlled by the property surfaceRendering, which can be assigned any of the values shown in Table 6.5. The value `None` causes nothing to be rendered, while `Shaded` causes a mesh to be rendered using the standard rendering properties appropriate to that mesh. For polygonal meshes (which include the surface mesh), this will be done according to the *face* rendering properties in same way as for rigid bodies (Section 3.2.8). These properties include faceColor, shading, alpha, faceStyle, drawEdges, edgeWidth, and edgeColor (or lineColor if the former is not set). The following code fragment sets an FEM surface to be rendered with blue-gray faces and dark blue edges (Figure 6.20, left):

```
import java.awt.Color;
import maspack.render.RenderProps;
import artisynth.core.femmodels.FemModel.SurfaceRender;
   ...

   FemModel3d fem;
   ...

   fem.setSurfaceRendering (SurfaceRender.Shaded);
   RenderProps.setFaceColor (fem, new Color (0.7f. 0.7f, 1f));
   RenderProps.setDrawEdges (fem, true);
   RenderProps.setEdgeColor (fem, Color.BLUE);
```

Table 6.5: Values of the surfaceRendering property controlling how a polygonal FEM is displayed. Scalar stress/strain measures are described in Section 6.11.2

| Value | Description |
|---|---|
| `None` | mesh is not rendered |
| `Shaded` | rendered as a mesh using the standard rendering properties |
| `Stress` | color map of the von Mises stress |
| `Strain` | color map the von Mises strain |
| `MAPStress` | color map of the maximum absolute value principal stress |
| `MAPStrain` | color map of the maximum absolute value principal strain |
| `MaxShearStress` | color map of the maximum shear stress |
| `MaxStearStrain` | color map of the maximum sheer strain |
| `EnergyDensity` | color map of the strain energy density |

Other values of surfaceRendering cause polygonal meshes to be displayed as a color map showing the various stress or strain measures shown in Table 6.5 and described in greater detail in Section 6.11.2. The fragment below sets an FEM

surface to be rendered to show the von Mises stress (Figure 6.20, right), while also rendering the edges as dark blue:

```
import java.awt.Color;
import maspack.render.RenderProps;
import artisynth.core.femmodels.FemModel.SurfaceRender;
   ...

  FemModel3d fem;
   ...

   fem.setSurfaceRendering (SurfaceRender.Stress);
   RenderProps.setDrawEdges (fem, true);
   RenderProps.setEdgeColor (fem, Color.BLUE);
```

The color maps used in stress/strain rendering are controlled using the following additional properties:

**stressPlotRange**

The range of numeric values associated with the color map. These are either fixed or updated automatically, according to the property stressPlotRanging. Values outside the range are truncated.

**stressPlotRanging**

Describes if and how stressPlotRange is updated:

`Fixed`

Range does not change and should be set by the application.

`Auto`

Range automatically expands to contain the most recently computed values. Note that this does *not* cause the range to contract.

**colorMap**

Value is a delegate object that converts stress and strain values to colors. Various types of maps exist, including HueColorMap (the default), GreyscaleColorMap, RainbowColorMap, and JetColorMap. These all implement the ColorMap interface.

All of these properties are inherited and exported both by FemModel3d *and* the individual mesh components (which are instances of FemMeshComp), allowing mesh rendering to be controlled on a per-mesh basis.

### 6.12.4 FEM-based muscles

FemMuscleModel and its subcomponents MuscleBundle export additional properties to control the rendering of muscle bundles and their associated fibre directions. These include:

**directionRenderLen**

A scalar in the range $[0,1]$, which if $> 0$ causes the fibre directions to be rendered within the elements associated with a `MuscleBundle` (Section 6.9.3). The directions are rendered as line segments, controlled by the standard *line* render properties lineColor and lineWidth, and the value of directionRenderLen specifies the length of this segment relative to the size of the element.

**directionRenderType**

If directionRenderLen $> 0$, this property specifics *where* the fibre directions should be rendered within muscle bundle elements. The value `ELEMENT` causes a single direction to be rendered at the element center, while `INTEGRATION_POINTS` causes directions to be rendered at each of the element's integration points.

**elementWidgetSize**

A scalar in the range $[0,1]$, which if $> 0$ causes the elements within a muscle bundle to be rendered using an element widget (Section 6.12.2).

Since these properties are inherited and exported by both `FemMuscleModel` and `MuscleBundle`, they can be set for the FEM muscle model as a whole or for individual muscle bundles.

The code fragment below sets the element fibre directions for two muscle bundles to be drawn, one in red and one in green, using a directionRenderLen of 0.5. Each bundle runs horizontally along an FEM beam, one at the top and the other at the bottom (Figure 6.21, left):

```
FemModel3d fem;
...

// set line width and directionRenderLen for all bundles and lines
RenderProps.setLineWidth (fem, 2);
fem.setDirectionRenderLen (0.5);
// set line color for FEM elements
RenderProps.setLineColor (fem, new Color(0.7f, 0.7f, 1f));

// set line colors for top and bottom bundles
MuscleBundle top = fem.getMuscleBundles().get("top");
MuscleBundle bot = fem.getMuscleBundles().get("bot");
RenderProps.setLineColor (top, Color.RED);
RenderProps.setLineColor (bot, new Color(0f, 0.8f, 0f));
```

By default, directionRenderType is set to `ELEMENT` and so directions are drawn at the element centers. Setting directionRenderType to `INTEGRATION_POINT` causes the directions to be drawn at each integration points (Figure 6.21, right), which is useful when directions are specified at integration points (Section 6.9.3).

If we are only interested in seeing the elements associated with a muscle bundle, we can use its elementWidgetSize property to draw the elements as widgets (Figure 6.22, left). For this, the last two lines of the fragment above could be replaced with

```
RenderProps.setFaceColor (top, Color.RED);
top.setElementWidgetSize (0.6);
RenderProps.setFaceColor (bot, new Color(0f, 0.8f, 0f));
bot.setElementWidgetSize (0.6);
```

Finally, if the muscle bundle contains point-to-point fibres (Section 6.9.2), these can be rendered by setting the *line* properties of the standard render properties. For example, the fibre rendering of Figure 6.22 (right) can be set up using the code

```
RenderProps.setSpindleLines (top, /*radius=*/0.13, Color.RED);
RenderProps.setSpindleLines (bot, /*radius=*/0.13, new Color(0f, 0.8f, 0f));
```

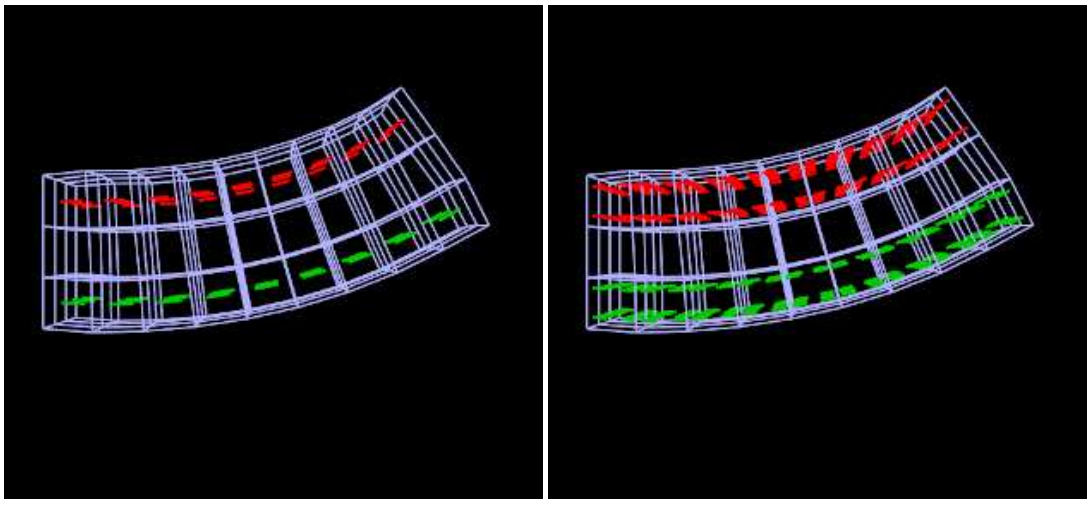


Figure 6.21: Rendering the element fibre directions for two muscle bundles, one in red and one in green, with directionRenderLen of 0.5 and directionRenderType set to `ELEMENT` (left) and `INTEGRATION_POINT` (right).

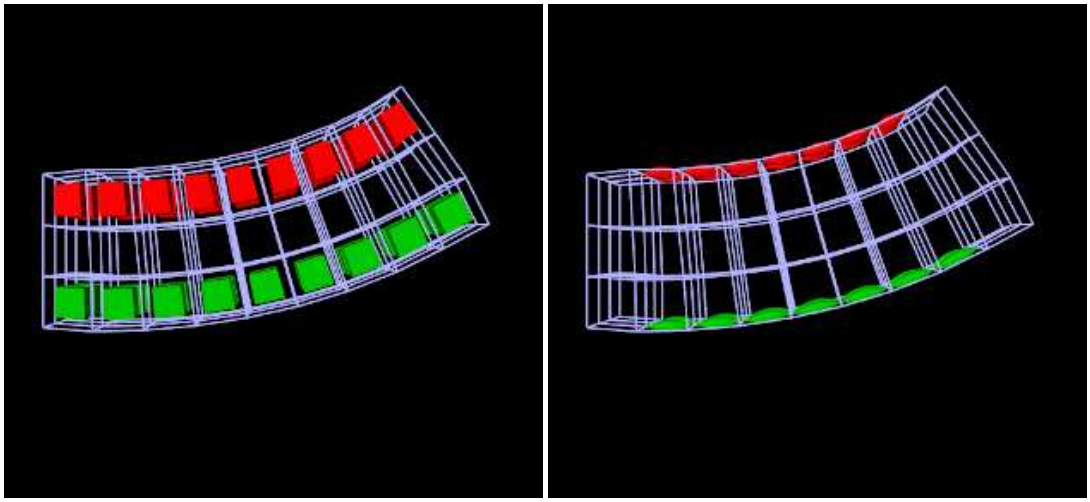


Figure 6.22: Left: rendering muscle bundle elements using element widgets with elementWidgetSize set to 0.6 (left). Right: rendering muscle bundle fibres.

### 6.12.5 Color bars

To display values corresponding to colors, a ColorBar needs to be added to the `RootModel`. Color bars are general `Renderable` objects that are only used for visualizations. They are added to the display using the

```
addRenderable (Renderable r)
```

method in `RootModel`. Color bars also have a `ColorMap` associated with it. The following functions are useful for controlling its visualization:

```
setNumberFormat (String fmtStr );    // C-like numeric format specification
populateLabels (double min, double max, int tick );     // initialize labels
updateLabels (double min, double max );                 // update existing labels

setColorMap (ColorMap map );                            // set color map

// Control position/size of the bar
setNormalizedLocation (double x, double y, double width, double height);
setLocationOverride (double x, double y, double width, double height)
```

The normalized location specifies sizes relative to the screen size (1 = screen width/height). The location override, if values are non-zero, will override the normalized location, specifying values in absolute pixels. Negative values for position correspond to distances from the left/top. For instance,

```
setNormalizedLocation(0, 0.1, 0, 0.8);  // set relative positions
setLocationOverride(-40, 0, 20, 0);     // override with pixel lengths
```

will create a bar that is 10% up from the bottom of the screen, 40 pixels from the right edge, with a height occupying 80% of the screen, and width 20 pixels.

Note that the color bar is not associated with any mesh or finite element model. Any synchronization of colors and labels must be done manually by the developer. It is recommended to do this in the `RootModel`'s `prerender(...)` method, so that colors are updated every time the model's rendering configuration changes.

### 6.12.6 Example: stress/strain plotting with color bars

The following model extends `FemBeam` to render stress, with an added color bar. The loaded model is shown in Figure 6.23.

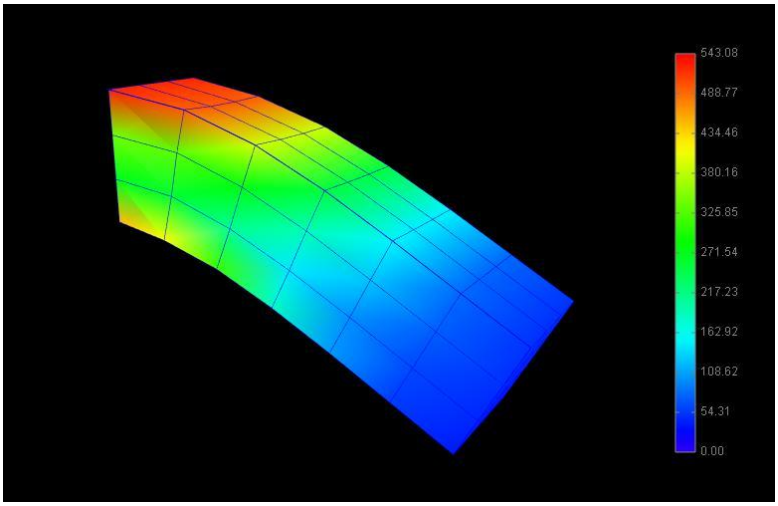



Figure 6.23: FemBeamColored model loaded into ArtiSynth.

```
1  package artisynth.demos.tutorial;
2
3  import java.io.IOException;
4
5  import maspack.render.RenderList;
6  import maspack.util.DoubleInterval;
7  import artisynth.core.femmodels.FemModel.Ranging;
8  import artisynth.core.femmodels.FemModel.SurfaceRender;
9  import artisynth.core.renderables.ColorBar;
10
11 public class FemBeamColored extends FemBeam {
12
13    @Override
14    public void build(String[] args) throws IOException {
15       super.build(args);
16
17       // Show stress on the surface
18       fem.setSurfaceRendering(SurfaceRender.Stress);
19       fem.setStressPlotRanging(Ranging.Auto);
20
21       // Create a colorbar
22       ColorBar cbar = new ColorBar();
23       cbar.setName("colorBar");
24       cbar.setNumberFormat("%.2f");      // 2 decimal places
25       cbar.populateLabels(0.0, 1.0, 10); // Start with range [0,1], 10 ticks
26       cbar.setLocation(-100, 0.1, 20, 0.8);
27       addRenderable(cbar);
28
29    }
30
31    @Override
32    public void prerender(RenderList list) {
33       super.prerender(list);
34       // Synchronize color bar/values in case they are changed. Do this *after*
35       // super.prerender(), in case values are changed there.
36       ColorBar cbar = (ColorBar)(renderables().get("colorBar"));
37       cbar.setColorMap(fem.getColorMap());
38       DoubleInterval range = fem.getStressPlotRange();
39       cbar.updateLabels(range.getLowerBound(), range.getUpperBound());
40
41
42    }
43
```

```
44 }
```

### 6.12.7 Cut planes

In addition to stress/strain visualization on its meshes, FEM models can be supplied with one or more *cut planes* that allow stress or strain values to be visualized on the cross section of the model with the plane.

Cut plane components are implemented by FemCutPlane and can be created with the following constructors:

| | |
|---|---|
| FemCutPlane() | Creates a cut plane aligned with the world origin. |
| FemCutPlane (double res) | Creates an origin-aligned cut plane with grid resolution `res`. |
| FemCutPlane (RigidTransform3d TPW) | Creates a cut plane with pose `TPW` a specified grid resolution. |
| FemCutPlane (double res, RigidTransform3d TPW) | Creates a cut plane with pose `TPW` and grid resolution `res`. |

The *pose* and *resolution* are described further below. Once created, a cut plane can be added to an FEM model using its `addCutPlane()` method:

```
FemModel3d fem;
RigidTransform3d TPW; // desired pose of the plane

... initialize fem and TPW ...

FemCutPlane cplane = new FemCutPlane (TPW);
fem.addCutPlane (cplane);
```

The FEM model methods for handling cut planes include:

| | |
|---|---|
| void addCutPlane(FemCutPlane cp) | Adds a cut plane to the model. |
| int numCutPlanes() | Queries number of cut planes in the model. |
| FemCutPlane getCutPlanes(int idx) | Gets the `idx`-th cut plane in the model. |
| boolean removeCutPlane(FemCutPlane cp) | Removes a cut plane from the model. |
| void clearCutPlanes() | Removes all cut planes from the model. |

As with rigid and mesh bodies, the *pose* of the cut plane gives its position and orientation relative to world coordinates and is described by a RigidTransform3d. The plane itself is aligned with the $x$-$y$ plane of this local coordinate system. More specifically, if the pose is given by $\mathbf{T}_{PW}$ such that

$$\mathbf{T}_{PW} = \begin{pmatrix} \mathbf{R}_{PW} & \mathbf{p}_{PW} \\ 0 & 1 \end{pmatrix} \tag{6.54}$$

then the plane passes through point $\mathbf{p}_{PW}$ and its normal is given by the $z$ axis (third column) of $\mathbf{R}_{PW}$. When rendered in the viewer, FEM model stress/strain values are displayed as a color map within the polygonal region formed by the intersection between the plane and the model's surface mesh.

The following properties of `FemCutPlane` are used to control how it is rendered in the viewer:

**squareSize**

A double value, which if $> 0$ specifies the size of a square that shows the position of the plane.

**resolution**

A double value, which if $> 0$ specifies an explicit size (in units of distance) for the grid cells created within the FEM surface/plane intersection to interpolate stress/strain values. Smaller values will give more accurate results but may slow down the rendering time. Accuracy will also not be improved if the resolution is significantly less that the size of the FEM elements. If resolution $\leq 0$, the grid size is determined automatically based on the element sizes.

**axisLength, axisDrawStyle**

Identical in function to the axisLength and axisDrawStyle properties for rigid bodies (Section 3.2.8). Specifies the length and style for rendering the local coordinate frame of the cut plane.

**surfaceRendering**

Describes what is rendered on the surface/plane intersection polygon, according to table 6.5.

**stressPlotRange, stressPlotRanging, colorMap**

Identical in function to the stressPlotRange, stressPlotRanging and colorMap properties exported by `FemModel3d` and `FemMeshComp` (Section 6.12.3). Controls the range and color map associated with the surface rendering.

As with all properties, the values of the above can be accessed either in code using set/get accessors named after the property (e.g., `setSurfaceRendering()`, `getSurfaceRendering()`), or within the GUI by selecting the cut plane and then choosing Edit properties ... from the context menu.

### 6.12.8 Example: FEM model with a cut plane

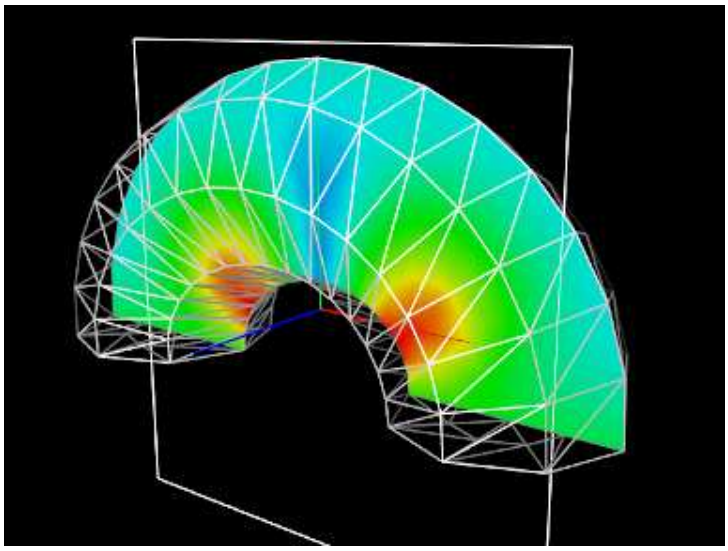


Figure 6.24: FemCutPlaneDemo, showing stress values within the plane after the model has run and settled into an equilibrium state.

Cut planes are illustrated by the application model defined in

```
artisynth.demos.tutorial.FemCutPlaneDemo
```

which creates a simple FEM model in the shape of a half-torus, and then adds a cut plane to render its internal stresses. Its `build()` method is give below:

```
1     public void build (String[] args) {
2        // create a MechModel to contain the FEM model
3        MechModel mech = new MechModel ("mech");
4        addModel (mech);
5
6        // create a half-torus shaped FEM to illustrate the cut plane
7        FemModel3d fem = FemFactory.createPartialHexTorus (
8          null, 0.1, 0.0, 0.05, 8, 16, 3, Math.PI);
9        fem.setMaterial (new LinearMaterial (20000, 0.49));
10       fem.setName ("fem");
11       mech.addModel (fem);
12       // fix the bottom nodes, which lie on the z=0 plane, to support it
13       for (FemNode3d n : fem.getNodes()) {
14          Point3d pos = n.getPosition();
15          if (Math.abs(pos.z) < 1e-8) {
16             n.setDynamic (false);
17          }
18       }
```

```
19
20       // create a cut plane, with stress rendering enabled and a pose that
21       // situates it in the z-x plane
22       FemCutPlane cplane =
23          new FemCutPlane (new RigidTransform3d (0,0,0.03,  0,0,Math.PI/2));
24       fem.addCutPlane (cplane);
25
26       // set stress rendering with a fixed range of (0, 1500)
27       cplane.setSurfaceRendering (SurfaceRender.Stress);
28       cplane.setStressPlotRange (new DoubleInterval (0, 1500.0));
29       cplane.setStressPlotRanging (Ranging.Fixed);
30
31       // create a panel to control cut plane properties
32       ControlPanel panel = new ControlPanel();
33       panel.addWidget (cplane, "squareSize");
34       panel.addWidget (cplane, "axisLength");
35       panel.addWidget (cplane, "stressPlotRanging");
36       panel.addWidget (cplane, "stressPlotRange");
37       panel.addWidget (cplane, "colorMap");
38       addControlPanel (panel);
39
40       // set other render properites ...
41       // make FEM line color white:
42       RenderProps.setLineColor (fem, Color.WHITE);
43       // make FEM elements invisible so they're not in the way:
44       RenderProps.setVisible (fem.getElements(), false);
45       // render FEM using a wireframe surface mesh so we can see through it:
46       fem.setSurfaceRendering (SurfaceRender.Shaded);
47       RenderProps.setDrawEdges (fem.getSurfaceMeshComp(), true);
48       RenderProps.setFaceStyle (fem.getSurfaceMeshComp(), FaceStyle.NONE);
49       RenderProps.setEdgeWidth (fem.getSurfaceMeshComp(), 2);
50       // render cut plane using both square outline and its axes:
51       cplane.setSquareSize (0.12); // size of the square
52       cplane.setAxisLength (0.08); // length of the axes
53       RenderProps.setLineWidth (cplane, 2); // boost line width for visibility
54    }
```

After a `MechModel` is created (lines 27-29), an FEM model consisting of a half-torus is created, with the bottom nodes set non-dynamic to provide support (lines 31-43). A cut plane is then created with a pose that centers it on the world origin and aligns it with the world *z*-*x* plane (lines 47-49). Surface rendering is set to `Stress`, with a fixed color plot range of $(0, 1500)$ (lines 52-54). A control panel is created to expose various properties of the plane (lines 57-63), and then other rendering properties are set: the default FEM line color is made white (line 67); to avoid visual clutter FEM elements are made invisible and instead the FEM is rendered using a wireframe representation of its surface mesh (lines 69-74); and in addition to its render surface, the cut plane is also displayed using its coordinate axes (with length 0.08) and a square of size 0.12 (lines 76-78).

To run this example in ArtiSynth, select All demos > tutorial > FemCutPlaneDemo from the Models menu. When loaded and run, the model should appear as in Figure 6.24. The plane can be repositioned by selecting it in the viewer (by clicking on its square, coordinate axes, or render surface) and then using one of the transformer tools to change its position and/or orientation (see the section "Transformer Tools" in the ArtiSynth User Interface Guide).

# Chapter 7

# Fields

In modeling applications, particularly those employing FEM methods, situations often arise where it is necessary to describe numeric quantities that vary over some spatial domain. For example, the stiffness parameters of an FEM material may vary at different points within the volumetric mesh. When modeling muscles, the activation direction vector will also typically vary over the mesh.

ArtiSynth provides *field components* which can be used to represent such spatially varying quantities, together with mechanisms to attach these to certain properties within various materials. Field components can implement either *scalar* or *vector* fields. Scalar field components implement the interface ScalarFieldComponent and supply the method

```
double getValue (Point3d p)
```

while vector fields implement VectorFieldComponent and supply the method

```
T getValue (Point3d p)
```

where `T` parameterizes a class implementing maspack.matrix.VectorObject. In both cases, the idea is to provide values at arbitrary points over some spatial domain.

For vector fields, the `VectorObject` represented by `T` can generally be any *fixed-size* vector or matrix located in the package `maspack.matrix`, such as Vector2d, Vector3d, or Matrix3d. `T` can *also* be one of the variable-sized objects VectorNd or MatrixNd, although this requires using special wrapper classes and the sizing must remain constant within the field (Section 7.4).

> Both `ScalarFieldComponent` and `VectorFieldComponent` are subclassed from `FieldComponent`. The reason for separating scalar and vector fields is simply efficiency: having scalar fields work with the primitive type `double`, instead of the object `Double`, requires considerably less storage and somewhat less computational effort.

A field is typically implemented by specifying a finite set of values at discrete locations on an underlying spatial grid and then using an interpolation method to determine values at arbitrary points. The field components themselves are defined within the package `artisynth.core.fields`.

At present, three types of fields are implemented:

**Grid fields**

The most basic type of field, in which the values are specified at the vertices of a regular Cartesian grid and then interpolated between vertices. As discussed further in Section 7.1, there are two main types grid field component: ScalarGridField and VectorGridField<T>.

**FEM fields**

Fields for which values are specified at the features of an FEM mesh (e.g., nodes, elements or element integration points). As discussed further in Section 7.2, there are six primary types of FEM field component: ScalarNodalField and VectorNodalField<T> (nodes), ScalarElementField and VectorElementField<T> (elements), and ScalarSubElemField and VectorSubElemField<T> (integration points).

**Mesh fields**

Fields in which values are specified for either the vertices or faces of a triangular mesh. As discussed further in Section 7.3, there are four primary types of mesh field component: ScalarVertexField and VectorVertexField<T> (vertices), and ScalarFaceField and VectorFaceField<T> (faces).

Within an application model, field deployment typically involves the following steps:

1. Create the field component and add it to the model. Grid and mesh fields are usually added to the *fields* component list of a MechModel, while FEM fields are added to the *fields* list of the FemModel3d for which the field is defined.
2. Specify field values for the appropriate features (e.g., grid vertices, FEM nodes, mesh faces).
3. If necessary, bind any needed material properties to the field, as discussed further in Section 7.5.

As a simple example of the first two steps, the following code fragment constructs a scalar nodal field associated with an FEM model named `fem`:

```
FemModel3d fem;
// ... build the fem ...
ScalarNodalField field = new ScalarNodalField ("stiffness", fem, 0);
for (FemNode3d n : fem.getNodes()) {
   double value = ... // compute value for the node
   field.setValue (n, value);
}
fem.addField (field); // add the field to the fem
```

More details on specific field components are given in the sections below.

## 7.1 Grid fields

A grid field specifies scalar or vector values at the vertices of a regular Cartesian grid and interpolates values between vertices. ArtiSynth currently provides two grid field components in `artisynth.core.fields`:

```
ScalarGridField

VectorGridField<T>
```

where `T` is any class implementing maspack.matrix.VectorObject. For both grid types, the value returned by `getValue(p)` is determined by finding the grid cell containing **p** and then using trilinear interpolation of the surrounding nodal values. If **p** lies *outside* the grid volume, then `getValue(p)` either returns the value at the nearest grid point $\mathbf{p}'$ (if the component property clipToGrid is set to `true`), or else returns a special value indicating that **p** is outside the grid. This special value is `ScalarGridField.OUTSIDE_GRID` for scalar fields or `null` for vector fields.

Grid field components are implemented as wrappers around the more basic objects ScalarGrid and VectorGrid<T> defined in `maspack.geometry`. Applications first create one of these primary grid objects and then use it to create the field component. Instances of `ScalarGrid` and `VectorGrid<T>` can be created with constructors such as

```
ScalarGrid (Vector3d widths, Vector3i res)
ScalarGrid (Vector3d widths, Vector3i res, RigidTransform3d TCL)

VectorGrid (Class<T> type, Vector3d widths, Vector3i res)
VectorGrid (Class<T> type, Vector3d widths, Vector3i res, RigidTransform3d TCL)
```

where `widths` gives the grid widths along each of the $x$, $y$ and $z$ axes, `res` gives the number of cells along each axis, and for vector grids `type` is the class type of the maspack.matrix.VectorObject object parameterized by `T`. `TCL` is an optional argument which, if not `null`, describes the position and orientation of the grid center with respect to local coordinates; otherwise, in local coordinates the grid is centered at the origin and aligned with the $x$, $y$ and $z$ axes.

For the vector grid constructors above, `T` should be a fixed-size vector or matrix, such as `Vector3d` or `Matrix3d`. The variable sized objects `VectorNd` and `MatrixNd` can also be used with the aid of special wrapper classes, as described in Section 7.4.

By default, grids are axis-aligned and centered at the origin of the world coordinate system. A transform `TLW` can be specified to place the grid at a different position and/or orientation. `TLW` represents the transform from local to world coordinates and can be controlled with the methods

```
void setLocalToWorld (RigidTransform3d TLW)
RigidTransform3d getLocalToWorld()
```

If not specified, `TLW` is the identity and local and world coordinates are the same.

Once a grid is created, it can be used to instantiate a grid field component using one of the constructors

```
ScalarGridField (ScalarGrid grid)
ScalarGridField (String name, ScalarGrid grid)

VectorGridField (VectorGrid grid)
VectorGridField (String name, VectorGrid grid)
```

where `grid` is the primary grid and `name` is an optional component name. Note that the primary grid is not copied, so any subsequent changes to it will be reflected in the enclosing field component.

Once the grid field component is created, its values can be set by specifying the values at its vertices. The methods to query and set vertex values for scalar and vector fields are

```
int numVertices()

// scalar fields:
double getVertexValue (int xi, int yj, int zk)
double getVertexValue (int vi)
void setVertexValue (int xi, int yj, int zk, double value)
void setVertexValue (int vi, double value)

// vector fields:
T getVertexValue (int xi, int yj, int zk)
T getVertexValue (int vi)
void setVertexValue (int xi, int yj, int zk, T value)
void setVertexValue (int vi, T value)
```

where `xi`, `yj`, and `zk` are the vertex's indices along the x, y, and z axes, and `vi` is a general index that should be in the range `0` to `field.numVertices()-1` and is related to `xi`, `yj`, and `zk` by

```
vi = xi + nx*yj + (nx*ny)*zk,
```

where `nx` and `ny` are the number of vertices along the $x$ and $y$ axes.

When computing a grid value using `getValue(p)`, the point `p` is assumed to be in either grid local or world coordinates, depending on whether the field component's property localValuesForField is `true` or `false` (local and world coordinates are the same unless the primary grid's local-to-world transform `TLW` has been set as described above).

To find the spatial position of a vertex within a grid field component, one may use the methods

```
Vector3d getVertexPosition (xi, yj, zk)
Vector3d getVertexPosition (vi)
```

which return the vertex position in either local or world coordinates depending on the setting of localValuesForField.

The following code example shows the creation of a `ScalarGridField`, with widths $5 \times 5 \times 10$ and a cell resolution of $10 \times 10 \times 20$, centered at the origin and whose vertex values are set to their distance from the origin, in order to create a simple distance field:

```
// create a 5 x 5 x 10 grid with resolution 10 x 10 x 20 centered on the
// origin
ScalarGrid grid = new ScalarGrid (
   new Vector3d (5, 5, 10), new Vector3i (10, 10, 20));

// use this to create a scalar grid field where the value at each vertex
// is the distance from the origin
ScalarGridField field = new ScalarGridField (grid);

// iterate through all the vertices and set the value for each to its
// distance from the origin, as given by the norm of it's position
for (int vi=0; vi<grid.numVertices(); vi++) {
   Vector3d vpos = grid.getVertexCoords (vi);
   grid.setVertexValue (vi, vpos.norm());
}
// add to a mech model:
mech.addField (field);
```

As shown in the example and as mentioned earlier, grid field are generally added to the `fields` list of a `MechModel`.

## 7.2 FEM fields

A FEM field specifies scalar or vector values for the features of an FEM mesh. These can be either the nodes (nodal fields), elements (element fields), or element integration points (sub-element fields). As such, FEM fields provide a way to augment the mesh to store application-specific quantities, and are typically used to bind properties of FEM materials that need to vary over the domain (Section 7.5).

To evaluate `getValue(p)` at an arbitrary point **p**, the field finds the FEM element containing **p** (or nearest to **p**, if **p** is outside the FEM mesh), and either interpolates the value from the surrounding nodes (nodal fields), uses the element value directly (element fields), or interpolates from the integration point values (sub-element fields).

> When finding a FEM field value at a point **p**, `getValue(p)` is evaluated with respect to the FEM model's current *spatial* position, as opposed to its *rest* position.

FEM fields maintain a *default value* which describes the value at features for which values are not explicitly set. If unspecified, the default value itself is zero.

In the remainder of this section, it is assumed that vector fields are constructed using fixed-size vectors or matrices, such as `Vector3d` or `Matrix3d`. However, it is also possible to construct fields using `VectorNd` or `MatrixNd`, with the aid of special wrapper classes, as described in Section 7.4. That section also details a convenience wrapper class for `Vector3d`.

The three field types are now described in detail.

### 7.2.1 Nodal fields

Implemented by ScalarNodalField, VectorNodalField<T>, or subclasses of the latter, nodal fields specify their values at the nodes of an FEM mesh. They can be created with constructors such as

```
// scalar fields:
ScalarNodalField (FemModel3d fem)
ScalarNodalField (FemModel3d fem, double defaultValue)
ScalarNodalField (String name, FemModel3d fem, double defaultValue)

// vector fields (with vector type parameterized by T):
VectorNodalField (Class<T> type, FemModel3d fem)
VectorNodalField (Class<T> type, (FemModel3d fem, T defaultValue)
VectorNodalField (String name, Class<T> type, FemModel3d fem, T defaultValue)
```

where `fem` is the associated FEM model, `defaultValue` is the default value, and `name` is a component name. For vector fields, the maspack.matrix.VectorObject type is parameterized by `T`, and `type` gives its actual class type (e.g., `Vector3d.class`).

Once the grid has been created, values can be queried and set at the nodes using methods such as

```
// scalar fields:
void setValue (FemNode3d node, double value) // set value for node
double getValue (FemNode3d node)             // get value for node
double getValue (int nodeNum)                // get value using node number

// vector fields:
void setValue (FemNode3d node, T value)      // set value for node
T getValue (FemNode3d node)                  // get value for node
T getValue (int nodeNum)                     // get value using node number

// all fields:
boolean isValueSet (FemNode3d node)          // query if value set for node
void clearValue (FemNode3d node)             // unset value for node
void clearAllValues()                        // unset values for all nodes
```

Here `nodeNum` refers to an FEM node's *number* (which can be obtained using `node.getNumber()`). Numbers are used instead of indices to identity FEM nodes and elements because they are guaranteed to be *persistent* in case of mesh editing, and will remain unchanged as long as the node or element is not removed from the FEM model. Note that values don't need to be set at all nodes, and set values can be unset using the `clear` methods. For nodes with no value set, the `getValue()` methods will return the default value.

As a simple example, the following code fragment constructs a `ScalarNodalField` for an FEM model `fem` that describes stiffness values at every node of an FEM:

```
// instantiate the field and set values for each node:
ScalarNodalField field = new ScalarNodalField ("stiffness", fem);
for (FemNode3d n : fem.getNodes()) {
   double stiffness = ... // compute stiffness value for the node
   field.setValue (n, stiffness);
}
fem.addField (field); // add the field to the fem
```

As noted earlier, FEM fields should be stored in the `fields` list of the associated FEM model.

Another example shows the creation of a field of 3D direction vectors:

```
VectorNodalField<Vector3d> field =
  new VectorNodalField<Vector3d> ("directions", Vector3d.class, fem);
Vector3d dir = new Vector3d();
for (FemNode3d node : fem.getNodes()) {
   ... // compute direction and store in dir
   field.setValue (node, dir);
}
```

When creating a vector field, the constructor needs the class type of the field's `VectorObject`. However, for the special case of `Vector3d`, one may also use the convenience wrapper class `Vector3dNodalField`, described in Section 7.4.

### 7.2.2 Element fields

Implemented by ScalarElementField, VectorElementField<T>, or subclasses of the latter, element fields specify their values at the elements of an FEM mesh, and these values are assumed to be *constant* within the element. To evaluate `getValue(p)`, the field finds the containing (or nearest) element for **p**, and then returns the value for that element.

> Because values are assume to be constant within each element, element fields are inherently discontinuous across element boundaries.

The constructors for element fields are analogous to those for nodal fields:

```
// scalar fields:
ScalarElementField (FemModel3d fem)
ScalarElementField (FemModel3d fem, double defaultValue)
ScalarElementField (String name, FemModel3d fem, double defaultValue)

// vector fields (with vector type parameterized by T):
VectorElementField (Class<T> type, FemModel3d fem)
VectorElementField (Class<T> type, (FemModel3d fem, T defaultValue)
VectorElementField (String name, Class<T> type, FemModel3d fem, T defaultValue)
```

and the methods for setting values at elements are similar as well:

```
// scalar fields:
void setValue (FemElement3dBase elem, double value) // set value for element
double getValue (FemElement3dBase elem)      // get value for element
double getElementValue (int elemNum)         // get value for volume element
double getShellElementValue (int elemNum)    // get value for shell element

// vector fields:
void setValue (FemElement3dBase elem, T value) // set value for element
T getValue (FemElement3dBase elem)           // get value for element
T getElementValue (int elemNum)              // get value for volume element
T getShellElementValue (int elemNum)         // get value for shell element

// all fields:
boolean isValueSet (FemElement3dBase elem)   // query if value set for element
void clearValue (FemElement3dBase elem)      // unset value for element
void clearAllValues()                        // unset values for all elements
```

The elements associated with an element field can be either volume *or* shell elements, which are stored in the FEM component lists `elements` and `shellElements`, respectively. Since volume and shell elements may share element numbers, the separate methods `getElementValue()` and `getShellElementValue()` are used to access values by element number.

The following code fragment constructs a `VectorElementField` based on `Vector2d` that stores a 2D coordinate value at all regular and shell elements:

```
VectorElementField<Vector2d> efield =
  new VectorElementField<Vector2d> ("coordinates", Vector2d.class, fem);
Vector2d coords = new Vector2d();
for (FemElement3d e : fem.getElements()) {
   ... // compute coordinate values and store in coords
   efield.setValue (e, coords);
}
for (ShellElement3d e : fem.getShellElements()) {
   ... // compute coordinate values and store in coords
   efield.setValue (e, coords);
}
```

### 7.2.3 Sub-element fields

Implemented by ScalarSubElemField, VectorSubElemField<T>, or subclasses of the latter, sub-element fields specify their values at the integration points within each element of an FEM mesh. These fields are used when we need precisely computed information at each of the element's integration points, and we can't assume that nodal interpolation will give an accurate enough approximation.

To evaluate `getValue(p)`, the field finds the containing (or nearest) element for **p**, extrapolating the integration point values back to the nodes, and then using nodal interpolation.

Constructors are similar to those for element fields:

```
// scalar fields:
ScalarSubElemField (FemModel3d fem)
```

```
ScalarSubElemField (FemModel3d fem, double defaultValue)
ScalarSubElemField (String name, FemModel3d fem, double defaultValue)

// vector fields:
VectorSubElemField (Class<T> type, FemModel3d fem)
VectorSubElemField (Class<T> type, (FemModel3d fem, T defaultValue)
VectorSubElemField (String name, Class<T> type, FemModel3d fem, T defaultValue)
```

Value accessors are also similar, except that an additional argument `k` is required to specify the index of the integration point:

```
// scalar fields:
void setValue (FemElement3dBase e, int k, double value) // set value for point
double getValue (int elemNum, int k)                    // get value for point
double getElementValue (FemElement3dBase e, int k) // get value for volume point
double getShellElementValue (int elemNum, int k)   // get value for shell point

// vector fields:
void setValue (FemElement3dBase e, T value, int k) // set value for point
T getValue (FemElement3dBase e, int k)             // get value for point
T getElementValue (int elemNum, int k)             // get value for volume point
T getShellElementValue (int elemNum, int k)        // get value for shell point

// all fields:
boolean isValueSet (FemElement3dBase e, int k)     // query if value set for point
void clearValue (FemElement3dBase e, int k)        // unset value for point
void clearAllValues()                              // unset values for all points
```

The integration point index `k` should be in the range 0 to $n-1$, where $n$ is the total number of integration points for the element in question and can be queried by the method numAllIntegrationPoints().

> The total number of integration points $n$ includes both the regular integration point as well as the *warping* point, which is located at the element center, is indexed by $n-1$, and is used for corotated linear materials. If a `SubElemField` is being using to supply mesh-varying values to one of the linear material parameters (Section 7.5), then it is important to supply values at the warping point.

The example below shows the construction of a `ScalarSubElemField`:

```
ScalarSubElemField field = new ScalarSubElemField ("modulus", fem);
for (FemElement3d e : fem.getElements()) {
   IntegrationPoint3d[] ipnts = e.getAllIntegrationPoints();
   for (int k=0; k<ipnts.length; k++) {
      double value = ... // compute value at integration point k
      field.setValue (e, k, value);
   }
}
fem.addField (field); // add the field to the fem
```

First, the field is instantiated with the name `"modulus"`. The code then iterates first through the FEM elements, and then through each element's integration points, computing values appropriate to each one. A list of each element's integration points is returned by `e.getAllIntegrationPoints()`. Having access to the actual integration points is useful in case information about them is needed to compute the field values. In particular, if it is necessary to obtain an integration point's rest or current (spatial) position, these can be queried as follows:

```
IntegrationPoint3d ipnt;
Point3d pos = new Point3d();
FemElement3d elem;

...

// compute spatial position:
ipnt.computePosition (pos, elem.getNodes());
```

```
        // compute rest position:
        ipnt.computeRestPosition (pos, elem.getNodes());
```

> The regular integration points, *excluding* the warping point, can be queried using `getIntegrationPoints()` and `numIntegrationPoints()` instead of `getAllIntegrationPoints()` and `numAllIntegrationPoints()`.

## 7.3 Mesh fields

A mesh field specifies scalar or vector values for the features of an FEM mesh. These can be either the vertices (vertex fields) or faces (face fields). Mesh fields have been introduced in particular to allow properties of contact force behaviors (Section 8.7.2), such as LinearElasticContact, to vary over the contact mesh. Mesh fields can currently be defined only for triangular polyhedral meshes.

To evaluate `getValue(p)` at an arbitrary point **p**, the field finds the mesh face nearest to **p**, and then either interpolates the value from the surrounding vertices (vertex fields) or uses the face value directly (face fields).

Mesh fields make use of the classes PolygonalMesh, Vertex3d, and Face, defined in the package `maspack.geometry`. They maintain a *default value* which describes the value at features for which values are not explicitly set. If unspecified, the default value itself is zero.

In the remainder of this section, it is assumed that vector fields are constructed using fixed-size vectors or matrices, such as `Vector3d` or `Matrix3d`. However, it is also possible to construct fields using `VectorNd` or `MatrixNd`, with the aid of special wrapper classes, as described in Section 7.4. That section also details a convenience wrapper class for `Vector3d`.

The two field types are now described in detail.

### 7.3.1 Vertex fields

Implemented by ScalarVertexField, VectorVertexField<T>, or subclasses of the latter, vertex fields specify their values at the vertices of a mesh. They can be created with constructors such as

```
    // scalar fields:
    ScalarVertexField (MeshComponent mcomp)
    ScalarVertexField (MeshComponent mcomp, double defaultValue)
    ScalarVertexField (String name, MeshComponent mcomp, double defaultValue)

    // vector fields (with vector type parameterized by T):
    VectorVertexField (Class<T> type, MeshComponent mcomp)
    VectorVertexField (Class<T> type, (MeshComponent mcomp, T defaultValue)
    VectorVertexField (String name, Class<T> type, MeshComponent mcomp, T defaultValue ↩
      )
```

where `mcomp` is a mesh component containing the mesh, `defaultValue` is the default value, and `name` is a component name. For vector fields, the maspack.matrix.VectorObject type is parameterized by `T`, and `type` gives its actual class type (e.g., `Vector3d.class`).

Once the field has been created, values can be queried and set at the vertices using methods such as

| Scalar fields: | |
|---|---|
| void setValue(Vertex3d vtx, double value) | Set value for vertex `vtx`. |
| double getValue(Vertex3d vtx) | Get value for vertex `vtx`. |
| double getValue(int vidx) | Get value for vertex at index `vidx`. |
| Vector fields with parameterized type T: | |
| void setValue(Vertex3d vtx, T value) | Set value for vertex `vtx`. |
| T getValue(Vertex3d vtx) | Get value for vertex `vtx`. |
| T getValue(int vidx) | Get value for vertex at index `vidx`. |
| All fields: | |
| boolean isValueSet(Vertex3d vtx) | Query if value set for vertex `vtx`. |
| void clearValue(Vertex3d vtx) | Unset value for vertex `vtx`. |
| void clearAllValues() | Unset values for all vertices. |

Note that values don't need to be set at all vertices, and set values can be unset using the `clear` methods. For vertices with no value set, `getValue()` will return the default value.

As a simple example, the following code fragment constructs a `ScalarVertexField` for a mesh contained in the MeshComponent `mcomp` that describes contact stiffness values at every vertex:

```
MechModel mech; // containing mech model

...

// instantiate the field and set values for each node:
ScalarVertexField field = new ScalarVertexField ("stiffness", mcomp);
// extract the mesh from the component
PolygonalMesh mesh = (PolygonalMesh)mcomp.getMesh();
for (Vertex3d vtx : mesh.getVertices()) {
   double stiffness = ... // compute stiffness value for the vertex
   field.setValue (vtx, stiffness);
}
mech.addField (field); // add the field to the mech model
```

As noted earlier, mesh fields are usually stored in the `fields` list of a MechModel.

Another example shows the creation of a field of 3D direction vectors:

```
VectorVertexField<Vector3d> field =
  new VectorVertexField<Vector3d> ("directions", Vector3d.class, mcomp);
Vector3d dir = new Vector3d();
for (Vertex3d vtx : mesh.getVertices()) {
   ... // compute direction and store in dir
   field.setValue (vtx, dir);
}
```

When creating a vector field, the constructor needs the class type of the field's `VectorObject`. However, for the special case of `Vector3d`, one may also use the convenience wrapper class `Vector3dVertexField`, described in Section 7.4.

### 7.3.2 Face fields

Implemented by ScalarFaceField, VectorFaceField<T>, or subclasses of the latter, face fields specify their values at the faces of a polygonal mesh, and these values are assumed to be *constant* within the face. To evaluate `getValue(p)`, the field finds the containing (or nearest) face for **p**, and then returns the value for that face.

> Because values are assume to be constant within each face, face fields are inherently discontinuous across face boundaries.

The constructors for face fields are analogous to those for vertex fields:

```
// scalar fields:
ScalarFaceField (MeshComponent mcomp)
ScalarFaceField (MeshComponent mcomp, double defaultValue)
ScalarFaceField (String name, MeshComponent mcomp, double defaultValue)

// vector fields (with vector type parameterized by T):
VectorFaceField (Class<T> type, MeshComponent mcomp)
VectorFaceField (Class<T> type, (MeshComponent mcomp, T defaultValue)
VectorFaceField (String name, Class<T> type, MeshComponent mcomp, T defaultValue)
```

and the methods for setting values at faces are similar as well, where `fidx` refers to the face index:

```
// scalar fields:
void setValue (Face face, double value)      // set value for face
double getValue (Face face)                  // get value for face
double getValue (int fidx)                   // get value using face index

// vector fields:
void setValue (Face face, T value)           // set value for face
T getValue (Face face)                       // get value for face
T getValue (int fidx)                        // get value using face index

// all fields:
boolean isValueSet (Face face)               // query if value set for face
void clearValue (Face face)                  // unset value for face
void clearAllValues()                        // unset values for all faces
```

The following code fragment constructs a `VectorFaceField` based on `Vector2d` that stores a 2D coordinate value at all faces of a mesh:

```
VectorFaceField<Vector2d> field =
  new VectorFaceField<Vector2d> ("coordinates", Vector2d.class, mcomp);
Vector2d coords = new Vector2d();
// extract the mesh from the component
PolygonalMesh mesh = (PolygonalMesh)mcomp.getMesh();
for (Face face : mesh.getFaces()) {
   ... // compute coordinate values and store in coords
   field.setValue (face, coords);
}
```

## 7.4 Fields for VectorNd, MatrixNd and Vector3d

The vector fields described above can be implemented for most fixed-size vectors and matrices defined in the package `maspack.matrix` (e.g., Vector2d, Vector3d, Matrix3d). However, if vectors or matrices with different size are needed, it is also possible to create vector fields using VectorNd and MatrixNd, provided that the sizing remain constant within any given field. This is achieved using special wrapper classes that contain the sizing information, which is supplied in the constructors. For convenience, wrapper classes are also provided for vector fields that use `Vector3d`, since that vector size is quite common.

For vector FEM and mesh fields, the complete set of wrapper classes is listed below:

| Class | base class | vector type T |
|---|---|---|
| Vector FEM fields: | | |
| Vector3dNodalField | VectorNodalField<T> | Vector3d |
| VectorNdNodalField | VectorNodalField<T> | VectorNd |
| MatrixNdNodalField | VectorNodalField<T> | MatrixNd |
| Vector3dElementField | VectorElementField<T> | Vector3d |
| VectorNdElementField | VectorElementField<T> | VectorNd |
| MatrixNdElementField | VectorElementField<T> | MatrixNd |
| Vector3dSubElemField | VectorSubElemField<T> | Vector3d |
| VectorNdSubElemField | VectorSubElemField<T> | VectorNd |
| MatrixNdSubElemField | VectorSubElemField<T> | MatrixNd |
| Vector mesh fields: | | |
| Vector3dVertexField | VectorVertexField<T> | Vector3d |
| VectorNdVertexField | VectorVertexField<T> | VectorNd |
| MatrixNdVertexField | VectorVertexField<T> | MatrixNd |
| Vector3dFaceField | VectorFaceField<T> | Vector3d |
| VectorNdFaceField | VectorFaceField<T> | VectorNd |
| MatrixNdFaceField | VectorFaceField<T> | MatrixNd |

These all behave identically to their base classes, except that their constructors omit the `type` argument (since this is built into the class definition) and, for `VectorNd` and `MatrixNd`, supply information about the vector or matrix sizes. For example, constructors for the FEM nodal fields include

```
Vector3dNodalField (FemModel3d fem)
Vector3dNodalField (FemModel3d fem, Vector3d defaultValue)
Vector3dNodalField (String name, FemModel3d fem, Vector3d defaultValue)

VectorNdNodalField (int vecSize, FemModel3d fem)
VectorNdNodalField (int vecSize, FemModel3d fem, VectorNd defaultValue)
VectorNdNodalField (
  int vecSize, String name, FemModel3d fem, VectorNd defaultValue)

MatrixNdNodalField (int rowSize, int colSize, FemModel3d fem)
MatrixNdNodalField (
  int rowSize, int colSize, FemModel3d fem, MatrixNd defaultValue)
MatrixNdNodalField (
  int rowSize, int colSize, String name, FemModel3d fem, MatrixNd defaultValue)
```

where `vecSize`, `rowSize` and `colSize` give the sizes of the `VectorNd` or `MatrixNd` objects within the field. Constructors for other field types are equivalent and are described in their API documentation.

For grid fields, one can use either VectorNdGrid or MatrixNdGrid, both defined in `maspack.geometry`, as the primary grid used to construct the field. Constructors for these include

```
VectorNdGrid (int VecSize, Vector3d widths, Vector3i res)
MatrixNdGrid (
  int rowSize int colSize, Vector3d widths, Vector3i res, RigidTransform3d TCL)
```

## 7.5 Binding material properties

A principal purpose of field components is to enable certain FEM material properties to vary spatially over the FEM geometry. Many material properties can be *bound* to a field, so that when they are queried internally by the solver at the integration points for each FEM element, the field value at that integration point is used instead of the regular property value. Likewise, some properties of contact force behaviors, such as the YoungsModulus and thickness properties of LinearElasticContact (Section 8.7.3) can be bound to a field that varies over the contact mesh geometry.

To bind a property to a field, it is necessary that

1. The type of the field matches the value of the property;
2. The property is itself *bindable*.

If the property has a `double` value, then it can be bound to any `ScalarFieldComponent`. Otherwise, if the property has a value `T`, where `T` is an instance of `maspack.matrix.VectorObject`, then it can be bound to a vector field.

Bindable properties export two methods with the signature

```
setXXXField (F field)

F getXXXField ()
```

where `XXX` is the property name and `F` is an appropriate field type.

> As long as a property XXX is bound to a field, its regular value will still appear (and can be set) through widgets in control or property panels, or via its `getXXX()` and `setXXX()` accessors. However, the regular value won't be used internally by the FEM simulation.

For example, consider the YoungsModulus property, which is present in several FEM materials, including LinearMaterial and NeoHookeanMaterial. This has a `double` value, and is bindable, and so can be bound to a `ScalarFieldComponent` as follows:

```
FemModel3d fem;
ScalarNodalField field;
NeoHookeanMaterial mat;

... other initialization ...

mat.setYoungsModulusField (field);  // bind to the field
fem.setMaterial (mat);   // set the material in the FEM model
```

> It is important to perform field bindings on materials **before** they are set in an FEM model (or one of its subcomponents, such as `MuscleBundles`). That's because the `setMaterial()` method *copies* the input material, and so any settings made on it afterwards won't be seen by the FEM:
>
> ```
> // works: field will be seen by the copied version of 'mat'
> mat.setYoungsModulusField (field);
> fem.setMaterial (mat);
>
> // does NOT work: field not seen by the copied version of 'mat'
> fem.setMaterial (mat);
> mat.setYoungsModulusField (field);
> ```

To unbind YoungsModulus, one can call

```
mat.setYoungsModulusField (null);
```

Usually, FEM fields are used to bind properties in FEM materials while mesh fields are used for contact properties, but other fields *can* be used, particularly grid fields. To understand this a bit better, we discuss briefly some of the internals of how binding works. When evaluating stresses, FEM materials have access to a FemFieldPoint object that provides information about the integration point, including its element, nodal weights, and integration point index. This can be used to rapidly evaluate values within nodal, element and sub-element FEM fields. Likewise, when evaluating contact forces, contact materials like LinearElasticContact have access to a MeshFieldPoint object that describes the contact point position as a weighted sum of mesh vertex positions, which can then be used to rapidly evaluate evaluate values within vertex or face mesh fields. However, all fields, regardless of type, implement methods for determining values at both `FemFieldPoints` and `MeshFieldPoints`:

```
T getValue (FemFieldPoint fp)

T getValue (MeshFieldPoint mp)
```

If necessary, these methods simply fall back on the `getValue(Point3d)` method, which is defined for all fields and works relatively fast for grid fields.

There are some additional details to consider when binding FEM material properties:

1. When binding to a grid field, one has the choice of whether to use the grid to represent values with respect to the FEM's *rest* position or *spatial* position. This can be controlled by setting the useFemRestPositions property of the grid field, the default value for which is `true`.
2. When binding the bulkModulus property of an incompressible material, it is best to use a nodal field if the FEM model is using nodal-based soft incompressibility (i.e., the model's softIncompMethod property is set to either `NODAL` or `AUTO`; Section 6.7.3). That's because soft nodal incompressibility require evaluating the bulk modulus property at nodes instead of integration points, which is much easier to do with a nodal-based field.

### 7.5.1 Example: FEM with variable stiffness

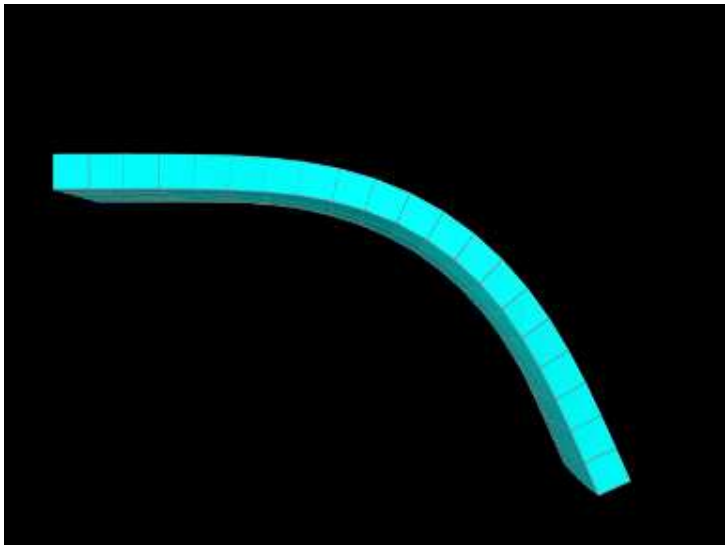


Figure 7.1: VariableStiffness model after being run in ArtiSynth.

A simple model demonstrating a stiffness that varies over an FEM mesh is defined in

```
artisynth.demos.tutorial.VariableStiffness
```

It consists of a simple thin hexahedral beam with a linear material for which the Young's modulus $E$ is made to vary nonlinearly along the x axis of the rest position according to the formula

$$E = \frac{10^8}{1 + 1000\,x^3} \tag{7.1}$$

The model's `build` method is given below:

```
1     public void build (String[] args) {
2        MechModel mech = new MechModel ("mech");
3        addModel (mech);
4
5        // create regular hex grid FEM model
6        FemModel3d fem = FemFactory.createHexGrid (
7           null, 1.0, 0.25, 0.05, 20, 5, 1);
8        fem.transformGeometry (new RigidTransform3d (0.5, 0, 0)); // shift right
9        fem.setDensity (1000.0);
10       mech.addModel (fem);
11
12       // fix the left-most nodes
```

```
13        double EPS = 1e-8;
14        for (FemNode3d n : fem.getNodes()) {
15           if (n.getPosition().x < EPS) {
16              n.setDynamic (false);
17           }
18        }
19        // create a scalar nodel field to make the stiffness vary
20        // nonlinearly along the rest position x axis
21        ScalarNodalField stiffnessField = new ScalarNodalField(fem, 0);
22        for (FemNode3d n : fem.getNodes()) {
23           double s = 10*(n.getRestPosition().x);
24           double E = 100000000*(1/(1+s*s*s));
25           stiffnessField.setValue (n, E);
26        }
27        fem.addField (stiffnessField);
28        // create a linear material, bind its Youngs modulus property to the
29        // field, and set the material in the FEM model
30        LinearMaterial linearMat = new LinearMaterial (100000, 0.49);
31        linearMat.setYoungsModulusField (stiffnessField); //, /*useRestPos=*/true);
32        fem.setMaterial (linearMat);
33
34        // set some render properties for the FEM model
35        fem.setSurfaceRendering (SurfaceRender.Shaded);
36        RenderProps.setFaceColor (fem, Color.CYAN);
37     }
```

Lines 6-10 create the hex FEM model and shift it so that the left side is aligned with the origin, while lines 12-17 fix the leftmost nodes. Lines 21-27 create a scalar nodal field for the Young's modulus, with lines 23-24 computing $E$ according to (7.1). The field is then bound to a linear material which is then set in the model (lines 30-32).

The example can be run in ArtiSynth by selecting All demos > tutorial > VariableStiffness from the Models menu. When run, the beam will bend under gravity, but mostly on the right side, due to the much higher stiffness on the left (Figure 7.1).

### 7.5.2 Example: specifying FEM muscle directions

Another example involves using a Vector3d field to specify the muscle activation directions over an FEM model and is defined in

```
artisynth.demos.tutorial.RadialMuscle
```

When muscles are added using MuscleBundles ( Section 6.9), the muscle directions are stored and handled internally by the muscle bundle itself. However, it is possible to add a MuscleMaterial directly to the elements of an FEM model, using a MaterialBundle (Section 6.8), in which case the directions need to be set explicitly using a field.

The model's `build` method is given below:

```
1     public void build (String[] args) {
2        MechModel mech = new MechModel ("mech");
3        addModel (mech);
4
5        // create a thin cylindrical FEM model with two layers along z
6        double radius = 0.8;
7        FemMuscleModel fem = new FemMuscleModel ("radialMuscle");
8        mech.addModel (fem);
9        fem.setDensity (1000);
10       FemFactory.createCylinder (fem, radius/8, radius, 20, 2, 8);
11       fem.setMaterial (new NeoHookeanMaterial (200000.0, 0.33));
12       // fix the nodes close to the center
13       for (FemNode3d node : fem.getNodes()) {
14          Point3d pos = node.getPosition();
15          double radialDist = Math.sqrt (pos.x*pos.x + pos.y*pos.y);
16          if (radialDist < radius/2) {
```

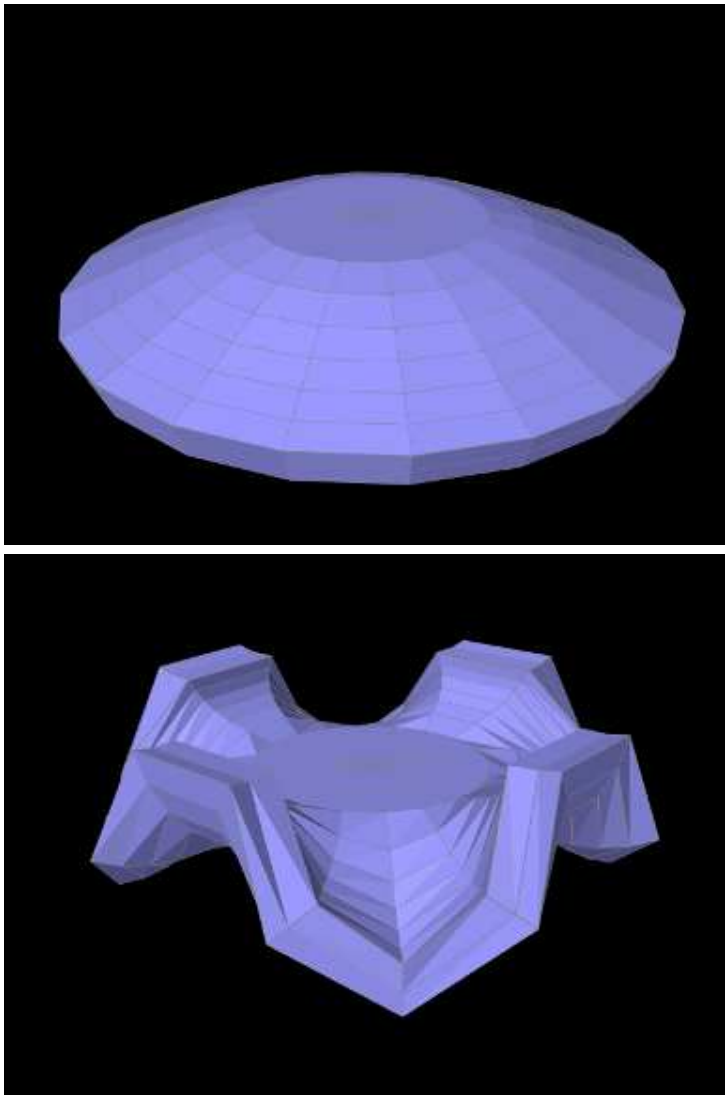


Figure 7.2: (Top) RadialMuscle model after being loaded into ArtiSynth and run with its excitation set to 0. (Bottom) RadialMuscle with its excitation set to around .375.

```
17              node.setDynamic (false);
18           }
19        }
20        // compute a direction field, with the directions arranged radially
21        Vector3d dir = new Vector3d();
22        Vector3dElementField dirField = new Vector3dElementField (fem);
23        for (FemElement3d elem : fem.getElements()) {
24           elem.computeCentroid (dir);
25           // set directions only for the upper layer elements
26           if (dir.z > 0) {
27              dir.z = 0; // remove z component from direction
28              dir.normalize();
29              dirField.setValue (elem, dir);
30           }
31        }
32        fem.addField (dirField);
33        // add a muscle material, and use it to hold a simple force
34        // muscle whose 'restDir' property is attached to the field
35        MaterialBundle bun = new MaterialBundle ("bundle",/*all elements=*/true);
36        fem.addMaterialBundle (bun);
37        SimpleForceMuscle muscleMat = new SimpleForceMuscle (500000);
38        muscleMat.setRestDirField (dirField);
```

```
39        bun.setMaterial (muscleMat);
40
41        // add a control panel to control the excitation
42        ControlPanel panel = new ControlPanel();
43        panel.addWidget (bun, "material.excitation", 0, 1);
44        addControlPanel (panel);
45
46        // set some rendering properties
47        fem.setSurfaceRendering (SurfaceRender.Shaded);
48        RenderProps.setFaceColor (fem, new Color (0.6f, 0.6f, 1f));
49     }
```

Lines 5-19 create a thin cylindrical FEM model, centered on the origin, with radius $r$ and height $r/8$, consisting of hexes with wedges at the center, with two layers of elements along the $z$ axis (which is parallel to the cylinder axis). Its base material is set to a neo-hookean material. To keep the model from falling under gravity, all nodes whose distance to the $z$ axis is less than $r/2$ are fixed.

Next, a Vector3d field is created to specify the directions, on a per-element basis, for the muscle material which will be added subsequently (lines 21-32). While we could create an instance of `VectorElementField<Vector3d>`, we use Vector3dElementField, since this is available and provides additional functionality (such as the ability to render the directions). Directions are set to lie outward in a radial direction perpendicular to the $z$ axis, and since the model is centered on the origin, they can be computed easily by first computing the element centroids, removing the $z$ component, and then normalizing. In order to give the muscle action an upward bias, we only set directions for elements in the upper layer. Direction values for elements in the lower layer will then automatically have a default value of 0, which will cause the muscle material to not apply any stress.

We next add to the model a muscle material whose directions will be determined by the field. To hold the material, we first create and add a `MaterialBundle` which is set to act on all elements (line 35-36). Then we set this bundle's material to `SimpleForceMuscle`, which adds a stress along the muscle direction that equals the excitation value times the value of its maxStress property, and bind the material's restDir property to the direction field (lines 37-39).

Finally, we create and add a control panel to allow interactive control over the muscle material's excitation property (lines 42-44), and set some rendering properties for the FEM model.

The example can be run in ArtiSynth by selecting All demos > tutorial > RadialMuscle from the Models menu. When it is first run, it falls around the edges under gravity (Figure 7.2, top). Applying an excitation causes a radial contraction which pulls the edges upward and, if high enough, causes then to buckle (Figure 7.2, bottom).

## 7.6 Visualizing fields

It is often useful to be able to visualize the contents of a field, particularly for testing and validation purposes. ArtiSynth field components export various properties that allow their values to be visualized in the viewer.

> As described in Section 7.6.3, the grid field components, `ScalarGridField` and `VectorGridField`, are not visible by default, and so must be made visible to enable visualization.

### 7.6.1 Scalar fields

Scalar fields are visualized by means of a color map that associates scalar values with colors, with the actually visualization typically taking the form of either a discrete set of colored points, or a colored surface embedded within the field. The color map and its associated visualization is controlled by the following properties:

**colorMap**

An instance of the ColorMap interface that controls how field values are mapped onto colors. Various types of maps exist, including GreyscaleColorMap, HueColorMap, RainbowColorMap, and JetColorMap, each containing subproperties controlling how its colors are generated.

**renderRange**

Composite property of the type ScalarRange that controls the range of field values used for color map rendering. Subproperties include interval, which gives the value range itself, and updating, which specifies how this interval is determined from the field: `FIXED` (interval can be set manually), `AUTO_EXPAND` (interval is expanded to accommodate all field values), and `AUTO_FIT` (interval is tightly fit to the field values). Note that for `AUTO_EXPAND` and `AUTO_FIT`, the interval is determined by the range of field values, and *not* the values that actually appear in the visualization. The latter can differ from the former when the surface does not cover the whole field, or lies outside the field, resulting in extrapolated values that fall outside the field's range. Setting updating to `FIXED` and manually setting the interval can be useful in such cases.

**visualization**

An enumerated type that specifies the actual type of rendering used to visualize the field (e.g., points, surface, or other). While its definition is specific to the field type (ScalarFemField.Visualization for FEM fields, ScalarMeshField.Visualization for mesh fields; ScalarGridField.Visualization for grid fields), overall it will have one of five values (`POINT`, `SURFACE`, `ELEMENT`, `FACE`, or `OFF`) described further below.

**volumeElemsVisible**

A boolean property, exported by `ScalarElementField` and `ScalarSubElemField`, which if `false` causes volumetric element values to be ignored in the visualization. The default value is `true`.

**shellElemsVisible**

A boolean property, exported by `ScalarElementField` and `ScalarSubElemField`, which if `false` causes shell element values to be ignored in the visualization. The default value is `true`.

**renderProps**

Basic rendering properties of the field component that are used to control some aspects of the rendering (Section 4.3), as described below.

The above properties can be accessed either interactively in the GUI, or in code, using the field's methods `getColorMap()`, `setColorMap(map)`, `getRenderRange()`, `setRenderRange(range)`, `getVisualization()`, and `setVisualization(type)`.

While the enumerated type associated with the visualization property is specific to the field type, all values together will be one of the following:

**POINT**

Field is visualized using colored points placed at the features used to define the field (e.g., nodes, element centers, or integration points for FEM fields; vertices and face centers for mesh fields; vertices for grid fields). How the points are displayed is controlled using the pointStyle subproperty of the grid's renderProps, with their size controlled either by pointRadius or pointSize (if the pointStyle is `POINT`).

**SURFACE**

Field is visualized using a colored surface, specified by one or more polygonal meshes associated with the field, where the values at mesh vertices are either determined directly, or interpolated from, the field. This type of visualization is available for `ScalarNodalField`, `ScalarSubElemField`, `ScalarVertexField` and `ScalarGridField`. For `ScalarVertexField`, the mesh is the mesh associated with the field itself. Otherwise, the meshes are supplied by external mesh components that are attached to the field as *render meshes*, as described in Section 7.6.4. For `ScalarNodalField` and `ScalarSubElemField`, these mesh components must be either `FemMeshComps` (Section 6.3) or `FemCutPlanes` (Section 6.12.7) associated with the FEM model, and may include its surface mesh; while for `ScalarGridField`, the mesh components are fixed mesh bodies (Section 3.8.1).

**ELEMENT**

Used only by `ScalarElementField`, allows the field to be visualized by element shaped widgets that are colored according to each element's field value and the color map. The size of the widget, relative to the true element size, is controlled by the property elementWidgetSize, which is a scalar in the range $[0, 1]$.

**FACE**

Used only by `ScalarFaceField`, allows the field to be visualized by rendering the associated field mesh, with each face colored according to its field value and the color map.

**OFF**

No visualization (the default value).

### 7.6.2 Vector fields

Vector fields can be visualized, when possible, by drawing three dimensional line segments, with a length proportional to the field value, originating at the features used to define the field (nodes, element centers, or integration points for FEM fields; vertices and face centers for mesh fields; vertices for grid fields). This can done for any field whose vector type is an instance of Vector (which includes Vector2d, Vector3d, and VectorNd) by mapping the vector values onto a 3-vector. This means ignoring element values for vectors whose size is greater than 3, and setting higher element values to 0 for vectors whose size is less than 3.

Vector field rendering is controlled by the following properties:

**renderScale**

A double which scales the vector field value to determine the length of the drawn line segment. The default value is 0, implying the no vectors are drawn.

**renderProps**

Basic rendering properties of the field component, as described in Section 4.3. How the lines are displayed is controlled by the line subproperties, with the color specified by lineColor, the style by lineStyle (`LINE`, `CYLINDER`, `SOLID_ARROW`, `SPINDLE`), and the size by either lineRadius or lineWidth (if the `lineStyle` is `LINE`).

### 7.6.3 Grid fields

As mentioned above, the grid field components, `ScalarGridField` and `VectorGridField`, are not visible by default, and so must be made visible to enable visualization:

```
RenderProps.setVisible (gridField, true);
```

By default, grid field components also render their grid edges, using the edge subproperties of renderProps (drawEdges, edgeWidth, and edgeColor). If edgeColor is not set (i.e., is set to `null`), the lineColor subproperty is used instead. Grid components also have a renderGrid property which can be set to `false` to disable rendering of the grid.

For `VectorGridField`, when rendering the grid and *and* visualizing the vectors, one can make them different colors by using the edgeColor subproperty of renderProps to set the grid color:

```
RenderProps.setLineColor (gridField, Color.BLUE); // vectors drawn in blue
RenderProps.setEdgeColor (gridField, Color.GRAY); // grid edges drawn in gray
```

### 7.6.4 Render meshes

The scalar fields `ScalarNodalField`, `ScalarSubElemField`, and `ScalarGridField` all make use of *render meshes* when being visualized using `SURFACE` visualization. These are a collection of one or more components, each providing a polygonal mesh that defines a surface for rendering a color map of the field, based on values at the mesh vertices that are themselves determined from the field. For `ScalarNodalField` and `ScalarSubElemField`), the mesh components must be either `FemMeshComp`s contained within the FEM's `meshes` list (Section 6.3) or `FemCutPlane`s contained within the FEM's `cutPlanes` list (Section 6.12.7), while for `ScalarGridField`, they are `FixedMeshBody`s (Section 3.8.1) that are usually contained within the `MechModel`.

Adding render meshes to a field must be done in code. For `ScalarNodalField` and `ScalarSubElemField`, the set of rendering meshes is controlled by the following methods:

| | |
|---|---|
| void addRenderMeshComp (FemMesh mcomp) | Add the render mesh component `mcomp`. |
| boolean removeRenderMeshComp (FemMesh mcomp) | Remove the render mesh component `mcomp`. |
| FemMesh getRenderMeshComp (idx) | Return the `idx`-th render mesh component. |
| int numRenderMeshComps() | Return the number of render mesh components. |
| void clearRenderMeshComps() | Clear all the render mesh components. |

Equivalent methods exist for `ScalarGridField`, using `FixedMeshBody` instead of `FemMesh`.

The following should be noted with respect to render meshes:

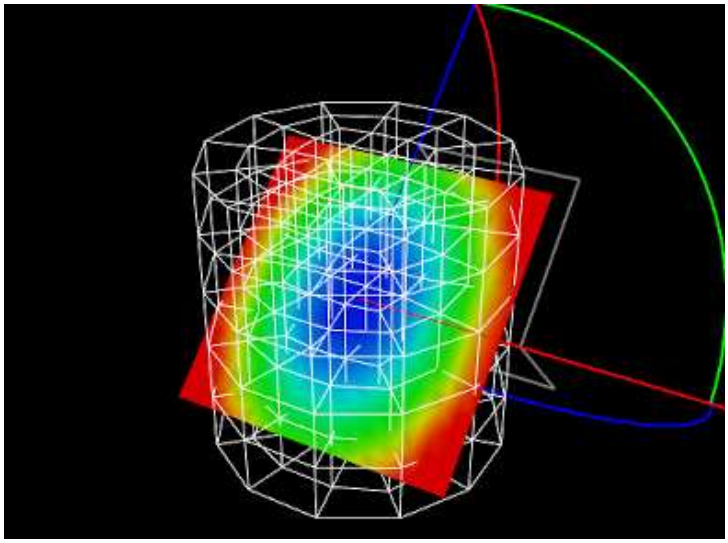


Figure 7.3: ScalarFieldVisualization model loaded into ArtiSynth.

- A render mesh can be created for the sole purpose of visualizing a field. A rectangle often does well for this purpose.
- Rendering of the visualization is done by the *field component*, not the render mesh component(s), and so it is usually necessary to make the latter invisible to avoid rendering conflicts.
- Render meshes should have a large enough number of vertices and triangles to support the resolution required to visualize the field properly.
- One can often use the GUI transformer tools to interactively adjust a render mesh's pose (see "Transformer tools" in the ArtiSynth User Interface Guide), allowing the user to move a render surface throughout the field. For `FixedMeshBody` and `FemCutPlane`, the tools change the component's pose, while for `FemMeshComp`, they change the location of its embedding within the FEM.

> Transformer tools have no effect on FEM meshes which are auto-generated, such as the default surface mesh.

As a simple example, the following code fragment creates a square render mesh for a `ScalarGridField`:

```
MechModel mech;
ScalarGridField field;

...

// create a square mesh, with size 1.0 x 1.0 and resolution 20 x 20, and
// use it to instantiate a FixedMeshBody which is added to the MechModel:
PolygonalMesh mesh = MeshFactory.createPlane (1.0, 1.0, 20, 20);
FixedMeshComp mcomp = new FixedMeshComp (mesh);
mech.addMeshBody (mcomp);

// set the mesh component invisible and add it to the field as a render mesh
RenderProps.setVisible (mcomp, false);
field.addRenderMeshComp (mcomp);
```

### 7.6.5 Example: Visualizing a scalar nodal field

A simple application that uses a `FemCutPlane` to visualize a `ScalarNodalField` is defined in

```
artisynth.demos.tutorial.ScalarFieldVisualization
```

The `build()` method for this is shown below:

```
1     public void build (String[] args) {
2        MechModel mech = new MechModel ("mech");
3        addModel (mech);
4
5        // create a hex FEM cylinder to use for the field
6        FemModel3d fem = FemFactory.createHexCylinder (
7           null, /*height=*/1.0, /*radius=*/0.5, /*nh=*/5, /*nt=*/10);
8        fem.setMaterial (new LinearMaterial (10000, 0.45));
9        fem.setName ("fem");
10       mech.addModel (fem);
11
12       // fix the top nodes of the FEM
13       for (FemNode3d n : fem.getNodes()) {
14          if (n.getPosition().z == 0.5) {
15             n.setDynamic (false);
16          }
17       }
18
19       // create a scalar field whose value is r^2, where r is the radial
20       // distance from FEM axis
21       ScalarNodalField field = new ScalarNodalField (fem);
22       fem.addField (field);
23       for (FemNode3d n : fem.getNodes()) {
24          Point3d pnt = n.getPosition();
25          double rsqr = pnt.x*pnt.x + pnt.y*pnt.y;
26          field.setValue (n, rsqr);
27       }
28
29       // create a FemCutPlane to provide the visualization surface, rotated
30       // into the z-x plane.
31       FemCutPlane cutplane = new FemCutPlane (
32          new RigidTransform3d (0,0,0, 0,0,Math.toRadians(90)));
33       fem.addCutPlane (cutplane);
34
35       // set the field's visualization and the cut plane to it as a render mesh
36       field.setVisualization (ScalarNodalField.Visualization.SURFACE);
37       field.addRenderMeshComp (cutplane);
38
39       // create a control panel to set properties
40       ControlPanel panel = new ControlPanel();
41       panel.addWidget (field, "visualization");
42       panel.addWidget (field, "renderRange");
43       panel.addWidget (field, "colorMap");
44       addControlPanel (panel);
45
46       // set render properties
47       // set FEM line color to render edges blue grey:
48       RenderProps.setLineColor (fem, new Color (0.7f, 0.7f, 1f));
49       // make cut plane visible via its coordinate axes; make surface invisible
50       // to avoid conflicting with field rendering:
51       cutplane.setSurfaceRendering (SurfaceRender.None);
52       cutplane.setAxisLength (0.4);
53       RenderProps.setLineWidth (cutplane, 2);
54       // for point visualization: render points as spheres with radius 0.01
55       RenderProps.setSphericalPoints (field, 0.02, Color.GRAY); // color ignored
56    }
```

After first creating a `MechModel` (lines 2-3), a cylindrical hexahedral FEM model is created to contain the field, with its top nodes fixed to allow it to deform under gravity (lines 13-17). A `ScalarNodalField` is then defined for this FEM, where the value at each node is set to $r^2$, with $r$ being the radial distance from the node to the FEM's central axis (lines 21-27).

To visualize the field, a `FemCutPlane` is created with its pose set to align it with the $z$-$x$ plane and then added to the FEM model (lines 31-33). The field's visualization is then set to `SURFACE` and the cut plane is added to it as a render

mesh (lines 36-37). At lines 40-44, a control panel is created to allow interactive adjustment of the field's visualization, renderRange, and colorMap properties. Finally, render properties are set: the FEM line color (used to render element edges) is set to blue-gray (line 48); the cut plane's surface rendering is set to `None` to avoid interfering with the field rendering, and axis rendering is enabled to make the field visible and selectable even if it doesn't intersect the FEM (lines 51-53); and, for `POINT` visualization, the field is set to render points as spheres with a radius of 0.02 (line 55).

To run this example in ArtiSynth, select All demos > tutorial > ScalarFieldVisualization from the Models menu. Users can employ the control panel to adjust the visualization, the render range, and the color map. When `SURFACE` visualization is selected, the field will be rendered onto the FEM/plane intersection defined by the cut plane. Simulating the model will cause the FEM to deform, deforming this intersection with it. Clicking on the intersection surface or the cut plane axes will cause the cut plane to be selected. Its pose can then be adjusted using the GUI transformer tools (see "Model Manipulation" in the ArtiSynth User Interface Guide), as shown in Figure 7.3.

> When the updating property of the render range is set to `AUTO_FIT`, the range will be automatically set to the range of values in the field, and *not* the values evaluated over the render surface. The latter may exceed the former due to extrapolation when the surface extends outside of the FEM, in which case the visualization will appear to saturate. While this is not generally a concern because FEM field values are unreliable outside of the FEM, one can override this effect by setting the updating property to `FIXED` and setting the range interval manually.

### 7.6.6 Examples: Visualizing other fields

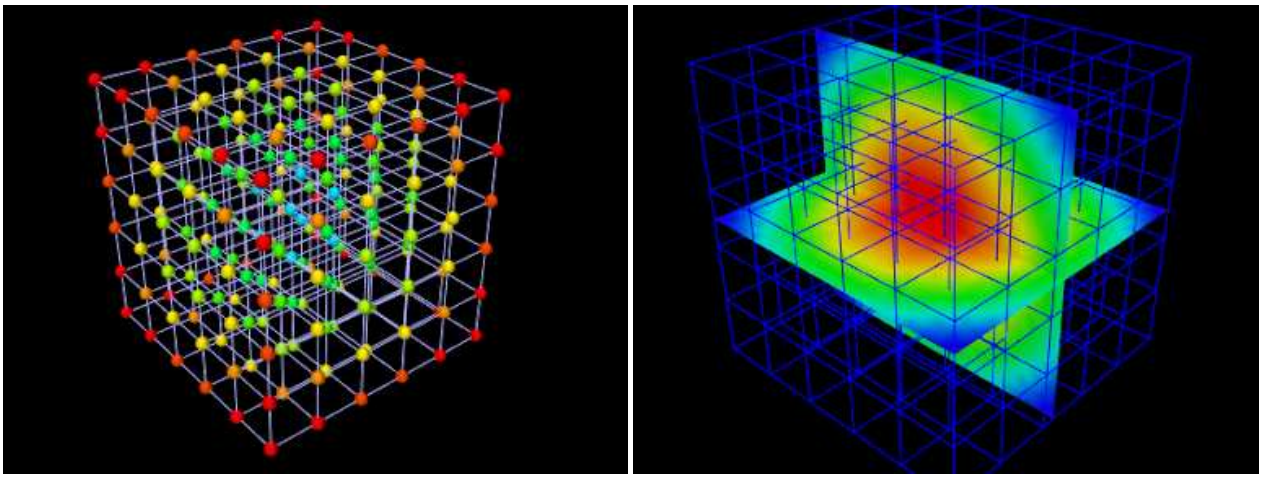


Figure 7.4: Left: `ScalarNodalFieldDemo` with `POINT` visualization. Right: `ScalarGridFieldDemo` with `SURFACE` visualization on two perpendicular meshes.

Numerous examples exist for creating and visualizing other field types:

```
artisynth.demos.fem.ScalarNodalFieldDemo
artisynth.demos.fem.ScalarElementFieldDemo
artisynth.demos.fem.ScalarSubElemFieldDemo
artisynth.demos.fem.VertexNodalFieldDemo
artisynth.demos.fem.VertexElementFieldDemo
artisynth.demos.fem.VertexSubElemFieldDemo

artisynth.demos.mech.ScalarVertexFieldDemo
artisynth.demos.mech.ScalarFaceFieldDemo
artisynth.demos.mech.ScalarGridFieldDemo
artisynth.demos.mech.VertexVertexFieldDemo
artisynth.demos.mech.VertexFaceFieldDemo
artisynth.demos.mech.VertexGridFieldDemo
```

Illustrations of some of these are shown in Figures 7.4, 7.5, and 7.6,

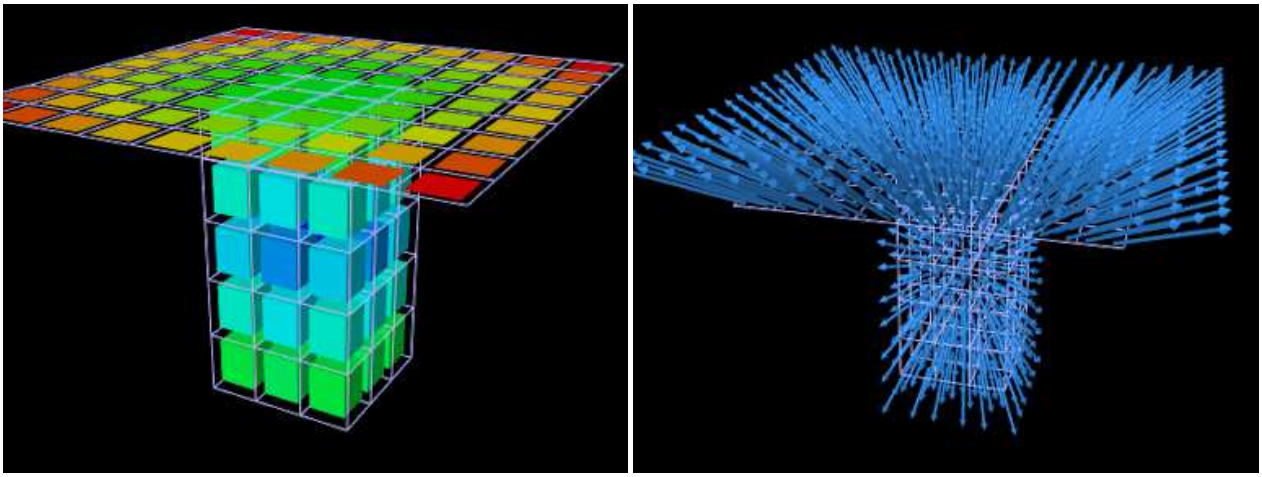


Figure 7.5: Left: `ScalarElementFieldDemo` with `ELEMENT` visualization. Right: `VectorSubElemFieldDemo` showing vectors in blue.

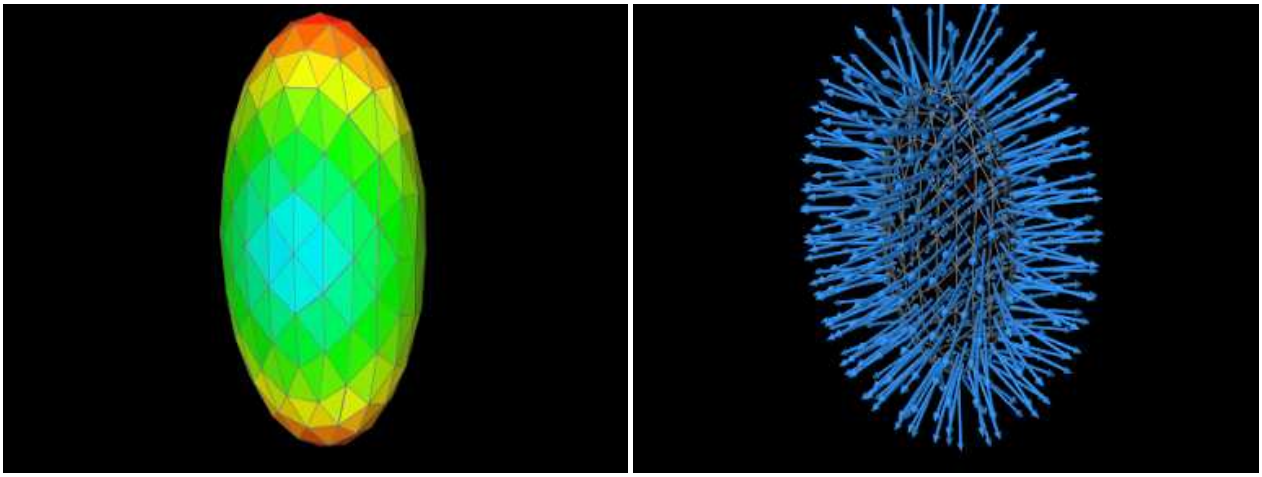


Figure 7.6: Left: `ScalarFaceFieldDemo` with `FACE` visualization. Right: `VectorFaceFieldDemo` showing vectors in blue.

# Chapter 8

# Contact and Collision

ArtiSynth supports contact and collisions between various collidable bodies, including rigid bodies and FEM models, through a collision handling mechanism build into `MechModel`. Collisions are disabled by default, but can be enabled between specific pairs of bodies, or between general categories of *rigid* and *deformable* bodies (Section 8.1). Collisions are supported between any body that implements the interface Collidable.

Collision detection works by finding the intersecting regions between the *collision meshes* of collidable bodies, and then using the intersection information to determine contact points and their associated constraints (Section 8.4). Collision meshes must be instances of PolygonalMesh and must be triangular; mesh requirements are described in more detail in Section 8.3. Typically, a collision mesh is the same as the body's surface mesh, but other meshes (or collections of meshes) can be specified instead (Section 8.3).

Detailed aspects of the collision behavior, such as friction, optional compliance and damping terms, methods used to determine contacts, and various tolerances, can be controlled using CollisionBehavior objects (Section 8.2). Collisions can be visualized by rendering contact normals, forces, intersection contours, and contact pressures (Section 8.5), and information about specific collisions, including contact data, forces, and pressures, can also be monitored as the simulation proceeds (Section 8.8).

It is also possible to explicitly control contact forces by making them an explicit function of penetration depth (Section 8.7); in particular, this capability is used to support elastic foundation contact (Section 8.7.3).

It should be understood that collision handling can be computationally expensive and, due to its discontinuous nature, may be less accurate than other aspects of the simulation. ArtiSynth therefore provides a number of ways to selectively control collision handling between different pairs of bodies. Collision handling is also challenging because if collisions are enabled among $n$ objects, then one needs to be able to easily specify the characteristics of up to $O(n^2)$ collision interactions, while also managing the fact that these interactions are highly transient. In ArtiSynth, collision handling is managed by a CollisionManager that is a subcomponent of each MechModel. The collision manager maintains default collision behaviors among certain groups of collidable objects, while also allowing the user to override the default behaviors by setting specific behaviors for any given pair of collidables.

> Within ArtiSynth, the terms collision and contact are used somewhat interchangeably, although we acknowledge that in the literature, collision is generally understood to refer to the transient process that occurs when bodies first come into contact, while contact refers to the more persistent situation as the bodies remain together.

## 8.1 Enabling collisions

> This section describes how to enable collisions in code. However, it is also possible to set some aspects of collision behavior using the ArtiSynth GUI. See the section "Collision handling" in the ArtiSynth User Interface Guide.

ArtiSynth can simulate collisions between bodies that implement the interface Collidable. Collidable bodies are further subdivided into those that are *rigid* and those that are *deformable*, according to whether their isDeformable() method returns `true`. Rigid collidables include RigidBody, while deformable collidables include FEM models (FemModel3d),

their mesh components (FemMeshComp), and skinned meshes (SkinMeshBody, Chapter 11). Because of the computational cost, collision detection is turned off by default and must be explicitly enabled when a model is built. Collisions can be enabled between specific pairs of bodies, or between general groupings of rigid and deformable bodies.

Collision handling is implemented by a model's MechModel, which contains a CollisionManager subcomponent that keeps track of which bodies are colliding and computes the constraints and forces needed to enforce the collision behaviors. The collision manager itself can accessed by the `MechModel` method

```
getCollisionManager()
```

and can be used to query and set various collision properties, as described further below.

### 8.1.1 Collisions between specific bodies

As indicated above, collidable bodies are typically components such as rigid bodies or FEM models. Given two collidable bodies `collidable0` and `collidable1`, the simplest way to enable collisions between them is with the `MechModel` methods

```
setCollisionBehavior (collidable0, collidable1, enabled)
setCollisionBehavior (collidable0, collidable1, enabled, mu)
```

The first method enables collisions *without* friction, while the second enables collisions with friction as specified by `mu`, which is the coefficient of Coulomb (or dry) friction. The `mu` value is ignored if `enabled` is `false`. Specifying a `mu` value of 0 disables friction, and the friction coefficient can also be left undefined by specifying a `mu` value less than 0, in which case the coefficient is inherited from the global friction value accessed by the `MechModel` methods

```
setFriction (mu)
double getFriction()
```

More detailed control over collision behavior can be achieved using the method

```
setCollisionBehavior (collidable0, collidable1, behavior)
```

where `behavior` is a CollisionBehavior object (Section8.2.1) that specifies both *enabled* and *mu*, along with other, more detailed collision properties (see Section 8.2.1).

The `setCollisionBehavior()` methods work by adding a `CollisionBehavior` object to the collision manager as a subcomponent. With the method `setCollisionBehavior(collidable0,collidable1,behavior)`, the behavior object is created and supplied by the application. With the other methods, the behavior object is created automatically and returned by the method. Once a behavior has been specified, it can then be queried using

```
CollisionBehavior getCollisionBehavior (collidable0, collidable1)
```

This method will return `null` if no behavior for the pair in question has been explicitly set using one of the `setCollisionBehavior()` methods.

> One should normally avoid enabling collisions between bodies that are otherwise connected, for example, adjacent bodies in a linkage connected by joints, in order to avoid conflicts between the connection and the collision behavior. If collision interaction is required between *parts* of two connected bodies, this can be achieved in various ways as described in Section 8.3.

> Because behaviors are proper components, it is *not* permissible to add them to the collision manager twice. Specifically, the following will produce an error:
>
> ```
> CollisionBehavior behav = new CollisionBehavior();
> behav.setDrawIntersectionContours (true);
> mech.setCollisionBehavior (col0, col1, behav);
> mech.setCollisionBehavior (col2, col3, behav); // ERROR
> ```
>
> However, if desired, a new behavior can be created from an existing one:

```
CollisionBehavior behav = new CollisionBehavior();
behav.setDrawIntersectionContours (true);
mech.setCollisionBehavior (col0, col1, behav);
behav = new CollisionBehavior(behav);
mech.setCollisionBehavior (col2, col3, behav); // OK
```

### 8.1.2 Default collisions between groups

For convenience, it is also possible to specific default collision behaviors between different *groups* of collidables.

The default collision behavior between *all* collidables can be controlled using the `MechModel` methods

```
setDefaultCollisionBehavior (enabled, mu)
setDefaultCollisionBehavior (behavior)
```

where `enabled`, `mu` and `behavior` act as described in Section 8.1.1.

Because of the possible computational expense of collision detection, default collision behaviors should be used with care.

In addition, a default collision behavior can be set for generic groups of collidables using the `MechModel` methods

```
setDefaultCollisionBehavior (group0, group1, enabled, mu)
setDefaultCollisionBehavior (group0, group1, behavior)
```

where `group0` and `group1` are static instances of Collidable.Group that represent the groups described in Table 8.1. The groups `Collidable.Rigid` and `Collidable.Deformable` denote collidables that are rigid and deformable, respectively. The group `Collidable.AllBodies` denotes both rigid and deformable bodies, and `Collidable.Self` is used to enable self-collision, which is described in greater detail in Section 8.3.2.

| Collidable group | description |
|---|---|
| Collidable.Rigid | rigid collidables (e.g., rigid bodies) |
| Collidable.Deformable | deformable collidables (e.g, FEM models) |
| Collidable.AllBodies | rigid and deformable collidables |
| Collidable.Self | enables self-intersection for compound deformable collidables |
| Collidable.All | rigid and deformable collidables and self-intersection |

Table 8.1: Collision group types.

A call to one of the `setDefaultCollisionBehavior()` methods will override the effects of previous calls. So for instance, the code sequence

```
mech.setDefaultCollisionBehavior (true, 0);
mech.setDefaultCollisionBehavior (
   Collidable.Deformable, Collidable.Rigid, false, 0);
mech.setDefaultCollisionBehavior (true, 0.2);
```

will initially enable collisions between all bodies with a friction coefficient of 0, then *disable* collisions between deformable and rigid bodies, and finally re-enable collisions between all bodies with a friction coefficient of 0.2.

The default collision behavior between any pair of collidable groups can be queried using

```
CollisionBehavior getDefaultCollisionBehavior (group0, group1)
```

where `group0` and `group1` are restricted to the primary groups `Collidable.Rigid`, `Collidable.Deformable`, and `Collidabe.Self`, since individual behaviors are not maintained for the composite groups `Collidable.AllBodies` and `Collidable.All`.

Specific collision behaviors set using the `setCollisionBehavior()` methods of Section 8.1.1 will override any default collision settings. In addition, the second argument `collidable1` of these methods can describe either an individual collidable, *or* one of the groups of Table 8.1. For example, the code fragment

```
MechModel mech;
RigidBody bodA;
FemModel3d femB;

...

mech.setCollisionBehavior (bodA, Collidable.Deformable, true, 0.1);
mech.setCollisionBehavior (femB, Collidable.AllBodies, true, 0.0);
mech.setCollisionBehavior (bodA, femB, false, 0.0);
```

will enable collisions between `bodA` and all deformable collidables (with friction 0.1), as well as `femB` and all deformable and rigid collidables (with friction 0.0), while specifically `disabling` collisions between `bodA` and `femB`.

To determine the actual behavior controlling collisions between two collidables (whether due to a default behavior or one specified using `setCollisionBehavior()`), one may use the method

```
getActingCollisionBehavior (collidable0, collidable1)
```

where `collidable0` and `collidable1` must both be specific collidable components and cannot be a group (such as `Collidable.Rigid` or `Collidable.All`). If one of the collidables is a *compound* collidable (Section 8.3.2), or has a collidability setting (Section 8.2.2) that prevents collisions, there may be no consistent acting behavior, in which case the method returns `null`.

Collision behaviors take priority over each other in the following order:

1. Behaviors specified using `setCollisionBehavior()` involving two *specific* collidables.
2. Behaviors specified using `setCollisionBehavior()` involving one *specific* collidable and a *group* of collidables (indicated by a `Collidable.Group`), with later specifications taking priority over earlier ones.
3. Default behaviors specified using `setDefaultCollisionBehavior()`.

A collision behavior specified with `setCollisionBehavior()` can later be removed using

```
clearCollisionBehavior (collidable0, collidable1)
```

and *all* such behaviors in a `MechModel` can be removed with

```
clearCollisionBehaviors()
```

Note that this latter call does *not* remove default behaviors specified with `setDefaultCollisionBehavior()`.

### 8.1.3 Example: collision with a plane

A simple model illustrating collision between two jointed rigid bodies and a plane is defined in

`artisynth.demos.tutorial.JointedCollide`

This model is simply a subclass of `RigidBodyJoint` that overrides the `build()` method to add an inclined plane and enable collisions between it and the two connected bodies:

```
1    public void build (String[] args) {
2
3       super.build (args);
4
5       bodyB.setDynamic (true);  // allow bodyB to fall freely
6
7       // create and add the inclined plane
8       RigidBody base = RigidBody.createBox ("base", 25, 25, 2, 0.2);
9       base.setPose (new RigidTransform3d (5, 0, 0, 0, 1, 0, -Math.PI/8));
10      base.setDynamic (false);
11      mech.addRigidBody (base);
```

```
12
13        // turn on collisions
14        mech.setDefaultCollisionBehavior (true, 0.20);
15        mech.setCollisionBehavior (bodyA, bodyB, false);
16     }
```

The superclass `build()` method called at line 3 creates everything contained in `RigidBodyJoint`. The remaining code then alters that model: `bodyB` is set to be dynamic (line 5) so that it will fall freely, and an inclined plane is created from a thin box that is translated and rotated and then set to be be non-dynamic (lines 8-11). Finally, collisions are enabled by setting the default collision behavior (line 14), and then specifically disabling collisions between `bodyA` and `bodyB` (line 15). As indicated above, the latter step is necessary because the joint would otherwise keep the two bodies in a permanent state of collision.

To run this example in ArtiSynth, select All demos > tutorial > JointedCollide from the Models menu. The model should load and initially appear as in Figure 8.1. Running the model (Section 1.5.3) will cause the jointed assembly to collide with and slide off the inclined plane.

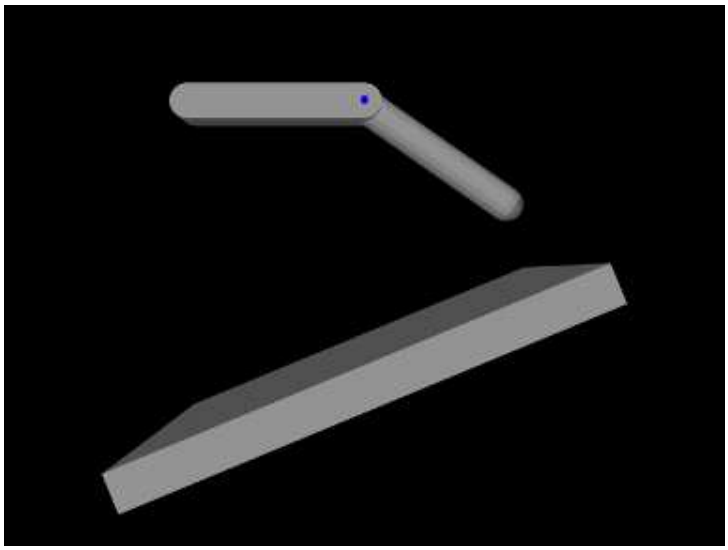


Figure 8.1: JointedCollide model loaded into ArtiSynth.

### 8.1.4 Collisions for FEM models

Both FemModel3d and FemMeshComp implement Collidable. By default, a `FemModel3d` uses its surface mesh as the collision surface, while `FemMeshComp` will uses the mesh it contains (although collisions will only occur if this is a triangular polygonal mesh).

Because `FemModel3d` contains other collidables as subcomponents, it is considered a `compound` collidable, as discussed further in Section 8.3.2. In particular, since `FemMeshComp` is also a `Collidable`, we can enable collisions with any embedded mesh inside an FEM. Any forces resulting from the collision are then automatically transferred back to the underlying nodes of the model using Equation (6.4).

**Note:** Collisions involving shell elements are not yet fully supported. This relates to the fact that shells are thin and can therefore pass through each other easily in a single time step, and also that meshes associated with shell elements are usually not closed. However, collisions *should* work properly if

1. The collision meshes do not pass completely through each other in a single time step;
2. Collisions do not occur near open mesh edges.

Restriction 2 can often be relaxed if the collider type is set to `TRI_INTERSECTION` (Section 8.4.2).

### 8.1.5 Example: FEM models and rigid bodies

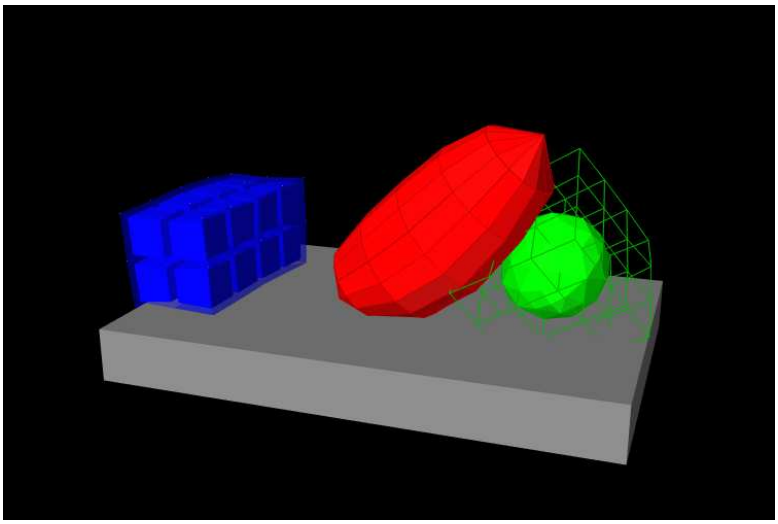


Figure 8.2: FemCollisions model loaded into ArtiSynth.

An example of FEM collisions is shown in Figure 8.2. The full source code can be found in the ArtiSynth repository under `artisynth.demos.tutorial.FemCollisions`. The model contains a rigid body table and three FEM models: a beam (blue), an `ellipsoid` (red), and a hex block containing an embedded spherical mesh (green). The collision-enabling code is as follows:

```
// Set up collisions
mech.setCollisionBehavior(ellipsoid, beam, true);    // beam-ellipsoid
mech.setCollisionBehavior(ellipsoid, table, true);   // ellipsoid-table
mech.setCollisionBehavior(table, beam, true);        // beam-table

FemMeshComp embeddedSphere =
   block.getMeshComp("embedded");                    // get embedded FemMeshComp
mech.setCollisionBehavior(embeddedSphere, table, true);      // sphere-table
mech.setCollisionBehavior(ellipsoid, embeddedSphere, true);  // sphere-ellipsoid
```

This enables collisions between the ellipsoid and the beam, table and embedded sphere, and between the table and the beam and embedded sphere. However, collisions are *not* enabled between the block itself and any other components; notice in the figure that the block surface passed through the table and ellipsoid.

## 8.2 Collision behaviors and collidability

### 8.2.1 Collision behaviors

As mentioned above, CollisionBehavior objects can be used to control other aspects of contact beyond friction and enabling. This may be done by setting the `CollisionBehavior` properties listed below. Except where otherwise indicated, these properties are also exported by the collision manager, where they can be used to provide global default settings for *all* collisions.

> In addition to these properties, `CollisionBehavior` and `CollisionManager` export other properties that can be used to control the rendering of collisions, as described in Section 8.5.

**enabled**

A boolean that determines if collisions are enabled. Not present in the collision manager.

**friction**

A double giving the coefficient of Coulomb friction, typically in the range $[0, 0.5]$. The default value is 0. Setting friction to a non-zero value increases the simulation time, since extra constraints must be added to the system to accommodate the friction.

**method**

An instance of CollisionBehavior.Method that controls how the contact constraints for the collision response are generated. The methods, described in more detail in Section 8.4.1, are:

| Method | Constraint generation |
|---|---|
| CONTOUR_REGION | constraints generated from planes fit to each mesh contact region |
| VERTEX_PENETRATION | constraints generated from penetrating vertices |
| VERTEX_EDGE_PENETRATION | constraints generated from penetrating vertices and edge-edge contacts |
| DEFAULT | method is determined automatically |
| INACTIVE | no constraints generated |

The default value is `DEFAULT`.

**vertexPenetrations**

An instance of CollisionBehavior.VertexPenetrations that controls the collidables for which vertex penetrations are computed when using vertex penetration contact (see Sections 8.4.1.2 and 8.4.1.3). The default value is `AUTO`.

**bilateralVertexContact**

A boolean which causes the system to handle vertex penetration contact using bilateral constraints instead of unilateral constraints. This usually improves computational performance significantly for collisions involving FEM models, but may result in overconstrained contact when vertex penetration contact is used between rigid bodies, as well as in some other circumstances (Section 8.6). The default value is `true`.

**colliderType**

An instance of CollisionManager.ColliderType that specifies the underlying mechanism used to determine the collision information between two meshes. The choice of collider may restrict which collision *methods* (described above) are allowed. Collider types are described in more detail in Section 8.4.1 and include:

| Type | Description |
|---|---|
| AJL_CONTOUR | uses mesh intersection contour to find penetrating regions and vertices |
| TRI_INTERSECTION | uses triangle intersections to find penetrating regions and vertices |
| SIGNED_DISTANCE | uses a signed distance field to find penetrating vertices |

The default value is `TRI_INTERSECTION`.

**reduceConstraints**

A boolean which, if `true`, indicates that the system should try to reduce the number of contacts between bodies in order to try and remove redundant contacts. See Section 8.6. The default value is `false`.

**compliance**

A double which adds a compliance (inverse stiffness) to the collision behavior, so that the contact has a "springiness". See Section 8.6. The default value for this is 0 (no compliance).

**damping**

A double which, if compliance is non-zero, specifies a damping to accompany the compliant behavior. See Section 8.6. The default value is 0. When compliance is specified, it is usually necessary to set the damping to a non-zero value to prevent bouncing.

**stictionCreep**

A double which, if non-zero, is used to regularize friction constraints. The default value is 0. It is usually only necessary to set this number when `MechModel`'s useImplicitFriction property is set to `true` (see Section 8.9.4), *and* when the contact constraints are redundant (Section 8.6). The number should be interpreted as a small "creep" speed with which contacts nominally held still by static friction are allowed to drift. The number should be as large as possible without seriously affecting the simulation.

**forceBehavior**

A composite property of type ContactForceBehavior specifying a contact force behavior, as described in Section 8.7.2. The default value is `null`.

**penetrationTol**

A double controlling the amount of interpenetration that is permitted in order to ensure contact stability (see Section 8.9). This property is not present in the collision manager. If unspecified, the system will inherit this property from the `MechModel`, which computes a default penetration tolerance based on the overall size of the model.

**rigidRegionTol**

A double which, for the `CONTOUR_REGION` contact method and the `TRI_INTERSECTION` collider type, specifies an overlap factor that is used to group individual triangle intersections into contact regions. If not explicitly specified, the system computes a default value based on the overall size of the model.

**rigidPointTol**

A double which, for the `CONTOUR_REGION` contact method, specifies a minimum distance between contact points that is used to reduce the number of contacts. If not explicitly specified, the system computes a default value for this based on the overall size of the model.

To set properties globally in the collision manager, one can access it using the `MechModel` method getCollisionManager(), and then employ a code fragment such as the following:

```
CollisionManager cm = mech.getCollisionManager();
cm.setReduceConstraints (true);
```

Since collision behaviors are subcomponents of the collision manager, properties set in the collision manager will be *inherited* by any behaviors for which they have not been explicitly set.

> One can also set properties using the GUI, by selecting the collision manager or a collision behavior in the navigation panel and then selecting Edit properties ... from the context menu. See "Property panels" and "Collision handling" in the ArtiSynth User Interface Guide.

To set properties for specific collidable pairs, one can call either `setDefaultCollisionBehavior()` or `setCollisionBehavior()` with an appropriately set `CollisionBehavior` object:

```
RigidBody bodA;

CollisionBehavior behav = new CollisionBehavior (enabled, mu);
behav.setPenetrationTol (0.001);
setDefaultCollisionBehavior (Collidable.Deformable, Collidable.Rigid, behav);

behav.setPenetrationTol (0.003);
setCollisionBehavior (bodA, Collidable.Rigid, behav);
```

For behaviors that are already set, one may use `getDefaultCollisionBehavior()` or `getCollisionBehavior()` to obtain the behavior and then set the desired properties directly:

```
RigidBody bodA;
CollisionBehavior behav;

behav = getDefaultCollisionBehavior (Collidable.Deformable, Collidable.Rigid);
behav.setPenetrationTol (0.001);
behav = getCollisionBehavior (bodA, Collidable.Rigid);
behav.setPenetrationTol (0.003);
```

Note however that `getDefaultCollisionBehavior()` only works for `Collidable.Rigid`, `Collidable.Deformable`, and `Collidable.Self`, and that `getCollisionBehavior()` only works for a collidable pair that has been previously specified with `setCollisionBehavior()`. One may also use `getActingCollisionBehavior()` (described above) to obtain the behavior (default or otherwise) responsible for a specific pair of collidables, although in some instances no such single behavior exists and the method will then return `null`.

There are two constructors for `CollisionBehavior`:

```
CollisionBehavior ()
CollisionBehavior (boolean enable, double mu)
```

The first creates a behavior with the enabled property set to false and other properties set to their default (generally inherited) values. The second creates a behavior with the enabled and friction properties explicitly specified, and other properties set to their defaults. If `mu` is less than zero or `enabled` is false, then the `friction` property is set to be inherited so that its value is controlled by the global friction property in the collision manager (and accessed using the MechModel methods `setFriction(mu)` and `getFriction()`).

### 8.2.2 Collidability

Each collidable component maintains a collidable property, which specifically enables or disables the ability of that collidable to collide with other collidables.

The `collidable` property value is of the enumerated type Collidable.Collidability and has four possible settings:

**OFF**

All collisions disabled: the collidable will not collide with anything.

**ALL**

All collisions (both internal and external) enabled: the collidable may collide with any other collidable.

**INTERNAL**

The collidable may *only* collide with other collidables that are inside the same collidable hierarchy of some compound collidable (Section 8.3.2).

**EXTERNAL**

The collidable may collide with any other collidable *except* those that are inside the same collidable hierarchy of some compound collidable.

Note that collidability only *enables* collisions. In order for collisions to actually occur between two collidables, a default or override collision behavior must also be specified for them in the MechModel.

## 8.3 Collision meshes and compound collidables

Contact between collidable bodies is determined by finding intersections between their collision meshes. Collision meshes must be instances of PolygonalMesh, and must also be triangular, manifold, oriented, and non-self-intersecting (at least in the region of contact). The oriented requirement helps the collision detector differentiate inside from outside. Collision meshes should also be closed, if possible, although collision may sometimes work with the open meshes (such as those that arise with shell elements) under conditions described in Section 8.1.4. Collision meshes do *not* need to be connected; a collision mesh may be composed of separate parts.

Commonly, a collidable body has a single collision mesh which is the same as its surface mesh. However, some collidables, such as rigid bodies and FEM models, allow an application to specify a collidable mesh that is different from its surface mesh. This can be useful in situations such as the following:

- Collisions should be enabled for only *part* of a collidable, perhaps because other parts are in contact due to joints or other types of attachment.
- The collision mesh requires a different resolution than the surface mesh, possibly for computational reasons.
- The collision mesh requires a different geometry, such as in situations where the collidable's physical surface is sufficiently thin that it may otherwise pass through other collidables undetected (Section 8.9.2).

For a rigid body, the collision mesh is returned by its `getCollisionMesh()` method (see Section 8.3.4). By default, this is the same as the surface mesh returned by `getSurfaceMesh()`. However, the collision mesh can be changed by adding one (or more) additional mesh components to the `meshes` component list and setting its isCollidable property to `true`, usually while also setting isCollidable to `false` for the surface mesh (Section 3.2.9). The collision mesh returned by `getCollisionMesh()` is then formed from the union of all rigid body meshes for which isCollidable is `true`.

The *sum* operation used to create the `RigidBody` collision mesh uses addMesh(), which simply adds all vertices and faces together. While the result works correctly for collisions, it does not represent a proper CSG union operation (such as that described in Section 2.5.7) and may contain interpenetrating and overlapping features. Care should be taken to ensure that the resulting mesh is not self-intersecting in the regions that will be exposed to contact.

For FEM models, the mechanism is a little different, as discussed below in Section 8.3.2. An FEM model can have *multiple* collision meshes, depending on the setting of the collidable property of each of its mesh components. The ability to have multiple collision meshes permits self-collision intersection of the FEM model with itself.

### 8.3.1 Example: redefining a rigid body collision mesh

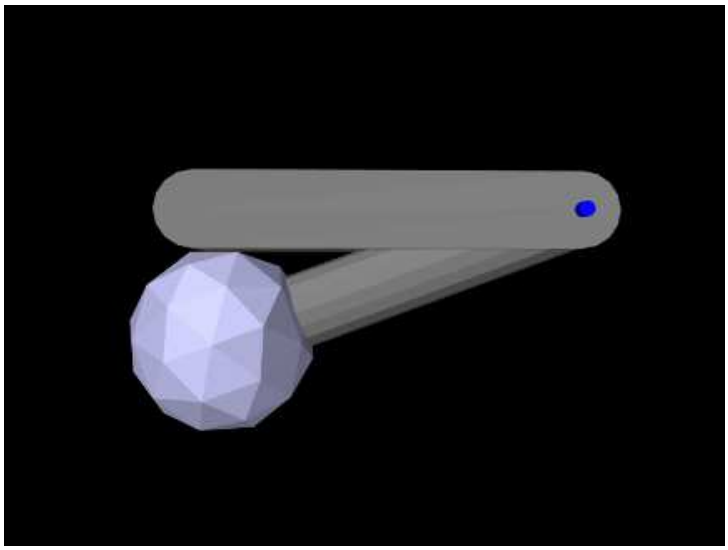


Figure 8.3: JointedBallCollide showing the ball at the tip of `bodyA` colliding with `bodyB`.

A model illustrating how to redefine the collision mesh for a rigid body is defined in

`artisynth.demos.tutorial.JointedBallCollide`

Like `JointedCollide` in Section 8.1.3, this model is simply a subclass of `RigidBodyJoint` that overrides the `build()` method, adding a ball to the tip of `bodyA` to enable it to collide with `bodyB`:

```
1     public void build (String[] args) {
2
3        super.build (args); // build the RigidBodyJoint model
4
5        // create a ball mesh
6        PolygonalMesh ball = MeshFactory.createIcosahedralSphere (2.5, 1);
7        // translate it to the tip of bodyA, add it to bodyA, and make it blue-gray
8        ball.transform (new RigidTransform3d (5, 0, 0));
9        RigidMeshComp mcomp = bodyA.addMesh (ball);
10       RenderProps.setFaceColor (mcomp, new Color (.8f, .8f, 1f));
11
12       // disable collisions for the main surface mesh of bodyA
13       bodyA.getSurfaceMeshComp().setIsCollidable (false);
14       // enable collisions between bodyA and bodyB
15       mech.setCollisionBehavior (bodyA, bodyB, true);
16    }
```

The superclass `build()` method called at line 3 creates everything contained in `RigidBodyJoint`. The remaining code then alters that model: A ball mesh is created, translated to the tip of `bodyA`, added as an additional mesh (Section

3.2.9), and set to render as blue-gray (lines 5-10). Next, collisions are disabled for `bodyA`'s main surface mesh by setting its isCollidable property to `false` (line 13); this will ensure that the collision mesh associated with `bodyA` will consist solely of the ball mesh, which is necessary because the surface mesh is permanently in contact with `bodyB`. Lastly, collisions are enabled between `bodyA` and `bodyB` (line 15).

To run this example in ArtiSynth, select All demos > tutorial > JointedBallCollide from the Models menu. Running the model will cause `bodyA` to fall, pivot about the hinge joint, and collide with `bodyB` (Figure 8.3).

### 8.3.2 Compound collidables and self-collision

An FEM model is an example of a *compound* collidable, which may contain *subcollidable* descendant components which are also collidable. Compound collidables are identified by having their isCompound() method return `true`. For an FEM model, the subcollidables are the mesh components in its `meshes` list. A non-compound collidable which is *not* a subcollidable of some other (compound) collidable is called *solitary*. If A is a subcollidable of a compound collidable C, then C is an *ancestor collidable* of A.

> Subcollidables do not need to be immediate child components of the compound collidable; they need only be descendants.

One of the main purposes of compound collidables is to enable self-collisions. While ArtiSynth does not currently support the detection of self-collisions within a single mesh, self-collision can be implemented within a compound collidable C by detecting collisions amount the (separate) meshes of its subcollidables.

> Compound collidables and their subcollidables are assumed to be deformable; otherwise, subcollision would not be possible.

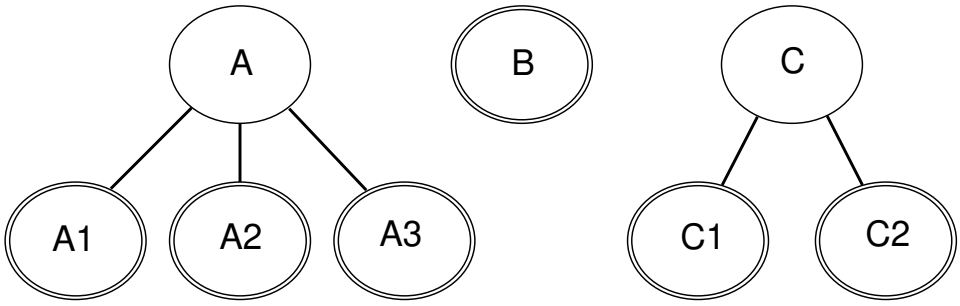



Figure 8.4: A collection of collidable components, where A is compound, with subcollidables A1, A2, and A3, B is solitary, and C is compound with subcollidables C1 and C2. The non-compound collidables, including the leaf nodes of the collidable trees, are also instances of CollidableBody, indicated by a double outline.

In general, an ArtiSynth model will contain a number of collidable components, some of which are compound (Figure 8.4). The non-compound collidables, including both solitary collidables and leaf nodes of a compound collidable's tree, are also instances of the subinterface CollidableBody, which provide the collision meshes, along with other information used to compute the collision response (Section 8.3.4).

Actual collisions happen only between `CollidableBody`s; compound collidables are used only for determining grouping and specifying collision behaviors. The rules for determining whether two collidable bodies A and B will actually collide are as follows:

1. *Internal (self) collisions:* If A and B are both subcollidables of some compound collidable C, then A and B will collide if (a) both their collidable properties are set to either `ALL` or `INTERNAL`, and (b) an explicit collision behavior is set between A and B, or self-collision is enabled for C (as described below).
2. *External collisions:* Otherwise, if A and B are either solitary *or* subcollidables of different compound collidables, then they will collide if (a) both their collidable properties are set to either `ALL` or `EXTERNAL`, and (b) a specific or default collision behavior is set between them or their collidable ancestors.

As mentioned above, the subcollidables of an FEM are its mesh components (Section 6.3), each of which is a collidable implemented by FemMeshComp. When a mech component is added to an FEM model, using either `addMeshComp()` or one of the `addMesh()` methods, its collidable property is automatically set to `INTERNAL`, so if a different setting is required, this must be specified *after* the component has been added to the model.

Subject to the above conditions, self-collision can be enabled for a specific compound collidable C by calling the `setCollisionBehavior()` methods with `collidable0` set to C and `collidable1` set to either C or `Collidable.SELF`, as in for example:

```
mech.setCollisionBehavior (C, Collidable.SELF, true, mu);

... OR ...

mech.setCollisionBehavior (C, C, true, mu);
```

It can also be enabled, by default, for all compound collidables by calling one of the `setDefaultCollisionBehavior()` methods with `collidable0` set to `Collidable.DEFORMABLE` and `collidable1` set to `Collidable.SELF`, e.g.:

```
mech.setDefaultCollisionBehavior (
   Collidable.DEFORMABLE, Collidable.SELF, true, mu);
```

For an example of how collision interactions can be set, refer to Figure 8.4, assume that components A, B and C are deformable, and that the collidable property for all collidables is set to `ALL` *except* for A3, where it is set to `EXTERNAL` (implying that A3 cannot self-collide with A1 and A2). Then if `mech` is the `MechModel` containing the collidables, the call

```
mech.setDefaultCollisionBehavior (
   Collidable.DEFORMABLE, Collidable.DEFORMABLE, true, 0.2);
```

will enable collisions between A1, A2, and A3 and each of B, C1, and C2, and between B and C1 and C2, but not among A1, A2, and A3 or C1 and C2. The subsequent calls

```
mech.setDefaultCollisionBehavior (
   Collidable.DEFORMABLE, Collidable.SELF, true, 0);
mech.setCollisionBehavior (B, A3, false);
```

will enable self-collision between A1 and A2 and C1 and C2 with zero friction, and disable collision between B and A3. Finally, the calls

```
mech.setCollisionBehavior (A3, C, true, 0.3);
mech.setCollisionBehavior (A, A, false);
```

will enable collision between A3 and each of C1 and C2 with friction 0.3, and disable all self-collisions among A1, A2 and A3.

### 8.3.3 Example: FEM model with self-collision

A model illustrating self-collision for an FEM model is defined in

```
artisynth.demos.tutorial.FemSelfCollide
```

It creates a simple FEM based on a partial torus that self intersects when it falls under gravity. Internal surface meshes are added to the left and right ends of the model to prevent interpenetration. The code for the `build()` method is listed below:

```
1 public class FemSelfCollide extends RootModel {
2
3    public void build (String[] args) {
4       MechModel mech = new MechModel ("mech");
5       addModel (mech);
```

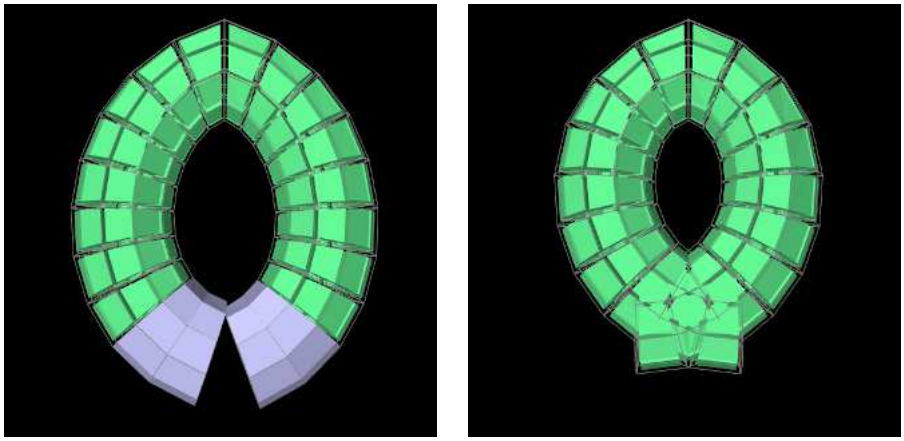


Figure 8.5: FemSelfCollide showing collision between the left and right contact meshes (left). Without these meshes, the FEM model intersects with itself as show on the right.

```
6
7       // create FEM model based on a partial (open) torus, with an
8       // opening (gap) of angle of PI/4.
9       FemModel3d ptorus = new FemModel3d("ptorus");
10      FemFactory.createPartialHexTorus (
11         ptorus, 0.15, 0.03, 0.06, 10, 20, 2, 7*Math.PI/4);
12      // rotate the model so that the gap is at the botom
13      ptorus.transformGeometry (
14         new RigidTransform3d (0, 0, 0, 0, 3*Math.PI/8, 0));
15      // set material and particle damping
16      ptorus.setMaterial (new LinearMaterial (5e4, 0.45));
17      ptorus.setParticleDamping (1.0);
18      mech.addModel (ptorus);
19
20      // anchor the FEM by fixing the top center nodes
21      for (FemNode3d n : ptorus.getNodes()) {
22         if (Math.abs(n.getPosition().x) < 1e-15) {
23            n.setDynamic(false);
24         }
25      }
26
27      // Create sub-meshes to resist collison at the left and right ends of the
28      // open torus. At each end, create a mesh component, and use its
29      // createVolumetricSurface() method to create the mesh from the
30      // elements near the end.
31      LinkedHashSet<FemElement3d> elems =
32         new LinkedHashSet<>(); // elements for mesh bulding
33      FemMeshComp leftMesh = new FemMeshComp (ptorus, "leftMesh");
34      // elements near the left end have numbers in the range 180 - 199
35      for (int n=180; n<200; n++) {
36         elems.add (ptorus.getElementByNumber(n));
37      }
38      leftMesh.createVolumetricSurface (elems);
39      ptorus.addMeshComp (leftMesh);
40
41      FemMeshComp rightMesh = new FemMeshComp (ptorus, "rightMesh");
42      elems.clear();
43      // elements at the right end have numbers in the range 0 - 19
44      for (int n=0; n<20; n++) {
45         elems.add (ptorus.getElementByNumber(n));
46      }
47      rightMesh.createVolumetricSurface (elems);
48      ptorus.addMeshComp (rightMesh);
49
```

```
50        // Create a collision behavior and use it to enable self collisions for
51        // the FEM model. Since the model has low resolution and sharp edges, use
52        // VERTEX_EDGE_PENETRATION, which requires the AJL_CONTOUR collider type.
53        CollisionBehavior behav = new CollisionBehavior (true, 0);
54        behav.setMethod (CollisionBehavior.Method.VERTEX_EDGE_PENETRATION);
55        behav.setColliderType (CollisionManager.ColliderType.AJL_CONTOUR);
56        mech.setCollisionBehavior (ptorus, ptorus, behav);
57
58        // render properties: render the torus using element widgets
59        ptorus.setElementWidgetSize (0.8);
60        RenderProps.setFaceColor (ptorus, new Color(.4f, 1f, .6f));
61        // enable rendering of the left and right contact meshes
62        leftMesh.setSurfaceRendering (SurfaceRender.Shaded);
63        rightMesh.setSurfaceRendering (SurfaceRender.Shaded);
64        RenderProps.setFaceColor (leftMesh, new Color(.78f, .78f, 1f));
65        RenderProps.setFaceColor (rightMesh, new Color(.78f, .78f, 1f));
66     }
67  }
```

The model creates an FEM model based on an open torus, using a factory method in FemFactory, and rotates it so that the gap is located at the bottom (lines 7-18). The torus is then anchored by fixing the nodes located at the top-center (lines 20-25). Next, mesh components are created to enforce self-collision at the left and right gap end points (lines 27-47). First, a FemMeshComp is created (Section 6.3), and then its `createVolumetricSurface()` method is used to create a local surface mesh wrapping the elements specified in `elems`. When selecting the elements, we use the convenient fact that for this particular FEM model, the elements near the left and right ends have numbers in the ranges 180…199 and 0…19, respectively.

Once the submeshes have been added to the FEM model, we create a collision behavior and use it to enable self-collisions (lines 49-55). An explicit behavior is created so that we can enable the `VERTEX_EDGE_PENETRATION` contact method (Section 8.4.1), because the meshes are coarse and the additional edge-edge collisions will improve behavior; this method also requires the `AJL_CONTOUR` collider type. While self-collision is enabled by calling

```
mech.setCollisionBehavior (ptorus, ptorus, behav);
```

it could also have been enabled by calling

```
mech.setCollisionBehavior (ptorus, Collidable.Self, behav);

... OR ...

mech.setCollisionBehavior (leftMesh, rightMesh, behav);
```

Note that there is no need to set the collidable property of the collision meshes since it is set to `INTERNAL` by default when they are added to the FEM model.

Render properties are set at lines 57-64, with the torus rendered as light green and the collision meshes as blue-gray. The torus is drawn using its elements widgets instead of its surface mesh, to prevent the latter from obscuring the collision meshes.

To run this example in ArtiSynth, select All demos > tutorial > FemSelfCollide from the Models menu. Running the model will cause the FEM model to self-collide as shown in Figure 8.5.

### 8.3.4 Collidable bodies

As mentioned in Section 8.3.2, non-compound collidables, including both solitary collidables and leaf nodes of a compound collidable's tree, are also instances of CollidableBody, which provide the actual collision meshes and other information used to compute the collision response. This is done using various methods, including:

- `PolygonalMesh getCollisionMesh()`

  Returns the actual surface mesh to be used for computing collisions.

- boolean hasDistanceGrid()

  If this method returns `true`, the body also maintains a signed distance grid for the mesh, which can be used by the collider type `SIGNED_DISTANCE` (Section 8.4.1).

- `DistanceGridComp getDistanceGridComp()`

  If `hasDistanceGrid()` returns `true`, this method return the distance grid.

- `double getMass()`

  Returns the mass of this body (or a suitable estimate), for use in automatically computing certain collision parameters.

- `int getCollidableIndex()`

  Returns the index of this collidable body within the set of all `CollidableBody`s in the `MechModel`. The index is determined by the body's depth-first ordering within the model component hierarchy. For components within the same component list, this ordering will be determined by the order in which the components are added in the model's `build()` method.

### 8.3.5 Nested MechModels

It is possible in ArtiSynth for one `MechModel` to contain other nested `MechModel`s within its component hierarchy. This raises the question of how collisions within the nested models are controlled. The general rule for this is the following:

> The collision behavior for two collidables `colA` and `colB` is determined by whatever behavior (either default or override) is specified by the lowest `MechModel` in the component hierarchy that contains both `colA` and `colB`.

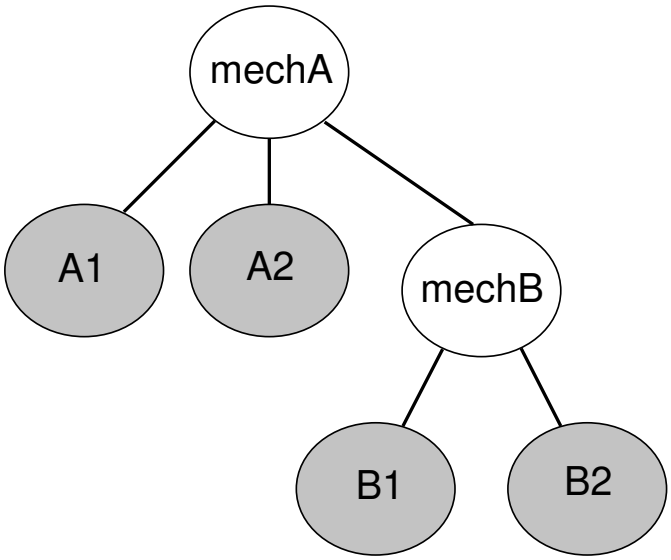



Figure 8.6: A `MechModel` containing collidables `B1` and `B2`, nested within another containing collidables `A1` and `A2`.

For example, consider Figure 8.6 where we have a `MechModel` (`mechB`) containing collidables `B1` and `B2`, nested within another `MechModel` (`mechA`) containing collidables `A1` and `A2`. Then consider the following code fragment:

```
mechB.setDefaultCollisionBehavior (true, 0.2);
mechA.setCollisionBehavior (B1, Collidable.AllBodies, true, 0.0);
mechA.setCollisionBehavior (B1, A1, false);
mechA.setCollisionBehavior (B1, B2, false); // Error!
```

The first line enables default collisions within `mechB` (with $\mu = 0$), controlling the interaction between `B1` and `B2`. However, collisions are still disabled within `mechA`, meaning `A1` and `A2` will not collide either with each other or with `B1` or `B2`. The second line enables collisions between `B1` and any other body within `mechA` (i.e., `A1` and `A2`), while the third

line disables collisions between B1 and A1. Finally, the fourth line results in an error, since B1 and B2 are both contained within mechB and so their collision behavior cannot be controlled from mechA.

While it is not legal to specify a specific behavior for collidables contained in a MechModel from a higher level MechModel, it *is* legal to create collision response components for the same pair within both models. So the following code fragment would be allowed and would create response components in both mechA and mechB:

```
mechB.setCollisionResponse (B1, B2);
mechA.setCollisionResponse (B1, B2);
```

## 8.4 Implementation

This section describes the technical details of the different ways in which collision are implemented in ArtiSynth. Knowledge of these details can be useful for choosing collision behavior property settings, understanding elastic foundation contact (Section 8.7.3), and interpreting the information provided by collision responses (Section 8.8.1) and collision rendering (Section 8.5).

The collision mechanism works by using a *collider* (described further below) to locate the *contact regions* where the (triangular) collision meshes of different collidables intersect each other. Figure 8.7 shows four contact regions between two meshes representing human teeth. Information about each contact region is then used to generate *contact constraints*, according to a *contact method*, that prevent further collision and resolve any existing interpenetration.

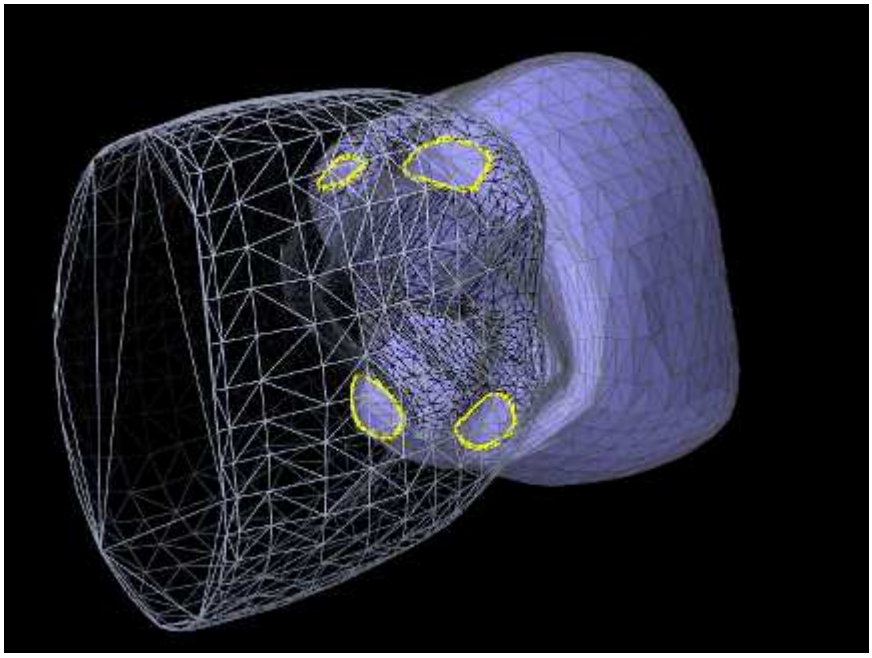


Figure 8.7: Collisions between collidables are found by locating contact regions (yellow contours) between their surface meshes.

### 8.4.1 Contact methods

Contact constraints are computed from mesh contact regions using one of two primary contact methods:

#### 8.4.1.1 Contour region

Contact constraints are generated for each contact region by first fitting a plane to the region, and then selecting contact points around its perimeter such that their 2D projection into the plane forms a convex hull (Figure 8.8). A contract constraint is assigned to each contact point, with a constraint direction parallel to the plane normal and a penetration

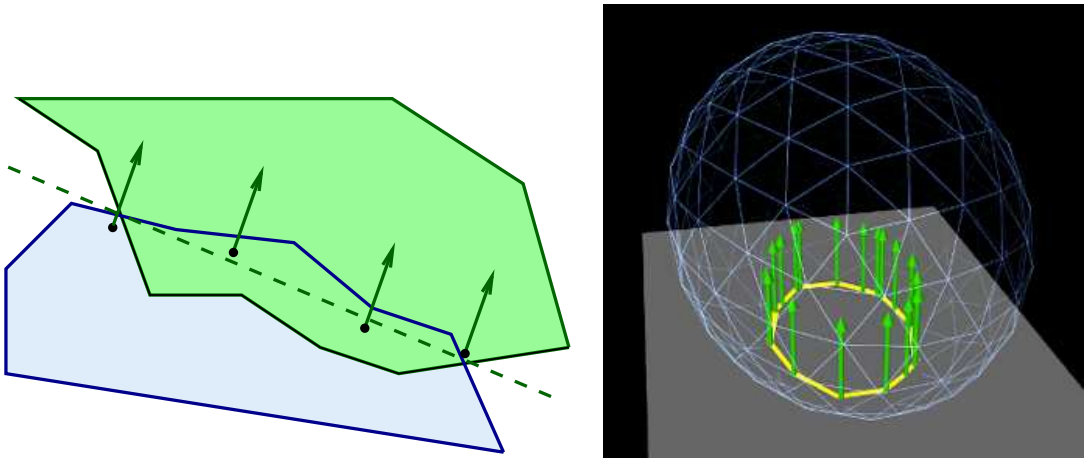


Figure 8.8: Contour region contact fits a plane to a contact region, and then chooses contact points around the region's perimeter such that their projection into the plane forms a convex hull.

distance given by the maximum interpenetration of the region. Note that the contact points do not necessarily lie *on* the plane, but they all have the same normal and penetration distance.

This is the default method used between rigid bodies, since it does not depend on mesh resolution, and rigid body contact behavior depends only on the convex hull of the contacting regions. Since the number of contact constraints may exceed the number of degrees of freedom (DOF) of the contacting bodies, the constraints are supplied to the physics solver as unilateral constraints (Section 1.2). Using contact region constraints when one of the collidables is an FEM model will result in an error.

#### 8.4.1.2 Vertex penetration

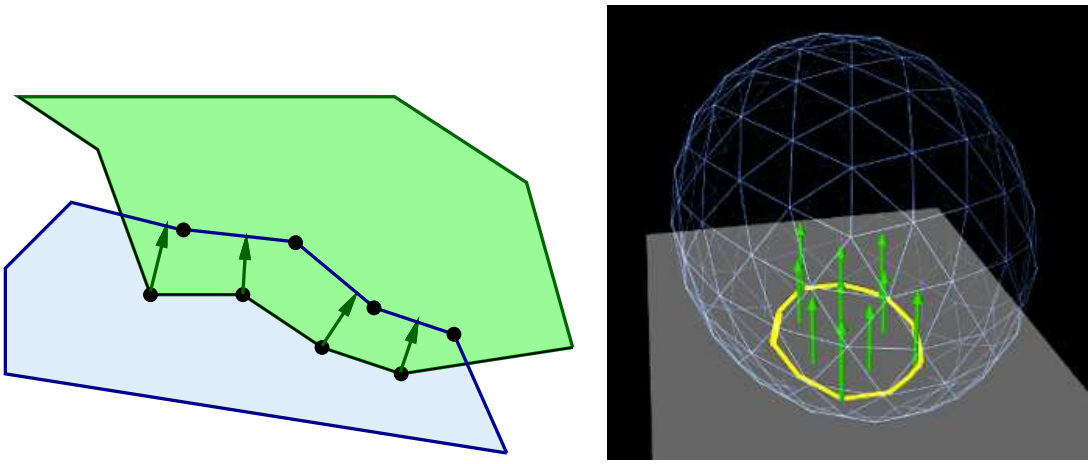


Figure 8.9: Vertex penetration contact creates a contact point for each penetrating vertex within a penetration region, with the contact normal directed toward to the nearest point on the opposite mesh.

Contacts are created for each mesh vertex that penetrates the other mesh, with a contact normal directed toward the nearest point on the opposing mesh (Figure 8.9). Contact constraints are generated for each contact, with the constraint direction parallel to the normal and the penetration distance equal to the distance to the opposing mesh.

This is the default contact method when one or both of the collidables is an FEM model. Also by default, if only one of the collidables is an FEM model, vertex penetrations are computed *only* for the FEM collidable; penetrations of the rigid body into the FEM are not considered. Otherwise, if both collidables are FEM models, then vertex penetrations are computed for *both* bodies, resulting in *two-way* contact (Figure 8.10).

Vertex penetration contact results in contact reaction forces being generated on the mesh vertices that are within or next to the intersection region (Figure 8.9). These appear on vertices of *both* intersecting meshes, regardless of whether

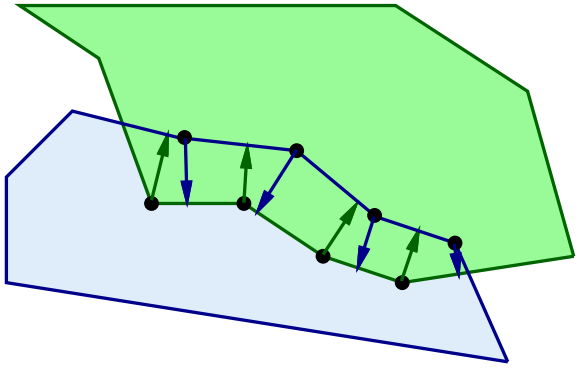


Figure 8.10: Two-way vertex penetration contact involves creating contact points from the penetrating vertices of *both* meshes.

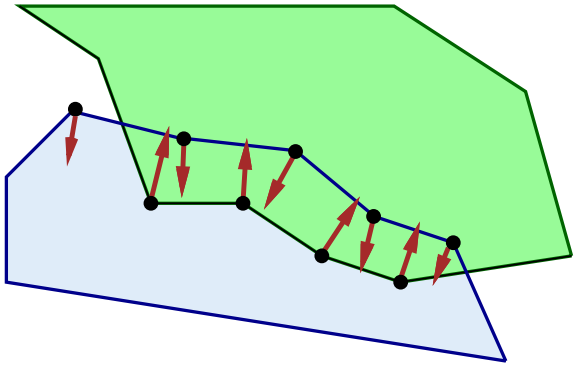


Figure 8.11: Vertex penetration contact generates contact reaction forces (red-brown arrows) on vertices of both intersecting meshes.

contact points are generated for one or both meshes, since each vertex associated with a contact imparts a reaction force on the vertices of the nearest face on the opposite mesh.

Because FEM models tend to have a large number of DOFs, the default behavior is to supply vertex penetration constraints to the physics solver as bilateral constraints (Section 1.2), removing them only between simulation time steps when the contact disappears or a separating force is detected. This results in much faster simulation times, particularly for FEM models, because it does not require solving a linear complementarity problem. However, this also means that using vertex penetration between rigid bodies, or other bodies with a low number of degrees of freedom, will typically result in an overconstrained system that must be handled using one of the techniques described in Section 8.6).

#### 8.4.1.3 Setting the contact method

Contact methods can be specified by the method property in either the collision manager or behavior (Section 8.2.1). The property's value is an instance of CollisionBehavior.Method, for which the standard settings are:

**CONTOUR_REGION**

Contact constraints are generated using contour regions, as described in Section 8.4.1.1. Setting this for contact that involves one or more FEM models will result in an error.

**VERTEX_PENETRATION**

Contact constraints are generated using vertex penetration, as described in Section 8.4.1.2. Setting this for contact between rigid bodies may result in an overconstrained system.

**VERTEX_EDGE_PENETRATION**

An experimental version of vertex penetration contact that supplements vertex constraints with extra constraints formed from edge-edge contact. Intended for collision meshes with low mesh resolutions and sharp edges. Available only when using the `AJL_CONTOUR` collider type (Section 8.4.2).

**DEFAULT**

Selects `CONTOUR_REGION` when both collidables are rigid bodies and `VERTEX_PENETRATION` otherwise.

**INACTIVE**

No constraints are generated. This will result in no collision response.

When vertex penetration contact is employed, the collidables for which penetrations are generated can also be controlled using the vertexPenetrations property, whose value is an instance of CollisionBehavior.VertexPenetrations:

**BOTH_COLLIDABLES**

Vertex penetrations are computed for both collidables.

**FIRST_COLLIDABLE**

Vertex penetrations are computed for the first collidable.

**SECOND_COLLIDABLE**

Vertex penetrations are computed for the second collidable.

**AUTO**

Vertex penetrations are determined automatically as follows: for both collidables if neither is a rigid body, for the non-rigid collidable if one collidable is a rigid body, and for the first collidable otherwise.

### 8.4.2 Collider types

The contact regions used by the contact method are generated by a collider, which is specified by the colliderType property in either the collision manager (as the default), or in the collision behavior for a specific pair of collidables. The property's value is an instance of CollisionManager.ColliderType, which describes the three collider types presently supported:

**TRI_INTERSECTION**

The original ArtiSynth collider which uses a bounding-box hierarchy to locate all triangle intersections between the two surface meshes. Contact regions are then estimated by grouping the intersection points together, and penetrating vertices are computed separately using point-mesh distance queries based on the bounding-box hierarchy. This latter step requires iterating over all mesh vertices, which may be slow for large meshes. `TRI_INTERSECTION` can often handle intersections near the edges of an open mesh, but does not support the `VERTEX_EDGE_PENETRATION` contact method.

**AJL_CONTOUR**

A bounding-box hierarchy is used to locate all triangle intersections between the two surface meshes, and the intersection points are then connected, using robust geometric predicates, to find the (piecewise linear) intersection contours. The contours are then used to identity the contact regions on each surface mesh, which are in turn used to determine the interpenetrating vertices and contact area. Intersection contours and the contact constraints generated from them are shown in Figure 8.12. `AJL_CONTOUR` supports the `VERTEX_EDGE_PENETRATION` contact method, but usually *cannot* handle intersections near the edges of an open mesh.

**SIGNED_DISTANCE**

Uses a grid-based signed distance field on one mesh to quickly determine the penetrating vertices of the opposite mesh, along with the penetration distance and normals. This is only available for collidable pairs where at least one of the bodies maintains a signed distance grid which can be obtained using getDistanceGridComp(). Advantages include speed (sometimes an order of magnitude faster than the other colliders) and the fact that the opposite mesh does *not* have to be triangular or manifold. However, signed distance fields can (at present) only be computed for fixed meshes, and so at least one colliding body must be rigid. The signed distance field also does not yet yield contact region information, and so the contact method is restricted to VERTEX_PENETRATION. Contacts generated from a signed distance field are illustrated in Figure Figure 8.13.

> Because the SIGNED_DISTANCE collider type currently supports only the VERTEX_PENETRATION method, its use between rigid bodies, or low DOF deformable bodies, will generally result in an overconstrained system unless measures are taken as described in Section 8.6.

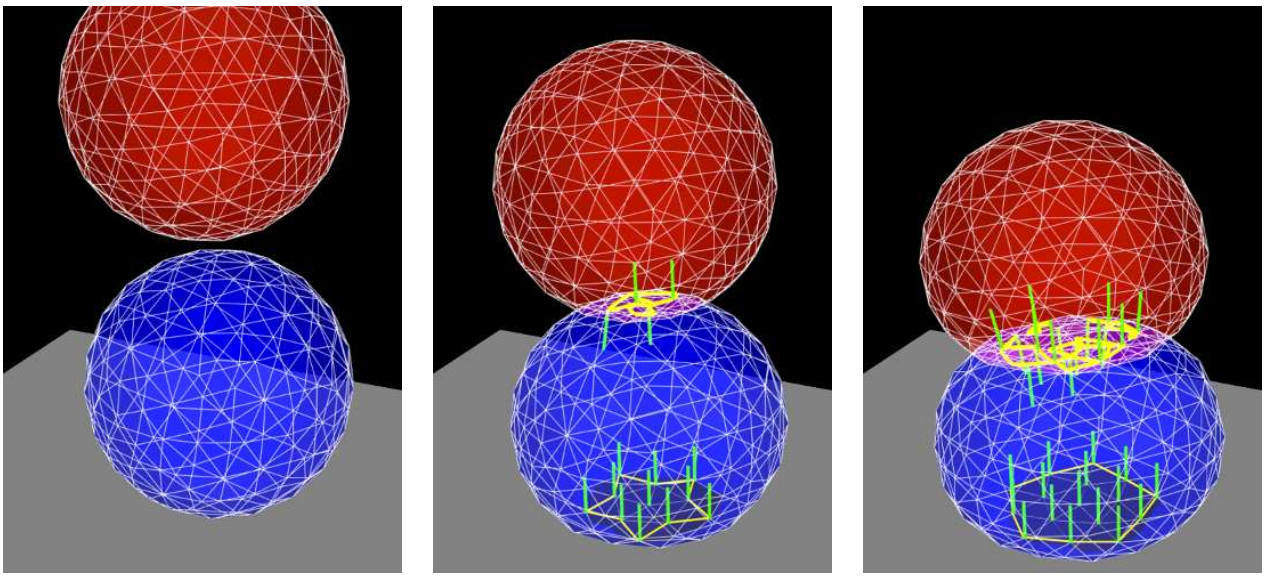



Figure 8.12: Time sequence of contact handling between two deformable models falling under gravity, showing the intersection contours (yellow) and the contact normals (green lines).

#### 8.4.2.1 Collision meshes and signed distance grids

As described in Section 8.3.4, collidable bodies declare the method getCollisionMesh(), which returns a mesh defining the collision surface. This is either used directly, or, for the SIGNED_DISTANCE collider, used to compute a signed distance grid which in turn in used to handle collisions.

For RigidBody objects, the mesh returned by getCollisionMesh() is typically the same as the surface mesh. However, it is possible to change this, by adding additional meshes to the body and modifying the meshes' isCollidable property (see Section 8.3). The collision mesh is then formed as the sum of all polygonal meshes in the body's meshes list whose isCollidable property is true.

Collidable bodies also declare the methods hasDistanceGrid() and getDistanceGridComp(). If the former returns true, then the body has a distance grid and the latter returns a DistanceGridComp containing it. A distance grid is a regular 3D grid, with uniformly arranged vertices and cells, that is used to represent a scalar distance field, with distance values stored implicitly at each vertex and interpolated within cells. Distance grids are used by the SIGNED_DISTANCE collider, and by default are generated on-demand, using an automatically chosen resolution and the mesh returned by getCollisionMesh() as the surface against which distances are computed.

When used for collision handling, values within a distance grid are interpolated trilinearly within each cell. This means that the effective collision surface is actually the trilinearly interpolated isosurface corresponding to a distance of 0. This surface will differ somewhat from the original surface returned by getCollisionMesh(), in a manner that depends on the grid resolution. Consequently, when using the SIGNED_DISTANCE collider, it is important to be able to visualize the trilinear isosurface, and possibly modify it by adjusting the grid resolution. The DistanceGridComp returned by getDistanceGridComp() exports properties to facilitate both of these requirements, as described in detail in Section 4.6.

## 8.5 Contact rendering

As mentioned in Section 8.2.1, additional collision behavior properties exist to enable and control the rendering of contact artifacts. These include intersection contours, contact points and normals, contact forces and pressures, and

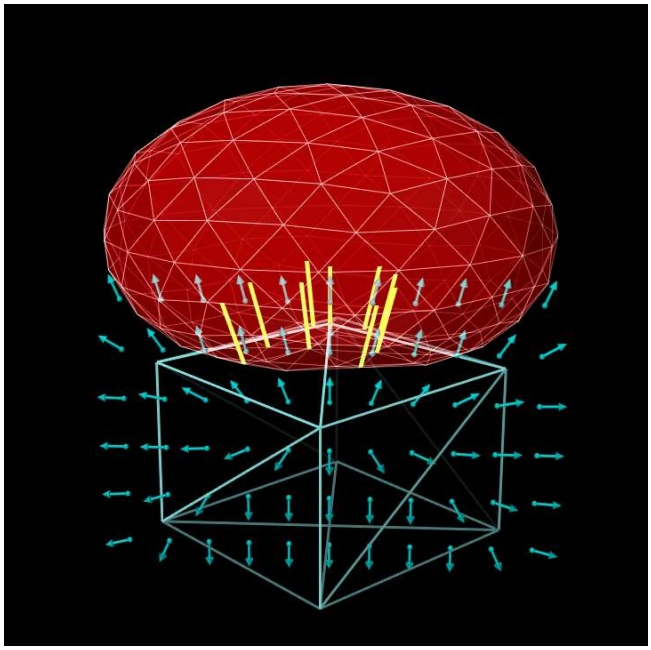


Figure 8.13: Contacts generated by a signed distance field. A deformable ellipsoid (red, top) is colliding with a solid prism (cyan, bottom). A signed distance field for the prism (the grid points and normals of which are shown partially by the dark cyan arrows) is used to locate penetrating vertices of the ellipsoid and hence generate contact constraints (yellow lines).

penetration depths. The properties that control these are supplemented by generic render properties (Section 4.3) to specify colors, line widths, etc.

By default, contact rendering is disabled. To enable it, one must set the collision manager to be *visible*, which can be done using a code fragment like the following:

```
RenderProps.setVisible (mechModel.getCollisionManager(), true);
```

CollisionManager and CollisionBehavior both export contact rendering properties. Setting these in the former controls rendering globally for all collisions, while setting them in the latter controls rendering for the collidable pair associated with the behavior. Some artifacts, like contact normals, forces, and color maps, must be drawn with respect to either the first or second collidable. By default, the first collidable is chosen, but the second collidable can be requested using the renderingCollidable property described below. Normals are drawn *toward*, and forces drawn such that they are acting *on*, the indicated collidable. For collision behaviors specified using the `setCollisionBehavior()` methods, the first and second collidables correspond to the `collidable0` and `collidable1` arguments. Otherwise, for default collision behaviors, the first collidable will be the one whose collidable bodies have the lowest collidable index (Section 8.3.4).

Properties exported by both `CollisionManager` and `CollisionBehavior` include:

**renderingCollidable**

Integer value of either 0 or 1 specifying the collidable for which forces, normals and color maps should be drawn. 0 or 1 selects either the first or second collidable. The default value is 0.

**drawIntersectionContours**

Boolean value requesting drawing of the intersection contours between collidable meshes, using the generic render properties edgeWidth and edgeColor. The default value is `false`.

**drawIntersectionPoints**

Boolean value requesting drawing of the interpenetrating vertices on each collidable mesh, using the generic render properties pointStyle, pointSize, pointRadius, and pointColor. The default value is `false`.

**drawContactNormals**

Boolean value requesting drawing of the normals associated with each contact constraint, using the generic render properties lineStyle, lineRadius, lineWidth, and lineColor. The default value is `false`. The length of the normals is controlled by the contactNormalLen property of the collision manager, which will be set to an appropriate default value if not set explicitly.

**drawContactForces**

Boolean value requesting drawing of the forces associated with each contact constraint, using the generic render properties lineStyle, lineRadius, edgeWidth, and edgeColor (or lineColor if edgeColor is `null`). The default value is `false`. The forces are drawn as line segments starting at each contact point and running parallel to the contact normal, with a length given by the current contact impulse value multiplied by the contactForceLenScale property of the collision manager (which has a default value of 1). The reason for using edgeWidth and edgeColor instead of lineWidth and lineColor is to allow the application to set the render properties such that both normals and forces can be visible if both are being rendered at the same time.

**drawFrictionForces**

Boolean value requesting the drawing of the forces associated with each contact's friction force, in the same manner and style as for drawContactForces. The default value is `false`. Note that unlike contact forces, friction forces are *perpendicular* to the contact normal.

**drawColorMap**

An enum of type CollisionBehavior.ColorMapType requesting that a color map be drawn over the contact regions showing a scalar value such as penetration depth or contact force pressure. The collidable (0 or 1) onto which the color map is drawn is controlled by the renderingCollidable property (described below). The range of values used for generating the map is controlled by the colorMapRange property of either the collision manager or the behavior, as described below. The values are mapped onto colors using the colorMap property of the collision manager.

Values of CollisionBehavior.ColorMapType include:

**NONE**

The color map is disabled. This is the default value.

**PENETRATION_DEPTH**

The color map shows penetration depth of one collidable mesh with respect to the other. If one or both collidable meshes are open, then the colliderType should be set to `AJL_CONTOUR` (Section 8.2.1).

**CONTACT_PRESSURE**

The color map shows the contact pressures over the contact region. Contact pressures are determined by examining the forces at the contact constraints and then distributing these over the surrounding faces. This information is most useful and accurate when using vertex penetration contact (Section 8.4.1.2).

The color map itself is drawn as a patch formed from the collidable's collision mesh, using faces and vertices associated with the collision region. The vertices of the patch are set to colors corresponding to the associated value (e.g., penetration depth or pressure) at that vertex, and the surrounding faces are colored appropriately. The resolution of the color map is thus determined by the resolution of the collision mesh, and so the application must ensure this is high enough to ensure proper results. If the mesh has only a few triangles (e.g., a rectangle with two triangles per face), the color interpolation may be spread over an unreasonably large area. Examples of color map usage are given in Sections 8.5.2 and 8.5.3.

**colorMapRange**

Composite property of the type ScalarRange that controls the range of values used for color map rendering. Subproperties include interval, which gives the value range itself, and updating, which specifies how this interval should be updated: `FIXED` (no updating), `AUTO_EXPAND` (interval is expanded as values increase or decrease), and `AUTO_FIT` (interval is reset to fit the values at every render step).

When the colorMapRange value for a `CollisionBehavior` is set to `null` (which is the default), the colorMapRange of the `CollisionManager` is used instead. This has the advantage of ensuring that the same color map range will be used across all collision interactions.

**colorMapInterpolation**

An enum of type Renderer.ColorInterpolation that explicitly specifies how colors in any rendered color map should be interpolated. `RGB` and `HSV` (the default) specify interpolation in RGB and HSV space, respectively. HSV interpolation is the default as it is generally better suited to rendering maps that are purely color-based.

In addition, the following properties are exported by the collision manager:

**contactNormalLen**

Double value specifying the length with which contact normals should be drawn when drawContactNormals is `true`. If unspecified, a default value is calculated by the system.

**contactForceLenScale**

Double value specifying the scaling factor for contact or friction force vectors when they are drawn in response to drawContactForces drawFrictionForces being `true`. The default value is 0.

**colorMap**

Composite property of type ColorMapBase specifying the actual color map used for color map rendering. The default value is HueColorMap.

Generic render properties within the collision manager can be set in the same way as the visibility, using the `RenderProps` methods presented in Section 4.3.2:

```
Renderable cm = mechModel.getCollisionManager();
RenderProps.setEdgeWidth (cm, 2);
RenderProps.setEdgeColor (cm, Color.Red);
```

As mentioned above, generic render properties can also be set individually for specific behaviors. This can be done using code fragments like this:

```
CollisionBehavior behav = mechModel.getCollisionBehavior (bodA, bodB);
RenderProps.setLineWidth (behav, 2);
RenderProps.setLineColor (behav, Color.Blue);
```

To access these properties on a read-only basis, one can do

```
RenderProps props = mechModel.getCollisionManager().getRenderProps();

... OR ...

RenderProps props = behav.getRenderProps();
```

Because of the manner in which ArtiSynth handles contacts, rendered contact information may sometimes appear to lag the simulation by one time step. That is because contacts are computed at the *end* of each time step $i$, and then used to compute the contact forces during the next step $i+1$. The contact information displayed at the end of step $i$ is thus based on contacts detected at the end of step $i-1$, along with contact forces that are computed during step $i$ and used to calculate the updated positions and velocities at the end of that step.

### 8.5.1 Example: rendering normals and contours

A simple model illustrating contact rendering is defined in

`artisynth.demos.tutorial.BallPlateCollide`

This shows a ball colliding with a plate, while rendering the resulting contact normals in red and the intersection contours in blue. This demo also allows the user to experiment with compliant contact (Section 8.7.1) by setting compliance and damping properties in a control panel.

The complete source code is shown below:

```
1 package artisynth.demos.tutorial;
2
3 import java.awt.Color;
4
5 import artisynth.core.gui.ControlPanel;
```

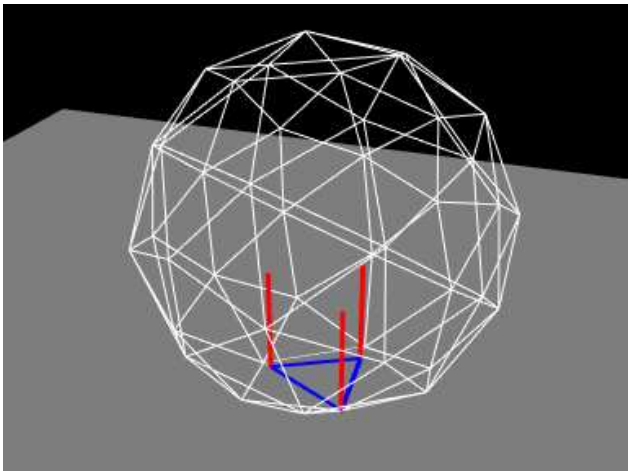


Figure 8.14: BallPlateCollide showing contact normals (red) and collision contour (blue) of the ball colliding with the plate.

```
import artisynth.core.mechmodels.CollisionManager;
import artisynth.core.mechmodels.MechModel;
import artisynth.core.mechmodels.RigidBody;
import artisynth.core.workspace.RootModel;
import maspack.matrix.RigidTransform3d;
import maspack.render.RenderProps;
import maspack.render.Renderer;

public class BallPlateCollide extends RootModel {

   public void build (String[] args) {

      // create MechModel and add to RootModel
      MechModel mech = new MechModel ("mech");
      addModel (mech);

      // create and add the ball and plate
      RigidBody ball = RigidBody.createIcosahedralSphere ("ball", 0.8, 0.1, 1);
      ball.setPose (new RigidTransform3d (0, 0, 2, 0.4, 0.1, 0.1));
      mech.addRigidBody (ball);
      RigidBody plate = RigidBody.createBox ("plate", 5, 5, 0.4, 1);
      plate.setDynamic (false);
      mech.addRigidBody (plate);

      // turn on collisions
      mech.setDefaultCollisionBehavior (true, 0.20);

      // make ball transparent so that contacts can be seen more clearly
      RenderProps.setFaceStyle (ball, Renderer.FaceStyle.NONE);
      RenderProps.setShading (ball, Renderer.Shading.NONE);
      RenderProps.setDrawEdges (ball, true);
      RenderProps.setEdgeColor (ball, Color.WHITE);

      // enable rendering of contacts normals and contours
      CollisionManager cm = mech.getCollisionManager();
      RenderProps.setVisible (cm, true);
      RenderProps.setLineWidth (cm, 3);
      RenderProps.setLineColor (cm, Color.RED);
      RenderProps.setEdgeWidth (cm, 3);
      RenderProps.setEdgeColor (cm, Color.BLUE);
      cm.setDrawContactNormals (true);
      cm.setDrawIntersectionContours (true);

```

```
49        // create a control panel to allow contact regularization to be set
50        ControlPanel panel = new ControlPanel();
51        panel.addWidget (mech.getCollisionManager(), "compliance");
52        panel.addWidget (mech.getCollisionManager(), "damping");
53        addControlPanel (panel);
54     }
55  }
```

The `build()` method starts by creating and adding a `MechModel` in the usual way (lines 19-20). The ball and plate are both created as rigid bodies (lines 22-28), with the ball pose set so that its origin is above the plate at (0, 0, 2) and its orientation is perturbed so that it will not land on the plate symmetrically (line 24). Collisions between the ball and plate are enabled at line 31, with a friction coefficient of 0.2. To allow better visualization of the contacts, the ball is made transparent by disabling the drawing of faces, and instead enabling the drawing of edges in white with no shading (lines 33-37).

Rendering of contacts and normals is established by setting the render properties of the collision manager (lines 39-47). First, the collision manager is set visible (which it is not by default). Then lines (used to render the contact normals) and edges (used to render to intersection contour) are set to red and blue, each with a pixel width of 3. Drawing of the normals and contour is enabled at lines 46-47.

Lastly, for interactively controlling contact compliance (Section 8.7.1), a control panel is built to allow users to adjust the collision manager's compliance and damping properties (lines 49-53).

To run this example in ArtiSynth, select All demos > tutorial > BallPlateCollide from the Models menu. When run, the ball will collide with the plate and the contact normals and collision contours will be drawn as shown in Figure 8.14.

To enable compliant contact, set the compliance to a non-zero value. A value of 0.001 (which corresponds to a contact stiffness of 1000) causes the ball to bounce considerably when it lands. To counteract this bouncing, the damping should be set to a non-zero value. Since the ball has a mass of 0.21, formula (8.2) suggests that critical damping (for which $\zeta = 1$) can be achieved with $D \approx 30$. This does in fact stop the bouncing. Increasing the compliance to 0.01 results in the ball penetrating the plate by a noticeable amount.

### 8.5.2 Example: rendering a color map

As described above, it is possible to use the drawColorMap property of the collision behavior to render a color map over the contact area showing a scalar value such as penetration depth or contact pressure. A simple example of this is defined in

```
artisynth.demos.tutorial.PenetrationRender
```

which sets drawColorMap to `PENETRATION_DEPTH` in order to display the penetration depth of one hemispherical mesh with respect to another.

> As mentioned above, proper results require that the collision mesh for the collidable on which the map is being drawn has a sufficiently high resolution.

The complete source code is shown below:

```
1  package artisynth.demos.tutorial;
2
3  import java.awt.Color;
4
5  import maspack.geometry.PolygonalMesh;
6  import maspack.geometry.MeshFactory;
7  import maspack.matrix.RigidTransform3d;
8  import maspack.render.*;
9  import maspack.render.Renderer.FaceStyle;
10 import maspack.render.Renderer.Shading;
11 import artisynth.core.mechmodels.*;
12 import artisynth.core.mechmodels.CollisionManager.ColliderType;
13 import artisynth.core.mechmodels.CollisionBehavior.ColorMapType;
```

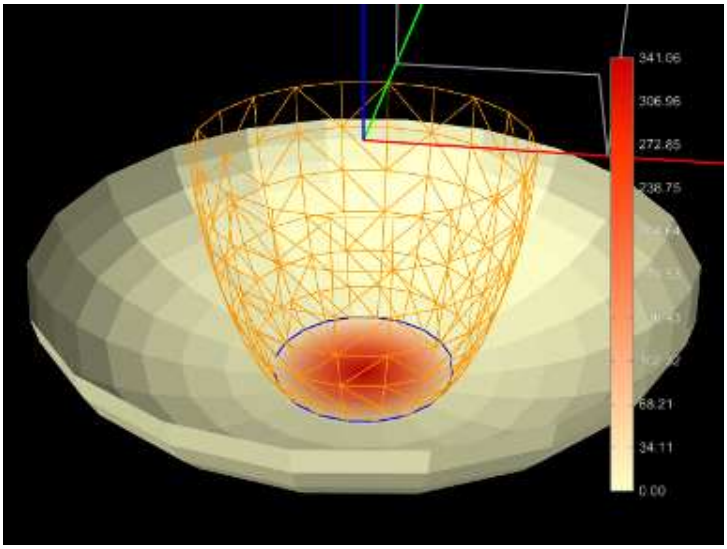



Figure 8.15: PenetrationRender showing the penetration depth of the bottom mesh with respect to the top, with red indicating greater depth. A translation dragger fixture at the top is being used to move the top mesh around, while the penetration range and associated colors are displayed on the color bar at the right.

```
14 import artisynth.core.util.ScalarRange;
15 import artisynth.core.workspace.RootModel;
16 import artisynth.core.renderables.ColorBar;
17 import maspack.render.color.JetColorMap;
18
19 public class PenetrationRender extends RootModel {
20
21    // Convenience method for creating colors from [0-255] RGB values
22    private static Color createColor (int r, int g, int b) {
23       return new Color (r/255.0f, g/255.0f, b/255.0f);
24    }
25
26    private static Color CREAM = createColor (255, 255, 200);
27    private static Color GOLD = createColor (255, 150, 0);
28
29    // Creates and returns a rigid body built from a hemispherical mesh.  The
30    // body is centered at the origin, has a radius of 'rad', and the z axis is
31    // scaled by 'zscale'.
32    RigidBody createHemiBody (
33       MechModel mech, String name, double rad, double zscale, boolean flipMesh) {
34
35       PolygonalMesh mesh = MeshFactory.createHemisphere (
36          rad, /*slices=*/20, /*levels=*/10);
37       mesh.scale (1, 1, zscale); // scale mesh in the z direction
38       if (flipMesh) {
39          // flip upside down is requested
40          mesh.transform (new RigidTransform3d (0, 0, 0, 0, 0, Math.PI));
41       }
42       RigidBody body = RigidBody.createFromMesh (
43          name, mesh, /*density=*/1000, /*scale=*/1.0);
44       mech.addRigidBody (body);
45       body.setDynamic (false);  // body is only parametrically controlled
46       return body;
47    }
48
49    // Creates and returns a ColorBar renderable object
50    public ColorBar createColorBar() {
51       ColorBar cbar = new ColorBar();
52       cbar.setName("colorBar");
```

```
53        cbar.setNumberFormat("%.2f");       // 2 decimal places
54        cbar.populateLabels(0.0, 1.0, 10); // Start with range [0,1], 10 ticks
55        cbar.setLocation(-100, 0.1, 20, 0.8);
56        cbar.setTextColor (Color.WHITE);
57        addRenderable(cbar);                // add to root model's renderables
58        return cbar;
59     }
60
61     public void build (String[] args) {
62        MechModel mech = new MechModel ("mech");
63        addModel (mech);
64
65        // create first body and set its rendering properties
66        RigidBody body0 = createHemiBody (mech, "body0", 2, -0.5, false);
67        RenderProps.setFaceStyle (body0, FaceStyle.FRONT_AND_BACK);
68        RenderProps.setFaceColor (body0, CREAM);
69
70        // create second body and set its pose and rendering properties
71        RigidBody body1 = createHemiBody (mech, "body1", 1, 2.0, true);
72        body1.setPose (new RigidTransform3d (0, 0, 0.75));
73        RenderProps.setFaceStyle (body1, FaceStyle.NONE); // set up
74        RenderProps.setShading (body1, Shading.NONE);     // wireframe
75        RenderProps.setDrawEdges (body1, true);           // rendering
76        RenderProps.setEdgeColor (body1, GOLD);
77
78        // create and set a collision behavior between body0 and body1, and make
79        // collisions INACTIVE since we only care about graphical display
80        CollisionBehavior behav = new CollisionBehavior (true, 0);
81        behav.setMethod (CollisionBehavior.Method.INACTIVE);
82        behav.setDrawColorMap (ColorMapType.PENETRATION_DEPTH);
83        behav.setRenderingCollidable (1); // show penetration of mesh 0
84        mech.setCollisionBehavior (body0, body1, behav);
85
86        CollisionManager cm = mech.getCollisionManager();
87        // works better with open meshes if AJL_CONTOUR is selected
88        cm.setColliderType (ColliderType.AJL_CONTOUR);
89        // set other rendering properities in the collision manager:
90        RenderProps.setVisible (cm, true);    // enable collision rendering
91        cm.setDrawIntersectionContours(true); // draw contours ...
92        RenderProps.setEdgeWidth (cm, 3);     // with a line width of 3
93        RenderProps.setEdgeColor (cm, Color.BLUE); // and a blue color
94        // create a custom color map for rendering the penetration depth
95        JetColorMap map = new JetColorMap();
96        map.setColorArray (
97           new Color[] {
98              CREAM,                         // no penetration
99              createColor (255, 204, 153),
100             createColor (255, 153, 102),
101             createColor (255, 102, 51),
102             createColor (255, 51, 0),
103             createColor (204, 0, 0),     // most penetration
104          });
105       cm.setColorMap (map);
106       // set color map range to "auto fit".
107       cm.setColorMapRange (new ScalarRange(ScalarRange.Updating.AUTO_FIT));
108
109       // create a separate color bar to show depth values associated with the
110       // color map
111       ColorBar cbar = createColorBar();
112       cbar.setColorMap (map);
113    }
114
115    public void prerender(RenderList list) {
116       super.prerender(list); // call the regular prerender method first
117
```

```
118         // Update the color bar labels based on the collison manager's
119         // color map range that was updated in super.prerender().
120         //
121         // Object references are obtained by name using 'findComponent'. This is
122         // more robust than using class member variables, since the latter will
123         // be lost if we save and restore this model from a file.
124         ColorBar cbar = (ColorBar)(renderables().get("colorBar"));
125         MechModel mech = (MechModel)findComponent ("models/mech");
126         RigidBody body0 = (RigidBody)mech.findComponent ("rigidBodies/body0");
127         RigidBody body1 = (RigidBody)mech.findComponent ("rigidBodies/body1");
128 
129         CollisionManager cm = mech.getCollisionManager();
130         ScalarRange range = cm.getColorMapRange();
131         cbar.updateLabels(0, 1000*range.getUpperBound());
132      }
133   }
```

To improve visibility, the example uses two rigid bodies, each created from an open hemispherical mesh using the method `createHemiBody()` (lines 32-47). Because this example is strictly graphical, the bodies are set to be non-dynamic so that they can be moved around using the viewer's graphical dragger fixtures (see the section "Transformer Tools" in the ArtiSynth User Interface Guide). Rendering properties for each body are set at lines 67-68 and 73-76, with the top body being rendered as a wireframe to improve visibility.

Lines 80-84 create and set a collision behavior between the two bodies with the drawColorMap property set to `PENETRATION_DEPTH`. Because for this example we want to show only the penetration and don't want a collision response, we set the contact method to be Method.INACTIVE. At line 88, the collider type used by the collision manager is set to ColliderType.AJL_CONTOUR, since this provides more reliable penetration calculations for open meshes. Other rendering properties are set for the collision manager at lines 90-107, including a custom color map that varies between `CREAM` (the color of the mesh) for no penetration and dark red for maximum penetration. The updating of the color map range in the collision manager is set to `AUTO_FIT` so that it will be recomputed at every time step. (Since the collision manager's color map range is set to "auto fit" by default, this is shown for illustrative purposes only. It is also possible to override the collision manager's color map range by setting the colorMapRange property in specific collision behaviors.)

At line 111, a color bar is created and added to the scene, using the method `createColorBar()` (lines 50-59), to explicitly show the depth that corresponds to the different colors. The color bar is given the same color map that is used to render the depth. Since the depth range is updated every time step, it is also necessary to update the corresponding labels in the color bar. This is done by overriding the root model's `prerender()` method (lines 115-132), where we obtain the color map range for the collision manager and use it to update the color bar labels. (Note that `super.prerender(list)` is called *first*, since the color map ranges are updated there.) References to the color bar, `MechModel`, and bodies are obtained using the CompositeComponent methods get() and findComponent(). This is more robust that storing these references in member variables, since the latter would be lost if the model is saved to and reloaded from a file.

To run this example in ArtiSynth, select All demos > tutorial > PenetrationRender from the Models menu. When run, the meshes will collide and render the penetration depth of the bottom mesh, as shown in Figure 8.15.

> When defining the color map for rendering (lines 99-108 in the example), it is recommended that the color corresponding to zero be set to the face color of the collidable mesh. This will allow the color map to blend properly to the regular mesh color.

### 8.5.3 Example: rendering contact pressures

Color map rendering can also be used to render contact pressures, which can be particularly useful for FEM models. A simple example is defined in

```
artisynth.demos.tutorial.ContactPressureRender
```

which sets the drawColorMap property of the collision behavior to `CONTACT_PRESSURE` in order to display a color map of the contact pressure. The example is similar to that of Section 8.5.2, which shows how to render penetration depth.

The caveats about color map rendering described in Section 8.5 apply. Pressure rendering is most useful and accurate when using vertex penetration contact. The resolution of the color map is limited by the resolution of the collision mesh for the collidable on which the map is drawn, and so the application should ensure that this is sufficiently fine. Also, to allow the map to blend properly with the rest of the collidable, the color corresponding to zero pressure should match the default face color for the collidable.

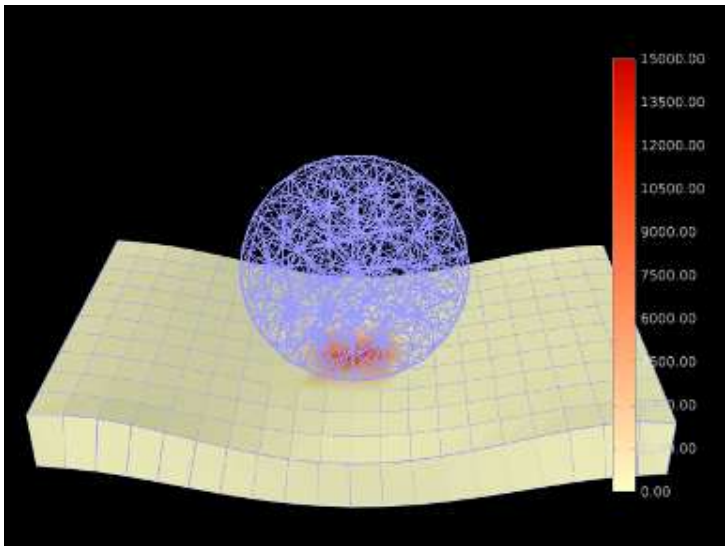



Figure 8.16: ContactPressureRender showing the contact pressure as a spherical FEM model falls onto an FEM sheet. The color map is drawn on the sheet model, with redder values indicating greater pressure.

The complete source code is shown below:

```
1  package artisynth.demos.tutorial;
2
3  import java.awt.Color;
4
5  import artisynth.core.femmodels.FemFactory;
6  import artisynth.core.femmodels.FemModel.SurfaceRender;
7  import artisynth.core.femmodels.FemModel3d;
8  import artisynth.core.femmodels.FemNode3d;
9  import artisynth.core.materials.LinearMaterial;
10 import artisynth.core.mechmodels.CollisionBehavior;
11 import artisynth.core.mechmodels.CollisionBehavior.ColorMapType;
12 import artisynth.core.mechmodels.CollisionManager;
13 import artisynth.core.mechmodels.MechModel;
14 import artisynth.core.renderables.ColorBar;
15 import artisynth.core.util.ScalarRange;
16 import artisynth.core.workspace.RootModel;
17 import maspack.matrix.RigidTransform3d;
18 import maspack.render.RenderList;
19 import maspack.render.RenderProps;
20 import maspack.render.color.JetColorMap;
21
22 public class ContactPressureRender extends RootModel {
23
24    double density = 1000;
25    double EPS = 1e-10;
26
27    // Convenience method for creating colors from [0-255] RGB values
28    private static Color createColor (int r, int g, int b) {
29       return new Color (r/255.0f, g/255.0f, b/255.0f);
30    }
31
```

```
32    private static Color CREAM = createColor (255, 255, 200);
33    private static Color BLUE_GRAY = createColor (153, 153, 255);
34
35    // Creates and returns a ColorBar renderable object
36    public ColorBar createColorBar() {
37       ColorBar cbar = new ColorBar();
38       cbar.setName("colorBar");
39       cbar.setNumberFormat("%.2f");       // 2 decimal places
40       cbar.populateLabels(0.0, 1.0, 10); // Start with range [0,1], 10 ticks
41       cbar.setLocation(-100, 0.1, 20, 0.8);
42       cbar.setTextColor (Color.WHITE);
43       addRenderable(cbar);                // add to root model's renderables
44       return cbar;
45    }
46
47    public void build (String[] args) {
48       MechModel mech = new MechModel ("mech");
49       addModel (mech);
50
51       // create FEM ball
52       FemModel3d ball = new FemModel3d("ball");
53       ball.setDensity (density);
54       FemFactory.createIcosahedralSphere (ball, /*radius=*/0.1, /*ndivs=*/2, 1);
55       ball.setMaterial (new LinearMaterial (100000, 0.4));
56       mech.addModel (ball);
57
58       // create FEM sheet
59       FemModel3d sheet = new FemModel3d("sheet");
60       sheet.setDensity (density);
61       FemFactory.createHexGrid (
62          sheet, /*wx*/0.5, /*wy*/0.3, /*wz*/0.05, /*nx*/20, /*ny*/10, /*nz*/1);
63       sheet.transformGeometry (new RigidTransform3d (0, 0, -0.2));
64       sheet.setMaterial (new LinearMaterial (500000, 0.4));
65       sheet.setSurfaceRendering (SurfaceRender.Shaded);
66       mech.addModel (sheet);
67
68       // fix the side nodes of the surface
69       for (FemNode3d n : sheet.getNodes()) {
70          double x = n.getPosition().x;
71          if (Math.abs(x-(-0.25)) <= EPS || Math.abs(x-(0.25)) <= EPS) {
72             n.setDynamic (false);
73          }
74       }
75
76       // create and set a collision behavior between the ball and surface.
77       CollisionBehavior behav = new CollisionBehavior (true, 0);
78       behav.setDrawColorMap (ColorMapType.CONTACT_PRESSURE);
79       behav.setRenderingCollidable (1); // show color map on collidable 1 (sheet);
80       behav.setColorMapRange (new ScalarRange(0, 15000.0));
81       mech.setCollisionBehavior (ball, sheet, behav);
82
83       CollisionManager cm = mech.getCollisionManager();
84       // set rendering properties in the collision manager:
85       RenderProps.setVisible (cm, true);    // enable collision rendering
86       // create a custom color map for rendering the penetration depth
87       JetColorMap map = new JetColorMap();
88       map.setColorArray (
89          new Color[] {
90             CREAM,                         // no penetration
91             createColor (255, 204, 153),
92             createColor (255, 153, 102),
93             createColor (255, 102, 51),
94             createColor (255, 51, 0),
95             createColor (204, 0, 0),       // most penetration
96          });
```

```
97        cm.setColorMap (map);
98
99        // create a separate color bar to show color map pressure values
100       ColorBar cbar = createColorBar();
101       cbar.updateLabels(0, 15000);
102       cbar.setColorMap (map);
103
104       // set color for all bodies
105       RenderProps.setFaceColor (mech, CREAM);
106       RenderProps.setLineColor (mech, BLUE_GRAY);
107    }
108 }
```

To begin, the demo creates two FEM models: a spherical ball (lines 52-56) and a rectangular sheet (lines 59-66), and then fixes the end nodes of the sheet (lines 69-74). Surface rendering is enabled for the sheet (line 65), but not for the ball, in order to improve the visibility of the color map.

Lines 77-81 create and set a collision behavior between the two models, with the drawColorMap property set to `CONTACT_PRESSURE`. Because for this example we want the color map to be drawn on the *second* collidable (the sheet), we set the setColorMapCollidable property to 1 (line 79); otherwise, the default value of 0 would cause the color map to be drawn on the first collidable (the ball). (Alternatively, we could have simply defined the collision behavior as being between the surface and the ball instead of the ball and the sheet.) The color map range is explicitly set to lie between $[0, 15000]$ (line 80); this is in contrast to the example in Section 8.5.2, where the range is auto-updated on each step. The color range is also set explicitly in the behavior, but if multiple objects were colliding it would likely be preferable to set it in the collision manager (with the behavior's value left as `null`) to ensure a uniform render range across all collisions. Other rendering properties are set for the collision manager at lines 83-97, including a custom color map that varies between `CREAM` (the color of the mesh) for no pressure and dark red for maximum pressure.

At line 100, a color bar is created and added to the scene, using the method `createColorBar()` (lines 36-45), to explicitly show the pressure that corresponds to the different colors. The color bar is given the same color map and value range used to render the pressure. Finally, default face and line colors for all components in the model are set at lines 105-106.

To run this example in ArtiSynth, select All demos > tutorial > ContactPressureRender from the Models menu. When run, the FEM models will collide and render the contact pressure on the sheet, as shown in Figure 8.16.

## 8.6 Overconstrained contact

When contact occurs between collidable bodies, the system creates a set of contact constraints to handle the collision and passes these to the physics solver (Section 1.2). When one or both of the contacting collidables are FEM models, the number of constraints is usually less than the DOF count of the collidables. However, if the both collidables are rigid bodies, then the number of constraints often *exceeds* the DOF count, a situation which is referred to as *redundant contact*. Redundant contact can also occur in other collision situations, such those involving embedded meshes (Section 6.3.2) when the mesh has a finer resolution than the embedding FEM, or skinned meshes (Chapter 11) when the number of contacts exceeds the DOF count of the underlying master bodies.

Redundant contact is not usually a problem in collisions between rigid bodies, because by default ArtiSynth handles these using `CONTOUR_REGION` contact (Section 8.4.1), for which contacts are supplied to the physics solver as *unilateral* constraints which are solved using complementarity techniques that can handle redundant constraints. However, as mentioned in Section 8.4.1.2, the default implementation for vertex penetration contact is to present the contacts to the physics solver as *bilateral* constraints (Section 1.2), which are removed between simulation steps. This results in much faster simulation times, and usually works well in the case of FEM models, where the DOF count almost always exceeds the number of constraints. However, when vertex penetration contact is employed between rigid bodies, and sometimes in other situations as described above, redundant contact can arise, and then the use of bilateral constraints results in *overconstrained contact* for which the solver has difficulty finding a proper solution.

A common indicator of overconstrained contact is for an error message to appear in ArtiSynth's console output, which typically looks like this:

```
Pardiso: num perturbed pivots=12
```

The simulation may also go unstable.

At present there are three general ways to manage overconstrained contact:

1. Requiring the system to use *unilateral* constraints for vertex penetration contact. This can be done by setting the property bilateralVertexContact (Section 8.2.1) to `false` in either the collision manager or behavior, but the resulting hit to computational performance may be large.
2. Constraint reduction, described in Section 8.6.1.
3. Regularizing the contact, using one of the methods of Section 8.7.

> When using the new implicit friction integration feature (Section 8.9.4), if redundant contacts arise, it is necessary to regularize both the contact constraints, as per Section 8.7, as well as the friction constraints, by setting the stictionCreep property of either the collision manager or behavior to a non-zero value, as described in Section 8.2.1.

### 8.6.1 Constraint reduction

Constraint reduction involves having the collision manager explicitly try to reduce the number of collision constraints to match the available DOFs, and is enabled by setting the reduceConstraints property for either the collision manager *or* the CollisionBehavior for a particular collision pair to `true`. The default value of reduceConstraints is `false`. As with all properties, it can be set interactively in the GUI, or in code using the property's accessor methods,

```
boolean getReduceContraints()
void setReduceContraints (boolean enable)
```

as illustrated by the following code fragments:

```
MechModel mech;
...

// enable constraint reduction for all collisions:
mech.getCollisionManager().setReduceContraints (true);

// enable constraint reduction for collisions between bodyA and bodyB:
CollisionBehavior behav =
   mech.setCollisionBehavior (bodyA, bodyB, true);
behav.setReduceContraints (true);
```

## 8.7 Contact regularization

Contact regularization is a technique in which contact constraints are made "soft" by adding compliance and damping. This means that they no longer remove DOFs from the system, and so overconstraining cannot occur, but contact penetration is no longer strictly enforced and there may be an increase in the computation time required for the simulation.

### 8.7.1 Compliant contact

Compliant contact is a simple form of regularization that is analogous to that used for joints and connectors, discussed in Section 3.3.8. Compliance and damping parameters $C$ and $D$ can be specified between any two collidables, with $C$ being the *inverse* of a corresponding contact stiffness $K = 1/C$, such the contact forces $f_c$ acting along each contact normal are given by

$$f_c = -Kd - D\dot{d}, \tag{8.1}$$

where $d$ is the contact penetration distance and is *negative* when penetration occurs. Contact forces act to push the contacting bodies apart, so if body A is penetrating body B and the contact normal $\mathbf{n}$ is directed *toward* A, the forces acting on A and B at the contact point will be $f_c\mathbf{n}$ and $-f_c\mathbf{n}$, respectively. Since $C$ is the inverse of the contact stiffness

$K$, a value of $C = 0$ (the default) implies "infinitely" stiff contact constraints. Compliance is enabled whenever the compliance is set to a value greater than 0, with stiffness decreasing as compliance increases.

Compliance can be enabled by setting the compliance and damping properties of either the collision manager *or* the CollisionBehavior for a specific collision pair; these properties correspond to $C$ and $D$ in (8.1). While it is not required to set the damping property, it is usually desirable to set it to a value that approximates critical damping in order to prevent "bouncing" contact behavior. Compliance and damping can be set in the GUI by editing the properties of either the collision manager or a specific collision behavior, or set in code using the accessor methods

```
double getCompliance()
void setCompliance (double c)

double getDamping()
void setDamping (double d)
```

as is illustrated by the following code fragments:

```
MechModel mech;
double compliance;
double damping;

... determine compliance and damping values ...

// set damping and compliance for all collisions:
mech.getCollisionManager().setCompliance (compliance);
mech.getCollisionManager().setDamping (damping);

// enable compliance and damping between bodyA and bodyB:
CollisionBehavior behav =
   mech.setCollisionBehavior (bodyA, bodyB, true);
behav.setCompliance (compliance);
behav.setDamping (damping);
```

An important question is what values to choose for compliance and damping. At present, there is no automatic way to determine these, and so some experimental parameter tuning is often necessary. With regard to the contact stiffness $K$, appropriate values will depend on the desired maximum penetration depth and other loadings acting on the body. The mass of the collidable bodies may also be relevant if one wants to control how fast the contact stabilizes. (The mass of every CollidableBody can be determined by its `getMass()` method.) Since $K$ acts on a per-contact basis, the resulting total force will increase with the number of contacts. Once $K$ is determined, the compliance value $C$ is simply the inverse: $C = 1/K$.

Given $K$, the damping $D$ can then be estimated based on the desired damping ratio $\zeta$, using the formula

$$D = 2\zeta\sqrt{KM} \tag{8.2}$$

where $M$ is the combined mass of the collidable bodies (or the mass of one body if the other is non-dynamic). Typically, the desired damping will be close to critical damping, for which $\zeta = 1$.

An example that allows a user to play with contact regularization is given in Section 8.5.1.

### 8.7.2 Contact force behaviors

When regularizing contact through compliance, as described in Section 8.6, the forces $f_c$ at each contact are explicitly determined as per (8.1). As a broader alternative to this, ArtiSynth allows applications to specify a *contact force behavior*, which allows $f_c$ to be computed as a more generalized function of $d$:

$$f_c = -F(d) - D(d)\dot{d}, \tag{8.3}$$

where $F(d)$ and $D(d)$ are functions returning the elastic force and the damping coefficient. The corresponding compliance is then a local function of $d$ given by $C(d) = 1/F'(d)$ (which is the inverse of the local stiffness).

Because $d$ is negative during contact, $F(d)$ should also be negative. $D(d)$, on the other hand, should be positive, so that the damping acts to reduce $\dot{d}$.

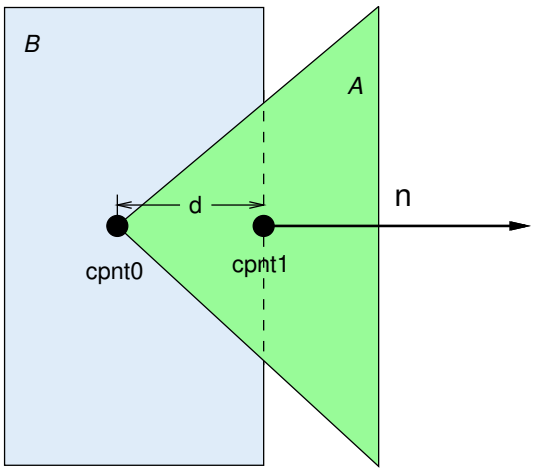



Figure 8.17: Highly magnified schematic of a single vertex penetration contact between body A (green) and body B (blue). `cpnt0` is the point on A penetrating B, while `cpnt1` is the corresponding nearest point on the surface of B. The contact normal **n** faces outward from the surface of B towards A, and the penetration distance $d$ is the (negative valued) distance along **n** from `cpnt1` to `cpnt0`.

Contact force behaviors are implemented as subclasses of the abstract class ContactForceBehavior. To enable them, the application sets the `forceBehavior` property of the CollisionBehavior controlling the contact interaction, using the method

```
setForceBehavior (ContactForceBehavior behav)
```

A `ContactForceBehavior` must implement the method `computeResponse()` to compute the contact force, compliance and damping at each contact:

```
void computeResponse (
    double[] fres, double dist, ContactPoint cpnt0, ContactPoint cpnt1,
    Vector3d nrml, double contactArea, int flags);
```

The arguments to this method supply specific information about the contact (see Figure 8.17):

**fres**

An array of length three that returns the contact force, compliance and damping in its first, second and third entries. The contact force is the $F(d)$ shown in (8.3), the compliance is $1/F'(d)$), and the damping is $D(d)$.

**dist**

The penetration distance $d$ (the value of which is negative).

**cpnt0, cpnt1**

ContactPoint objects containing information about the two points associated with the contact (Figure 8.17), including each point's position (returned by `getPoint()`) and associated vertices and weights (returned by `numVertices()`, `getVertices()`, and `getWeights()`).

**nrml**

The contact normal $n$.

**contactArea**

Average area per contact, computed by dividing the total area of the contact region by the number of penetrating vertices. This information is *only* available if the *colliderType* property of the collision behavior is set to `AJL_CONTOUR` (Section 8.2.1); otherwise, `contactArea` will be set to -1.

A collision force behavior may use its `computeResponse()` method to calculate the force, compliance and damping in any desired way. As per equation (8.3), these quantities are functions of the penetration distance $d$, supplied by the argument `dist`.

A very simple instance of `ContactForceBehavior` that just recreates the contact compliance of (8.1) is shown below:

```
class SimpleCompliantContact extends ContactForceBehavior {

   double myStiffness = 10000.0; // contact stiffness
   double myDamping = 10.0;

   public void computeResponse (
      double[] fres, double dist, ContactPoint cpnt0, ContactPoint cpnt1,
      Vector3d nrml, double regionArea, int flags) {

      fres[0] = dist*myStiffness;  // contact force
      fres[1] = 1/myStiffness      // compliance is inverse of stiffness
      fres[2] = myDamping;         // damping
   }
}
```

This example uses only the penetration distance information supplied by `dist`, dividing it by the compliance to determine the contact force. Since `dist` is negative when the collidables are interpenetrating, the returned contact force will also be *negative*.

#### 8.7.2.1 Computing forces based on pressure

Often, such as when implementing elastic foundation contact (Section 8.7.3), the contact force behavior is expressed in terms of *pressure*, given as a function of $d$:

$$p = G(d). \tag{8.4}$$

In that case, the contact force $F(d)$ is determined by multiplying $G(d)$ by the local area $A$ associated with the contact:

$$F(d) = A\,G(d). \tag{8.5}$$

> Since $d$ and $F(d)$ are both assumed to be negative during contact, we will assume here that $G(d)$ is negative as well.

To find the area $A$ within the `computeResponse()` method, one has two options:

A) Compute an estimate of the local contact area surrounding the contact, as discussed below. This may be the more appropriate option if the colliding mesh vertices are not uniformly spaced, or if the `contactArea` argument is undefined because the `AJL_CONTOUR` collider (Section 8.4.2) is not being used.

B) Use the `contactArea` argument if it is is defined. This gives an accurate estimate of the per-contact area on the assumption that the colliding mesh vertices are uniformly distributed. `contactArea` will be defined when using the `AJL_CONTOUR` collider (Section 8.4.2); otherwise, it will be set to -1.

For option (A), `ContactForceBehavior` provides the convenience method computeContactArea(cpnt0,cpnt1,nrml), which estimates the local contact area given the contact points and normal. If the above example of `computeResponse()` is modified so that the stiffness parameter controls pressure instead of force, then we obtain the following:

```
   public void computeResponse (
      double[] fres, double dist, ContactPoint cpnt0, ContactPoint cpnt1,
      Vector3d nrml, double regionArea, int flags) {

      // assume stiffness determines pressure instead of force,
      // so it needs to be scaled by the local contact area:
      double K = myStiffness * computeContactArea (cpnt0, cpnt1, nrml);

      fres[0] = dist*K;       // contact force
      fres[1] = 1/K;          // compliance
      fres[2] = myDamping;    // damping
   }
```

`computeContactArea()` only works for vertex penetration contact (Section 8.4.1.2), and will return -1 otherwise. That's because it needs the contact points' vertex information to compute the area. In particular, for vertex penetration contact, it associates the vertex with 1/3 of the area of each of its surrounding faces.

When computing contact forces, care must also be taken to consider whether the vertex penetrations are being computed for *both* colliding surfaces (two-way contact, Section 8.4.1.2). This means that forces are essentially being computed twice - once for each side of the penetrating surface. For forces based on pressure, the resulting contact forces should then be halved. To determine when two-way contact is in effect, the `flags` argument can be examined for the flag `TWO_WAY_CONTACT`:

```
public void computeResponse (
   double[] fres, double dist, ContactPoint cpnt0, ContactPoint cpnt1,
   Vector3d nrml, double regionArea, int flags) {

   double K = myStiffness * computeContactArea (cpnt0, cpnt1, nrml);
   if ((flags & TWO_WAY_CONTACT) != 0) {
     K *= 0.5;  // divide force response by 2
   }
   fres[0] = dist*K;       // contact force
   fres[1] = 1/K;          // compliance
   fres[2] = myDamping;    // damping
}
```

### 8.7.3 Elastic foundation contact

ArtiSynth supplies a built-in contact force behavior, LinearElasticContact, that implements elastic foundation contact (EFC) based on a linear material law as described in [3]. By default, the contact pressures are computed from

$$G(d) = -\frac{(1-\nu)E}{(1+\nu)(1-2\nu)} \ln\left(1+\frac{d}{h}\right), \tag{8.6}$$

where $E$ is Young's modulus, $\nu$ is Poisson's ratio, $d$ is the contact penetration distance, and $h$ is the foundation layer thickness. (Note that both $G(d)$ and $d$ are negated relative to the formulation in [3] because we assume $d < 0$ during contact.) The use of the $\ln()$ function helps ensure that contact does not penetrate the foundation layer. However, to ensure robustness in case contact *does* penetrate the layer (or comes close to doing so), the pressure relationship is linearized (with a steep slope) whenever $d < h(r-1)$, where $r$ is the *minimum thickness ratio* controlled by the `LinearElasticContact` property minThicknessRatio.

Alternatively, if a strictly linear pressure relationship is desired, this can be achieved by setting the property useLogDistance to `false`. The pressure relationship then becomes

$$G(d) = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)} \left(\frac{d}{h}\right), \tag{8.7}$$

where again $G(d)$ will be negative when $d < 0$.

Damping forces $f_d = -D(d)\dot{d}$ can be computed in one of two ways: either with $D(d)$ set to a constant $C$ such that

$$f_d = -C\dot{d}, \tag{8.8}$$

or with $D(d)$ set to a multiple of $C$ and the magnitude of the elastic contact force $|F(d)|$, such that

$$f_d = -C|F(d)|\dot{d}. \tag{8.9}$$

The latter is compatible with the EFC damping used by OpenSim (equation 24 in [22]).

Elastic foundation contact assumes the use of vertex penetration contact (Section 8.4.1) to achieve correct results; otherwise, a runtime error may result.

LinearElasticContact exports several properties to control the force response described above:

**youngsModulus**

$E$ in equations (8.7) and (8.6).

**poissonsRatio**

$\nu$ in equations (8.7) and (8.6).

**thickness**

$h$ in equations (8.7) and (8.6).

**minThicknessRatio**

$r$ value such that (8.6) is linearized if $d < h(r-1)$. The default value is 0.01.

**dampingFactor**

$C$ in equations (8.8) and (8.9).

**dampingMethod**

An enum of type ElasticContactBase.DampingMethod, for which `DIRECT` and `FORCE` cause damping to be implemented using (8.8) and (8.9), respectively. The default value is `DIRECT`.

**useLogDistance**

A boolean, which if `true` causes (8.6) to be used instead of (8.7). The default value is `true`.

**useLocalContactArea**

A boolean, which if `true` causes contact areas to be estimated from the vertex of the penetrating contact point (option A in Section 8.7.2.1). The default value is `true`.

### 8.7.4 Example: elastic foundation contact of a ball in a bowl

A simple example of elastic foundation contact is defined by the model

```
artisynth.demos.tutorial.ElasticFoundationContact
```

This implements EFC between a spherical ball and a bowl into which it is dropped. Pressure rendering is enabled (Section 8.5.3), and the bowl is made transparent, allowing the contact pressure to be viewed as the ball moves about.

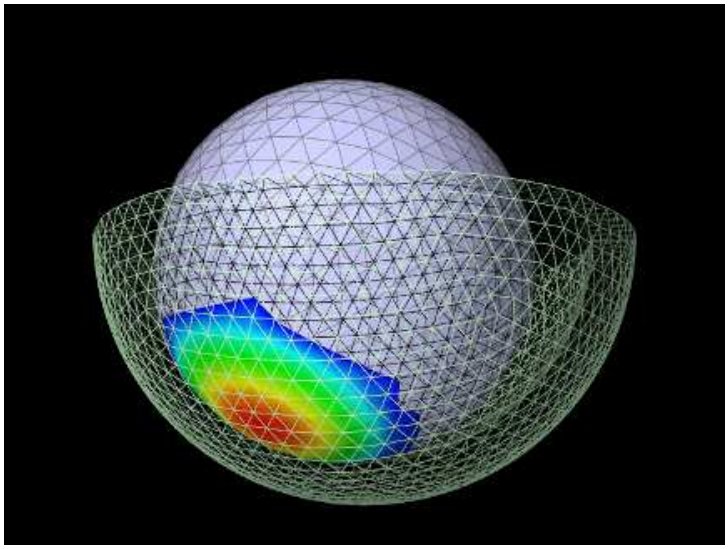


Figure 8.18: ElasticFoundationContact showing contact pressure between the ball and bowl during contact.

The source code for the build method is shown below:

```
1     public void build (String[] args) throws IOException {
2        MechModel mech = new MechModel ("mech");
3        mech.setGravity (0, 0, -9.8);
4        addModel (mech);
5
6        // read in the mesh for the bowl
7        PolygonalMesh mesh = new PolygonalMesh (
8           PathFinder.getSourceRelativePath (
9              ElasticFoundationContact.class, "data/bowl.obj"));
10
11       // create the bowl from the mesh and make it non-dynamic
12       RigidBody bowl =
13          RigidBody.createFromMesh (
14             "bowl", mesh, /*density=*/1000.0, /*scale=*/1.0);
15       mech.addRigidBody (bowl);
16       bowl.setDynamic (false);
17
18       // create another spherical mesh to define the ball
19       mesh = MeshFactory.createIcosahedralSphere (0.7, 3);
20       // create the ball from the mesh
21       RigidBody ball =
22          RigidBody.createFromMesh (
23             "ball", mesh, /*density=*/1000.0, /*scale=*/1.0);
24       // move the ball into an appropriate "drop" position
25       ball.setPose (new RigidTransform3d (0.1, 0, 0));
26       mech.addRigidBody (ball);
27
28       // Create a collision behavior that uses EFC. Set friction
29       // to 0.1 so that the ball will actually roll.
30       CollisionBehavior behav = new CollisionBehavior (true, 0.1);
31       behav.setMethod (Method.VERTEX_PENETRATION); // needed for EFC
32       // create the EFC and set it in the behavior
33       LinearElasticContact efc =
34          new LinearElasticContact (
35             /*E=*/100000.0, /*nu=*/0.4, /*damping=*/0.1, /*thickness=*/0.1);
36       behav.setForceBehavior (efc);
37
38       // set the collision behavior between the ball and bowl
39       mech.setCollisionBehavior (ball, bowl, behav);
40
41       // contact rendering: render contact pressures
42       CollisionManager cm = mech.getCollisionManager();
43       cm.setDrawColorMap (ColorMapType.CONTACT_PRESSURE);
44       RenderProps.setVisible (cm, true);
45       // mesh rendering: render only edges of the bowl so we can see through it
46       RenderProps.setFaceStyle (bowl, FaceStyle.NONE);
47       RenderProps.setLineColor (bowl, new Color (0.8f, 1f, 0.8f));
48       RenderProps.setDrawEdges (mech, true); // draw edges for all meshes
49       RenderProps.setFaceColor (ball, new Color (0.8f, 0.8f, 1f));
50
51       // create a panel to allow control over some of the force behavior
52       // parameters and rendering properties
53       ControlPanel panel = new ControlPanel("options");
54       panel.addWidget (behav, "forceBehavior");
55       panel.addWidget (behav, "friction");
56       panel.addWidget (cm, "colorMapRange");
57       addControlPanel (panel);
58    }
```

The demo first creates the ball and the bowl as rigid bodies (lines 6-26), with the bowl defined by a mesh read from the file `data/bowl.obj` relative to the demo source directory, and the ball defined as an icosahedral sphere. The bowl is fixed in place (`bowl.setDynamic(false)`), and the ball is positioned at a suitable location for dropping into the bowl when simulation begins.

Next, a collision behavior is created, with its collision method property set to `VERTEX_PENETRATION` (as required for EFC), and a friction coefficient of 0.1 (to ensure that the ball will actually roll) (lines 28-31). A `LinearElasticContact` is then constructed, with $E = 10^5$, $\nu = 0.4$, damping factor 0.1, and thickness 0.1, and added to the collision behavior using `setForceBehavior()` (lines 32-26). The behavior is then used to enable contact between the ball and bowl (line 39).

Lines 41-49 set up rendering properties: the collision manager is made visible, with color map rendering set to `CONTACT_PRESSURE`, and the ball and bowl are set to be rendered in pale blue and green, with the bowl rendered using only mesh edges so as to make it transparent.

Finally, a control panel is created allowing the user to adjust properties of the contact force behavior and the pressure rendering range (lines 51-57).

To run this example in ArtiSynth, select All demos > tutorial > ElasticFoundationContact from the Models menu. Running the model will cause the ball to drop into the bowl, with the resulting contact pressures rendered as in Figure 8.18.

### 8.7.5 Example: binding EFC properties to fields

It is possible to bind certain EFC material properties to a field, for situations where these properties vary over the surface of the contacting meshes. Properties of LinearElasticContact that can be bound to (scalar) fields include YoungsModulus and thickness, using the methods

```
setYoungsModulusField (ScalarFieldComponent fcomp)

setThicknessField (ScalarFieldComponent fcomp)
```

Most typically, the properties will be bound to a mesh field (Section 7.3) defined with respect to one of the colliding meshes.

When determining the value of either youngsModulus or thickness within its `computeResponse()` method, `LinearElasticContact` checks to see if the property is bound to a field. If it is, the method first checks if the field is a mesh field associated with the meshes of either `cpnt0` or `cpnt1`, and if so, extracts the value using the vertex information of the associated contact point. Otherwise, the value is extracted from the field using the position of `cpnt0`.

A simple example of binding the thickness property to a field is given by

```
artisynth.demos.tutorial.VariableElasticContact
```

This implements EFC between a beveled cylinder and a plate on which it is resting. The plate is made transparent, allowing the contact pressure to be viewed from below.

The model's code is similar to that for `ElasticFoundationContact` (Section 8.7.4), except that the collidables are different and the EFC thickness property is bound to a field. The source code for the first part of the build method is shown below:

```
1    public void build (String[] args) throws IOException {
2       MechModel mech = new MechModel ("mech");
3       mech.setGravity (0, 0, -9.8);
4       addModel (mech);
5
6       // read in the mesh for the top cylinder
7       PolygonalMesh mesh = new PolygonalMesh (
8          PathFinder.getSourceRelativePath (
9             VariableElasticContact.class, "data/beveledCylinder.obj"));
10      // create the cylinder from the mesh
11      RigidBody cylinder =
12         RigidBody.createFromMesh (
13            "cylinder", mesh, /*density=*/1000.0, /*scale=*/1.0);
14      mech.addRigidBody (cylinder);
15
16      // create a plate from a box mesh and make it non-dynamic
17      RigidBody plate =
18         // box has widths 1.5 x 1.5 x 0.5 and mesh resolutions 10 x 10 x 4
```

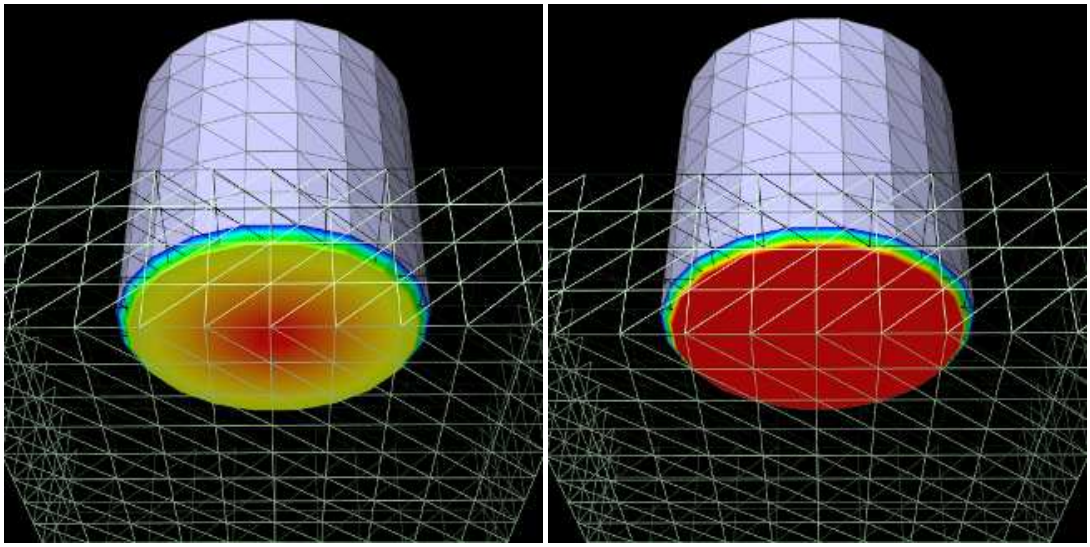


Figure 8.19: VariableElasticContact showing the contact pressure decreasing radially from the cylinder center, as the EFC thickness property increases due to its field binding (left). Without the field binding, the thickness and the pressure are constant across the cylinder bottom (right).

```
19            RigidBody.createBox (
20               "plate", 1.5, 1.5, 0.5, 10, 10, 4, /*density=*/1000.0, false);
21        plate.setPose (new RigidTransform3d (0, 0, -0.75));
22        plate.setDynamic (false);
23        mech.addRigidBody (plate);
24
25        // enable vertex penetrations (required by EFC)
26        CollisionManager cm = mech.getCollisionManager();
27        cm.setMethod (Method.VERTEX_PENETRATION);
28        // create the EFC
29        double thickness = 0.1;
30        LinearElasticContact efc =
31           new LinearElasticContact (
32              /*E=*/100000.0, /*nu=*/0.4, /*damping=*/0.1, thickness);
33
34        // Create a thickness field for the cylinder mesh. Use a scalar vertex
35        // field, with the thickness value h defined by h = thickness * (1 + r),
36        // where r is the radial distance from the cylinder axis.
37        ScalarVertexField field =
38           new ScalarVertexField (cylinder.getSurfaceMeshComp());
39        for (Vertex3d vtx : mesh.getVertices()) {
40           Point3d pos = vtx.getPosition();
41           double r = Math.hypot (pos.x, pos.y);
42           field.setValue (vtx, thickness*(1+r));
43        }
44        // add the field to the MechModel, and bind the EFC thickness property
45        mech.addField (field);
46        efc.setThicknessField (field);
47
48        // create a collision behavior that uses the EFC
49        CollisionBehavior behav = new CollisionBehavior (true, /*friction=*/0.1);
50        // Important: call setForceBehavior() *after* properties have been bound
51        behav.setForceBehavior (efc);
52        // enable cylinder/plate collisions using the behavior
53        mech.setCollisionBehavior (cylinder, plate, behav);
```

First, the cylinder and the plate are created as rigid bodies (lines 6-23), with the cylinder defined by a mesh read from the file `data/beveledCylinder.obj` relative to the demo source directory. The plate is fixed in place and positioned so that it is directly under the cylinder.

Next, the contact method is set to `VERTEX_PENETRATION` (as required for EFC), this time by setting it for all collisions in the collision manager, and a `LinearElasticContact` is constructed, with $E = 10^5$, $\nu = 0.4$, damping factor 0.1, and thickness 0.1 (lines 25-32). Then in lines 34-43, a `ScalarMeshField` is created for the cylinder surface mesh, with vertex values set so that the field defines a thickness value $h$ that increases with the radial distance $r$ from the cylinder axis according to:

$$h = \text{thickness}\,(1 + r)$$

This field is then added to the `fields` list of the `MechModel` and the EFC thickness property is bound to it (lines 44-46).

Lastly, a collision behavior is created, with the EFC added to it as a force behavior, and used to enable cylinder/plate collisions (lines 48-53).

> It is important that EFC properties are bound to fields *before* the behavior method `setForceBehavior()` is called, because that method *copies* the force behavior and so subsequent field bindings would not be noticed. If for some reason the bindings must be made after the call, they should be applied to the copied force behavior, which can be obtained using `getForceBehavior()`.

To run this example in ArtiSynth, select All demos > tutorial > VariableElasticContact from the Models menu. Running the model will result in contact pressures seen in the left of Figure 8.19, with the pressures decreasing radially in response to the increasing thickness. With no field binding, the pressures are constant across the cylinder bottom (Figure 8.19, right).

## 8.8 Monitoring collisions

Sometimes, an application may wish to know details about the collision interactions between a specific collidable and one or more other collidables. These details may include items such as contact forces or the penetration regions between the opposing meshes. This section describes ways in which these details can be observed through the use of *collision responses*.

### 8.8.1 Collision responses

Applications can track collision information by creating CollisionResponse components for the collidables in question and adding them to the `MechModel` using setCollisionResponse():

```
CollisionResponse resp = new CollisionResponse();
mech.setCollisionResponse (collidable0, collidable1, resp);
```

An alternate version of `setCollisionResponse()` creates the response component internally and returns it:

```
CollisionResponse resp = mech.setCollisionResponse (collidable0, collidable1);
```

During every simulation step, the `MechModel` will update its response components to reflect the current collision conditions between the associated collidables.

The first collidable specified for a collision response must be a specific collidable component, while the second may be either another collidable or a group of collidables represented by a Collidable.Group:

```
CollisionResponse r0, r1, r2;
RigidBody bodA;
FemModel3d femB;

// collision information between bodA and femB only:
r0 = setCollisionResponse (bodA, femB);
// collision information between femB and all rigid bodies:
r1 = setCollisionResponse (femB, Collidable.Rigid);
// collision information between femB and all bodies and self-collisions:
r2 = setCollisionResponse (femB, Collidable.All);
```

When a compound collidable is specified, the response component will collect collision information for all its subcollidables.

As as with collision behaviors, the same response cannot be added to an application twice:

```
CollisionResponse resp = new CollisionResponse();
mech.setCollisionResponse (femA, femB, resp);
mech.setCollisionResponse (femC, femD, resp); // ERROR
```

The complete set of methods for managing collision responses are similar to those for behaviors,

```
getCollisionResponse (collidable0, collidable1)
setCollisionResponse (collidable0, collidable1, response)
setCollisionResponse (collidable0, collidable1)

clearCollisionResponse (collidable0, collidable1)
clearCollisionResponses ()
```

where getCollisionResponse() and clearCollisionResponse() respectively return and clear response components that have been previously set using one of the `setCollisionResponse()` methods.

### 8.8.2 Available information

Information that can be obtained from a CollisionResponse component includes whether or not the collidable is in collision, the current contact points, forces and pressures acting on the vertices of the colliding meshes, and the underlying CollisionHandler objects that maintain the contact constraints between each colliding mesh. Much of the available information is similar to that which can be displayed using collision rendering (Section 8.5).

The information methods provided by CollisionResponse are listed below. Many take a `cidx` argument to request information for either the *first* or *second* collidable, which refer, respectively, to the collidables specified by the arguments `collidable0` and `collidable1` in the `setCollisionResponse()` methods.

- `boolean inContact()`

  Queries if the collidables associated with the response are in contact. Note that this may be true even if the method `getContactData()` returns an empty list; this can occur, for instance, for vertex penetration contact when the collision meshes intersect in a small region without any interpenetrating vertices.

- `List<ContactData> getContactData()`

  Returns a list of the most recently computed contacts between the collidables. Each contact is described by a `ContactData` object supplying information about the contact points on each collidable, the normal, and contact and friction forces. If there are no current contacts, the returned list is empty. For more details on this information, consult the API documentation for ContactData and ContactPoint.

- `Map<Vertex3d,Vector3d> getContactForces(int cidx)`

  Returns a map of the contact forces acting the on the vertices of the collision meshes of either the first or second collidable, as specified by setting `cidx` to 0 or 1. This information is most useful and accurate when using vertex penetration contact (Section 8.4.1.2).

- `Map<Vertex3d,Vector3d> getContactForces(int cidx)`

  Returns a map of the contact pressures acting the on the vertices of the collision meshes of either the first or second collidable, as specified by setting `cidx` to 0 or 1. This information is most useful and accurate when using vertex penetration contact (Section 8.4.1.2).

- `ArrayList<PenetrationRegion> getPenetrationRegions(int cidx)`

  Returns a list of all the penetration regions on either the first or second collidable, as specified by setting `cidx` to 0 or 1. Penetration regions are available only if the collision manager's collider type is set to `AJL_CONTOUR` (Section 8.4.2).

- `ArrayList<CollisionHandler> getHandlers()`

  Returns the CollisionHandlers for all currently active collisions associated with the collidables of the response.

A typical usage scenario for collision responses is to create them before the simulation is run, possibly in the root model `build()` method, and then query them when the simulation is running, such from the `apply()` method of a Monitor (Section 5.3). For example, in the root model `build()` method, the response could be created with the call

```
CollisionResponse resp = mech.setCollisionResponse (femA, femB);
```

and then used in some runtime code as follows:

```
Map<Vertex3d,Vector3d> collMap = resp.getContactForces (0);
```

If for some reason it is difficult to store a reference to the response between its construction and its use, then `getCollisionResponse()` can be used to retrieve it:

```
CollisionResponse resp = mech.getCollisionResponse (femA, femB);
Map<Vertex3d,Vector3d> collMap = resp.getContactForces (0);
```

As with contact rendering, the information returned by a collision response may sometimes appear to lag the simulation by one time step. That is because contacts are computed at the *end* of each time step $i$, and then used to compute the contact forces during the next step $i+1$. The information returned by a response at the end of step $i$ is thus based on contacts detected at the end of step $i-1$, along with contact forces that are computed during step $i$ and used to calculate the updated positions and velocities at the end of that step.

### 8.8.3 Example: monitoring contact forces

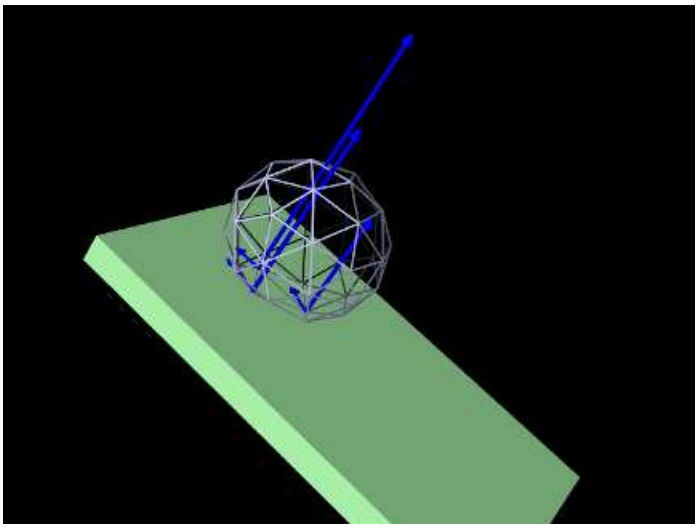


Figure 8.20: ContactForceMonitor showing the contact forces as the ball collides with the plane.

A simple example of using a `CollisionResponse` is given by

```
artisynth.demos.tutorial.ContactForceMonitor
```

This shows an FEM model of a ball rolling down an inclined plane, with a collision response combined with a Monitor used to print out the contact positions and forces at each time step. Contact forces are also rendered in the viewer. The model's source code, excluding `include` statements, is shown below:

```
1 public class ContactForceMonitor extends RootModel {
2
3    CollisionResponse myResp;
4
5    private class ContactMonitor extends MonitorBase {
6       public void apply (double t0, double t1) {
```

```
7           // get the contacts from the collision response and print their
8           // positions and forces.
9           List<ContactData> cdata = myResp.getContactData();
10          if (cdata.size() > 0) {
11             System.out.println (
12                "num contacts: "+ cdata.size() + ", time=" + t0);
13             for (ContactData cd : cdata) {
14                System.out.print (
15                   " pos:   " + cd.getPosition0().toString("%8.3f"));
16                System.out.println (
17                   ", force: " + cd.getContactForce().toString("%8.1f"));
18             }
19          }
20       }
21    }
22
23    public void build (String[] args) {
24       MechModel mech = new MechModel ("mech");
25       addModel (mech);
26
27       // create an FEM ball model
28       FemModel3d ball =
29          FemFactory.createIcosahedralSphere (
30             null, /*radius=*/0.7, /*ndivisions=*/1, /*quality=*/2);
31       ball.setName ("ball");
32       ball.setSurfaceRendering (SurfaceRender.Shaded);
33       mech.addModel (ball);
34       // reposition the ball to an appropriate "drop" position
35       ball.transformGeometry (new RigidTransform3d (-0.5, 0, 2.5, 0, 0.1, 0.1));
36
37       // create an inclined plane for the ball to collide with
38       RigidBody plane = RigidBody.createBox (
39          "plane", 4.0, 2.5, 0.25, /*density=*/1000);
40       plane.setPose (new RigidTransform3d (0, 0, 0, 0, Math.PI/5, 0));
41       plane.setDynamic (false);
42       mech.addRigidBody (plane);
43
44       // Enable collisions between the ball and the plane. Specifying the ball
45       // first means contact forces will be rendered in the direction acting on
46       // the ball.
47       mech.setCollisionBehavior (ball, plane, true, 0.3);
48       myResp = mech.setCollisionResponse (ball, plane);
49
50       // add a monitor to print out contact positions and forces
51       addMonitor (new ContactMonitor());
52
53       // Render properties: set the collision manager to render contact and
54       // friction forces, with a scale factor of 0.0001
55       CollisionManager cm = mech.getCollisionManager();
56       cm.setDrawContactForces (true);
57       cm.setDrawFrictionForces (true);
58       cm.setContactForceLenScale (0.0001);
59       RenderProps.setVisible (cm, true);
60       RenderProps.setSolidArrowLines (cm, 0.02, Color.BLUE);
61       // Render properties: for the ball, make the elements invisible, and
62       // render its surface as a wire frame to make it easy to see through
63       RenderProps.setVisible (ball.getElements(), false);
64       RenderProps.setLineColor (ball, new Color (.8f, .8f, 1f));
65       RenderProps.setFaceStyle (ball, FaceStyle.NONE);
66       RenderProps.setDrawEdges (ball, true);
67       RenderProps.setEdgeWidth (ball, 2); // wire frame edge width
68       RenderProps.setFaceColor (plane, new Color (.7f, 1f, .7f));
69    }
70 }
```

The build method creates a simple FEM ball and inclined plane, and positions the ball to an appropriate drop position (lines 27-42). The collisions are enabled between the ball and the plate, along with a CollisionResponse that is stored in the global reference `myResp` to allow it to be accessed from the monitor (lines 47-48).

The monitor itself is implemented by the class `ContactMonitor`, which is created by subclassing MonitorBase and overriding the `apply()` method to print out the contact information (lines 5-21). It does this by using the response's `getContactData()` method to obtain a list of the contacts, and if there are any, printing the number of contacts, the time step start time (`t0`), and the position and force of each contact. An instance of the monitor is created and added to the root model at line 51.

Rendering is set so that the collision manager renders both the contact and friction forces in the viewer (lines 55-60), using blue arrows with a radius of 0.02 (line 60) and a scale factor for the arrow length of 0.0001 (line 58). To make the ball easy to see through, its elements are made invisible, and instead is rendered using only its surface mesh, with face rendering disabled and edges drawn with a width of 2 (lines 63-67).

To run this example in ArtiSynth, select All demos > tutorial > ContactForceMonitor from the Models menu. Running the model will result in the ball colliding with the plane, showing the contact and friction forces as blue arrows (Figure 8.20, while the monitor prints out the contact positions and forces to the console at each time step, producing output like this:

```
num contacts: 2, time=0.64
 pos:     -0.918    0.059    0.821, force:  17508.5      0.0  24098.4
 pos:     -0.760   -0.285    0.706, force:  11607.9      0.0  15976.9
num contacts: 2, time=0.65
 pos:     -0.918    0.059    0.821, force:  16989.6      0.0  23384.2
 pos:     -0.760   -0.285    0.706, force:  11955.8      0.0  16455.7
num contacts: 3, time=0.66
 pos:     -0.918    0.059    0.821, force:  16465.4      0.0  22662.7
 pos:     -0.760   -0.285    0.706, force:  11880.9      0.0  16352.7
 pos:     -0.695    0.404    0.659, force:    661.1      0.0    910.0
```

### 8.8.4 Example: forces and pressures on mesh vertices

In the case of vertex penetration contact, contact forces are generated on the mesh vertices of both collidables in the vicinity of the intersection (Section 8.4.1.2 and Figure 8.11). The `CollisionResponse` methods getContactForces() and getContactPressures() can be used to obtain these forces, and their associated pressures, for the meshes of either the first or second collidable (corresponding to `collidable0` or `collidable1` in the `setCollisionResponse()` methods).

> `getContactForces()` and `getContactPressures()` will also work for contour region contact (Section 8.4.1.1), but the results may be somewhat approximated and less meaningful.

As an example, the `ContactForceMonitor` in the example of Section 8.8.3 can be modified to find the maximum vertex contact force for a specific collidable by adding the following to the `apply()` method:

```
int collidableIdx = 0;
Map<Vertex3d,Vector3d> map= myResp.getContactForces (collidableIdx);
double maxF = 0;
for (Map.Entry<Vertex3d,Vector3d> entry : map.entrySet()) {
   Vertex3d v = entry.getKey();
   Vector3d f = entry.getValue();
   double normF = f.norm();
   if (normF> maxF) {
      maxF=normF;
   }
}
```

`getContactForces()` returns a map of the contact forces on each mesh vertex of the collidable indicated by `collidableIdx` (with 0 and 1 denoting the first and second collidables). To find the maximum force, one can simply iterate over the map. Similarly, maximum contact pressure can be determined using a code fragment like this:

```
int collidableIdx = 0;
Map<Vertex3d,Double> pressures = myResp.getContactPressures (collidableIdx);
```

```
double maxP = 0;
for (Map.Entry<Vertex3d,Double> entry : pressures.entrySet()) {
   double pressure = entry.getValue();
   if (pressure > maxP) {
      maxP = pressure;
   }
}
```

Which collidable one chooses for evaluating contact forces or pressures will depend on the questions being asked by the modeling study. In the case of a FEM model colliding with a rigid body, one will almost certainly want to choose the FEM model as the collidable. In the case of two colliding FEMs (or two rigid bodies using vertex penetration contact), one may wish to use the collidable whose mesh has a higher resolution. While the forces and pressures on each collidable will be equal and opposite in aggregate, they will vary locally due to differences in the geometry and resolution of each collidable's mesh. For instance, a collidable with a finer mesh will typically have a greater number of vertex forces of lesser magnitude.

## 8.9 Tips and limitations

This section describes practical tips that can employed when using ArtiSynth's contact simulation, together with some of its limitations.

### 8.9.1 Contact jitter

ArtiSynth's attempt to resolve the interpenetration of colliding bodies may sometimes cause a jittering behavior around the colliding area, as the surface collides, separates, and re-collides. This can usually be stabilized by maintaining a certain interpenetration distance during contact. This distance is controlled by the `MechModel` property `penetrationTol`. ArtiSynth attempts to compute a suitable default value for this property, but for some applications it may be necessary to control the value explicitly using the `MechModel` methods

```
setPenetrationTol (double dist)

double getPenetrationTol()
```

### 8.9.2 Passing through objects

The ArtiSynth collision detection mechanism is *static*, which means that mesh intersections are computed at a fixed point in time and do not presently utilize velocity information. This means that if the colliding objects are fast enough or thin enough, it is possible for them to pass completely through each other during a simulation step (Figure 8.21, left). Similarly, even if objects do not pass completely through each other, the penetration may be large enough for contacts to be established with the wrong side (8.21, right).

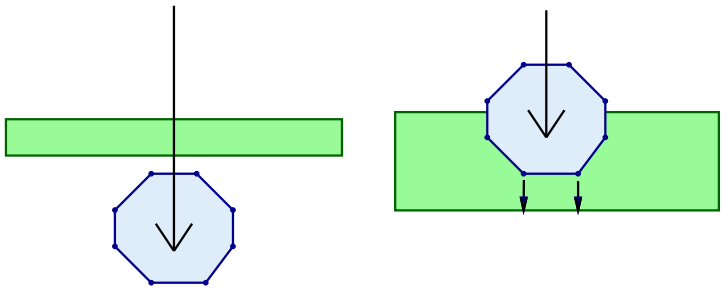


Figure 8.21: If object speeds are high enough or the objects thin enough, it is possible for collidables to pass completely through each other (left), or to penetrate sufficiently that contacts are established with the wrong side (right).

When this happens, there are two straightforward solutions:

1. Reduce the simulation step size.
2. Increase the thickness of one or more of the colliding meshes. This option is facilitated by the fact that, as mentioned in Section 8.3, collision meshes can be independent of a collidable's physical geometry.

> The problem of collidables passing through each other can be particularly acute in the case of shell elements, and consequently collisions involving shell elements are not fully supported. However, collisions involving shell elements will work in some circumstances, as described in Section 8.1.4.

### 8.9.3 Stray vertices

The stray vertex problem sometimes occurs when vertex penetration contact is used with objects with sharp edges. Because vertex penetration contacts are determined by finding the nearest face to each vertex on the opposing mesh, this may sometimes cause an inappropriate face to be selected if penetration is deep enough and near a sharp turn in the opposing mesh (Figure 8.22).

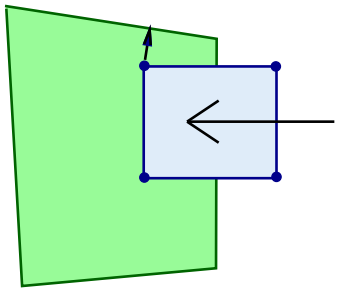


Figure 8.22: Vertex penetration contact near a sharp turn in the opposing mesh may sometimes cause the contact to be directed toward the wrong face, as seen here for the upper left vertex of the penetrating square.

Possible solutions include:

1. Reduce the simulation step size.
2. Use the `VERTEX_EDGE_PENETRATION` contact method (Section 8.4.1.3).
3. Adjust the mesh geometry or increase the mesh resolution; higher mesh resolutions will increase the number of contacts and reduce the impact of stray vertices.

### 8.9.4 Coulomb friction and stability

ArtiSynth uses a "box" friction approximation [10] to compute Coulomb (dry) friction, which allows for a less expensive and more robust computation at the expense of some accuracy. Box friction computes friction forces in two orthogonal directions in the plane perpendicular to the contact normal, using a fixed normal force taken from the previous solve, instead of employing the more detailed polyhedralized friction cones commonly used in multibody dynamics [1, 20]. The resulting linear complementarity problem is convex and hence more easily solved. Errors can be minimized by ensuring that one of the friction directions is parallel to the tangential contact velocity.

By default, friction forces are computed *after* after the main velocity solve, in a secondary solve that does not use implicit integration techniques. This reduces compute time and works well for rigid bodies, or for FEM models when the friction forces are relatively small (which they often are in biomechanical applications). However, applications involving FEM models and large friction forces may suffer from instability. An example of this might be an FEM model resting on an inclined plane and relying on friction to remain stationary under a large load. To handle such situations, one can enable *implicit friction integration* by setting the useImplicitFriction property in `MechModel`; this can be done in code using the `MechModel` methods

```
setUseImplicitFriction (boolean enable)

boolean getUseImplicitFriction()
```

At the time of this writing, implicit friction integration is a new ArtiSynth feature, and should be considered to be in beta testing.

When using implicit friction integration, if redundant contacts arise (typically between rigid bodies), it may be necessary to regularize both the contact constraints, using one of the methods described in Section 8.7, as well as the friction constraints, by setting the stictionCreep property of either the collision manager or behavior to a non-zero value, as described in Section 8.2.1.

# Chapter 9

# Muscle Wrapping and Via Points

ArtiSynth provides support for multipoint springs and muscles, which are similar to axial springs and muscles (Sections 3.1.1 and 4.5), except that they can contain multiple via points and also wrap around obstacles. This allows the associated force directions to vary in response to obstacles and constraints in the model, which is particularly important in biomechanical models where point-to-point muscles need to wrap around anatomical structures such as bones. A schematic illustration is shown in Figure 9.1, where a single spring connects points $\mathbf{p}_0$ and $\mathbf{p}_2$, while passing through a single via point $\mathbf{p}_1$ and wrapping around obstacles $W_1$ and $W_2$. Figure 9.2 shows two examples involving a rigid body with fixed via points and a spring wrapping around three rigid bodies.

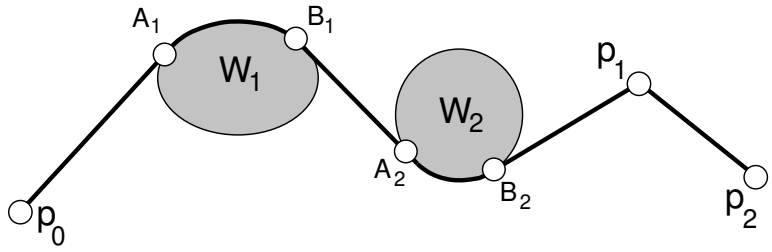



Figure 9.1: Schematic illustration of a multipoint spring passing through a via point $\mathbf{p}_1$ and wrapping around two obstacles $W_1$ and $W_2$. The points $A_1$, $B_1$ and $A_2$, $B_2$ denote the first and last locations where $W_1$ and $W_2$ make contact with the spring.

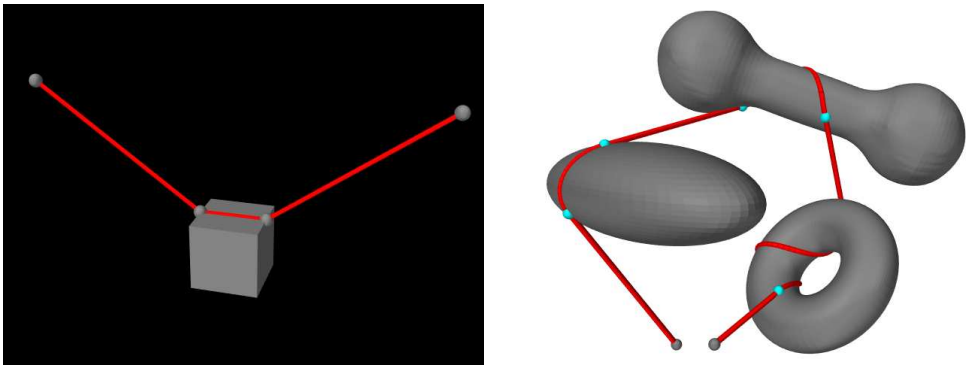


Figure 9.2: Left: A multipoint spring with two via points rigidly fixed to a box-shaped rigid body. Right: A multipoint spring wrapped around three obstacles.

As with axial springs and muscles, multipoint springs and muscles must have two points to denote their beginning and end. In between, they can have any number of *via* points, which are fixed locations which the spring must pass through in the specified order. Any ArtiSynth Point object may be specified as a via point, including particles and markers. The purpose of the via point is generally to direct the spring along some particular path. In particular, the path directions

before and after a via point will generally be different, and forces acting on the via point will be determined by the tension in the spring (or muscle) acting along these two different directions.

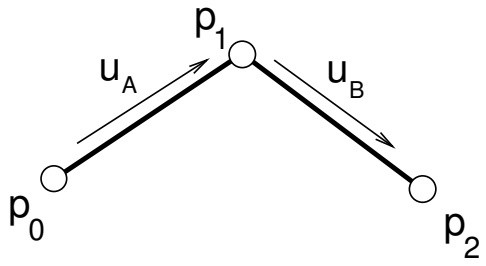



Figure 9.3: A multipoint spring with a single via point $\mathbf{p}_1$, showing the unit direction vectors $\mathbf{u}_A$ and $\mathbf{u}_B$ immediately before and after.

> Conceptually, the spring or muscle "slides" through its via points, which act analogously to virtual three dimensional pulleys. In particular, the proportional distance between via points does *not* remain fixed.

The tension $f$ within the spring or muscle is computed from its material, using the relation $f(l,\dot{l},a)$ described in Sections 3.1.1 and 4.5.1, where $l$ now denotes the *entire* length of the spring as it passes through the via points and wraps around obstacles. The total force $\mathbf{f}$ acting on each via point is then given by

$$\mathbf{f} = f \cdot (\mathbf{u}_B - \mathbf{u}_A)$$

where $\mathbf{u}_B$ and $\mathbf{u}_A$ are unit vectors giving the spring's direction immediately after and before the via point (Figure 9.3).

Multipoint springs can also be made to wrap around one or more *wrappable* objects. Unlike via points, wrappable objects can occur in any order along the spring and wrapping only occurs when the spring and the object actually collide. Any ArtiSynth object that implements Wrappable can be used as a wrapping object (currently, only RigidBody objects implement `Wrappable`). The forces acting on a wrappable are those generated by the forces $\mathbf{f}_A$ and $\mathbf{f}_B$ acting on the points A and B where the spring makes and leaves contact with the it (Figure 9.4). These forces are given by

$$\mathbf{f}_A = -f\mathbf{u}_A, \quad \mathbf{f}_B = f\mathbf{u}_B,$$

where $\mathbf{u}_B$ are $\mathbf{u}_A$ are unit vectors giving the spring's direction immediately before A and after B. Points A and B are collectively known as the A/B points.

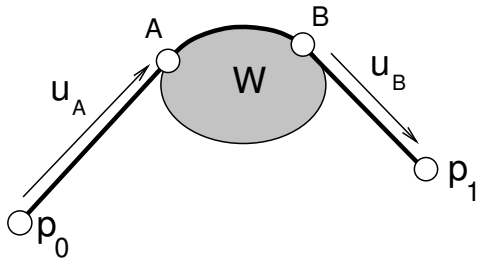



Figure 9.4: A multipoint spring wrapping around a single obstacle $W$, with initial and final contact at points A and B, and associated unit direction vectors $\mathbf{u}_A$ and $\mathbf{u}_B$.

## 9.1 Via Points

Multipoint springs and muscles are implemented by the classes MultiPointSpring and MultiPointMuscle, respectively. The relationship between MultiPointSpring and MultiPointMuscle is the same as that between AxialSpring and Muscle: The latter is a subclass of the former, and allows the creation of active tension forces in response to its excitation property.

An application allocates one of these components, sets the appropriate material properties for the tension forces, and then adds points and wrappable objects as desired.

Points can be added, queried, and removed using the methods

```
void addPoint (Point pnt)
Point getPoint (int idx)
int numPoints()
boolean removePoint (Point pnt)
```

As with AxialSpring, there must be at least two points anchoring the beginning and end of the spring. Any additional points will be *via points*.

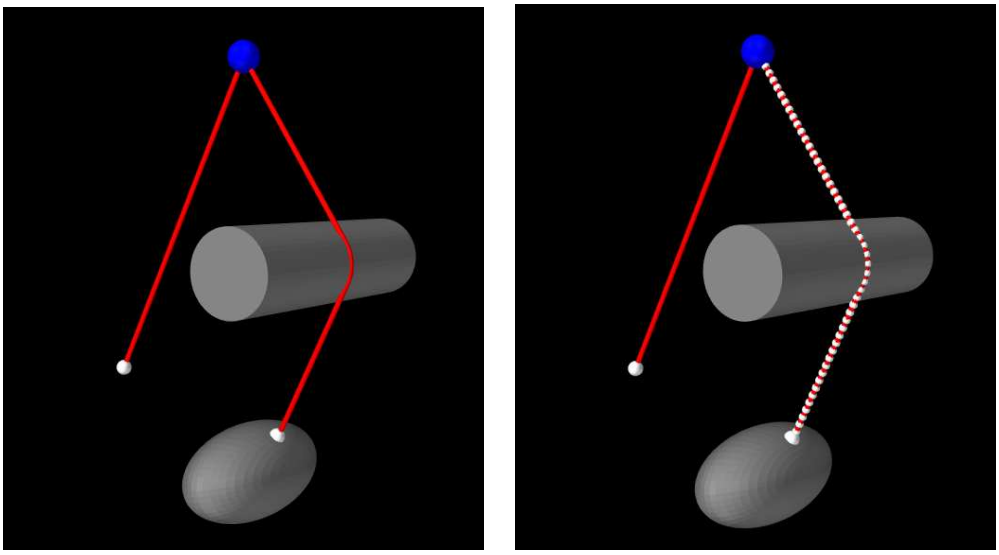


Figure 9.5: A multipoint spring with two segments, separated by a blue via point (top), with the rightmost segment set to be wrappable so that it can wrap around a cylinder. The right image shows the wrappable segment's knots.

The section of a multipoint spring between any two adjacent points is known as a *segment*. By default, each segment forms a straight line between the two points and does *not* interact with any wrappable obstacles. To interact with wrappables, a segment needs to be declared *wrappable*, as described in Section 9.2.

Spring construction is illustrated by the following code fragment:

```
MultiPoint spring = new MultiPointSpring();
spring.setMaterial (new LinearAxialMaterial (stiffness, damping));
spring.addPoint (p0); // start point
spring.addPoint (p1); // via point
spring.addPoint (p2); // via point
spring.addPoint (p3); // stop point
```

This creates a new `MultiPointSpring` and sets its material to a simple linear material with a specified stiffness and damping. Four points `p0`, `p1`, `p2`, `p3` are then added, forming a start point, two via points, and a stop point.

### 9.1.1 Example: a muscle with via points

A simple example of a muscle containing via points is given by `artisynth.demos.tutorial.ViaPointMuscle`. It consists of a MultiPointMuscle passing through two via points attached to a block. The code is given below:

```
1 package artisynth.demos.tutorial;
2
3 import java.awt.Color;
4
5 import artisynth.core.gui.ControlPanel;
6 import artisynth.core.materials.SimpleAxialMuscle;
```

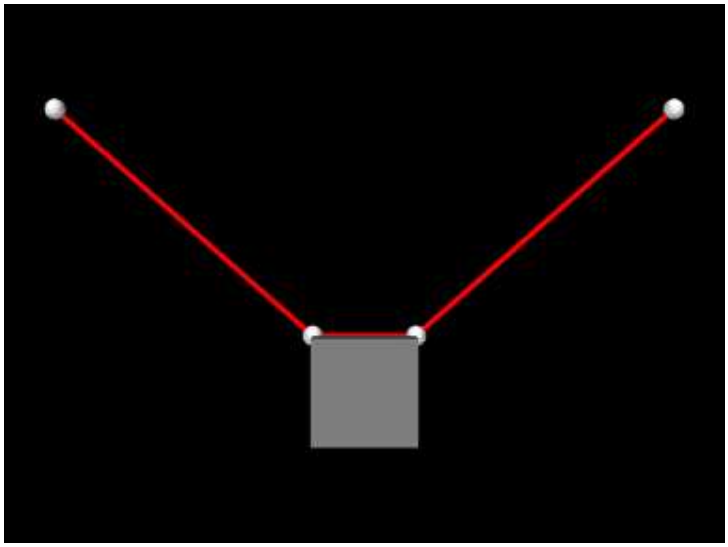


Figure 9.6: ViaPointMuscle model loaded into ArtiSynth.

```
7  import artisynth.core.mechmodels.FrameMarker;
8  import artisynth.core.mechmodels.MechModel;
9  import artisynth.core.mechmodels.MultiPointMuscle;
10 import artisynth.core.mechmodels.Particle;
11 import artisynth.core.mechmodels.RigidBody;
12 import artisynth.core.workspace.RootModel;
13 import maspack.matrix.Point3d;
14 import maspack.render.RenderProps;
15
16 public class ViaPointMuscle extends RootModel {
17
18    protected static double size = 1.0;
19
20    public void build (String[] args) {
21       MechModel mech = new MechModel ("mech");
22       addModel (mech);
23
24       mech.setFrameDamping (1.0); // set damping parameters
25       mech.setRotaryDamping (0.1);
26
27       // create block to which muscle will be attached
28       RigidBody block = RigidBody.createBox (
29          "block", /*widths=*/1.0, 1.0, 1.0, /*density=*/1.0);
30       mech.addRigidBody (block);
31
32       // create muscle start and end points
33       Particle p0 = new Particle (/*mass=*/0.1, /*x,y,z=*/-3.0, 0, 0.5);
34       p0.setDynamic (false);
35       mech.addParticle (p0);
36       Particle p1 = new Particle (/*mass=*/0.1, /*x,y,z=*/3.0, 0, 0.5);
37       p1.setDynamic (false);
38       mech.addParticle (p1);
39
40       // create markers to serve as via points
41       FrameMarker via0 = new FrameMarker();
42       mech.addFrameMarker (via0, block, new Point3d (-0.5, 0, 0.5));
43       FrameMarker via1 = new FrameMarker();
44       mech.addFrameMarker (via1, block, new Point3d (0.5, 0, 0.5));
45
46       // create muscle, set material, and add points
47       MultiPointMuscle muscle = new MultiPointMuscle ();
```

```
48        muscle.setMaterial (new SimpleAxialMuscle (/*k=*/1, /*d=*/0, /*maxf=*/10));
49        muscle.addPoint (p0);
50        muscle.addPoint (via0);
51        muscle.addPoint (via1);
52        muscle.addPoint (p1);
53        mech.addMultiPointSpring (muscle);
54
55        // set render properties
56        RenderProps.setSphericalPoints (mech, 0.1, Color.WHITE);
57        RenderProps.setCylindricalLines (mech, 0.03, Color.RED);
58
59        createControlPanel();
60     }
61
62     private void createControlPanel() {
63        // creates a panel to adjust the muscle excitation
64        ControlPanel panel = new ControlPanel ("options", "");
65        panel.addWidget (this, "models/mech/multiPointSprings/0:excitation");
66        addControlPanel (panel);
67     }
68  }
```

Lines 21-30 of the `build()` method create a `MechModel` and add a simple rigid body block to it. Two non-dynamic points (`p0` and `p1`) are then created to act as muscle end points (lines 33-38), along with two markers (`via0` and `via1`) which are attached to the block to act as via points (lines 41-44). The muscle itself is created by lines 42-53, with the end points and via points being added in order from start to end. The muscle material is a SimpleAxialMuscle, which computes tension according to the simple linear formula (4.1) described in Section 4.5.1. Lines 56-57 set render properties for the model, and line 59 creates a control panel (Section 5.1) that allows the muscle excitation property to be interactively controlled.

To run this example in ArtiSynth, select All demos > tutorial > ViaPointMuscle from the Models menu. The model should load and initially appear as in Figure 9.6. Running the model will cause the block to fall and swing about under gravity, while changing the muscle's excitation in the control panel will vary its tension.

## 9.2 Obstacle Wrapping

As mentioned in Section 9.1, segments between pairs of via points can be declared *wrappable*, allowing them to interact with wrappable obstacles. This can be done as via points are added to the spring, using the methods

```
void setSegmentWrappable (int numKnots)
void setSegmentWrappable (int numKnots, Point3d[] initialPoints)
```

These make *wrappable* the next segment to be created (i.e., the segment between the most recently added point and the next point to be added), with `numKnots` specifying the number of *knots* that should be used to implement the wrapping. Knots are points that divide the wrappable segment into a piecewise linear curve, and are used to check for collisions with the wrapping surfaces (Figure 9.5). The argument `initialPoints` used by the second method is an optional argument which, if non-null, can be used to specify intermediate guide points to give the segment an initial path around around any obstacles (for more details, see Section 9.4).

Each wrappable segment will be capable of colliding with any of the wrappable obstacles that are known to the spring. Wrappables can be added, queried and removed using the following methods:

```
void addWrappable (Wrappable wrappable)
Wrappable getWrappable (int idx)
int numWrappables()
boolean removeWrappable (Wrappable wrappable)
```

Unlike points, however, there is no implied ordering and wrappables can be added in any order and at any time during the spring's construction.

Wrappable spring construction is illustrated by the following code fragment:

```
MultiPoint spring = new MultiPointSpring();
spring.setMaterial (new LinearAxialMaterial (stiffness, damping));
spring.addPoint (p0); // start point
spring.setSegmentWrappable (50);  // wrappable segment
spring.addPoint (p1); // via point
spring.addPoint (p2); // end point
spring.addWrappable (wrappable1);
spring.addWrappable (wrappable2);
spring.updateWrapSegments(); // ``shrink wrap'' spring to the obstacles
```

This creates a new `MultiPointSpring` with a linear material and three points `p0`, `p1`, and `p2`, forming a start point, via point, and stop point. The segment between `p0` and `p1` is set to be wrappable with 50 knot points. Two wrappable obstacles are added next, each of which will interact with the `p0`-`p1` segment, but *not* with the non-wrappable `p1`-`p2` segment. Finally, `updateWrapSegments()` is called to do an initial solve for the wrapping segments, so that they will be "pulled tight" around any obstacles before simulation begins.

It is also possible to make a segment wrappable *after* spring construction, using the method

```
void setSegmentWrappable (int segIdx, int numKnots, Point3d[] initialPoints)
```

where `segIdx` identifies the segment between points `segIdx` and `segIdx` + 1.

How many knots should be specified for a wrappable segment? Enough so that the resulting piecewise-linear approximation to the wrapping curve is sufficiently "smooth", and also enough to adequately detect contact with the obstacles without passing through them. Values between 50 and 100 generally give good results. Obstacles that are small with respect to the segment length may necessitate more knots. Making the number of knots very large will slow down the computation (although the computational cost is only $O(n)$ with respect to the number of knots).

At the time of this writing, ArtiSynth implements two types of Wrappable object, both of which are instances of RigidBody. The first are specialized *analytic* subclasses of `RigidBody`, listed in Table 9.1, which define specific geometries and use analytic methods for the collision handling with the knot points. The use of analytic methods allows for greater accuracy and (possibly) computational efficiency, and so because of this, these special geometry wrappables should be used whenever possible.

| Wrappable | Description |
|---|---|
| RigidCylinder | A cylinder with a specified height and radius |
| RigidSphere | A sphere with a specified radius |
| RigidEllipsoid | An ellipsoid with specified semi-axis lengths |
| RigidTorus | A torus with specified inner and outer radii |

Table 9.1: Specialized analytic subclasses of `RigidBody`

The second are general rigid bodies which are *not* analytic subclasses, and for which the wrapping surface is determined directly from the geometry of its collision mesh returned by getCollisionMesh(). (Typically the collision mesh corresponds to the surface mesh, but it is possible to specify alternates; see Section 3.2.9.) This is useful in that it permits wrapping around *arbitrary* mesh geometries (Figure 9.7), but in order for the wrapping to work well, these geometries should be smooth, without sharp edges or corners. Wrapping around general meshes is implemented using a quadratically interpolated signed-distance grid (Section 4.6), and the resolution of this grid also affects the effective smoothness of the wrapping surface. More details on this are given in Section 9.3.

### 9.2.1 Example: wrapping around a cylinder

A example showing multipoint spring wrapping is given by `artisynth.demos.tutorial.CylinderWrapping`. It consists of a `MultiPointSpring` passing through a single via point, with both segments on either side of the point made wrappable. Two analytic wrappables are used: a fixed `RigidCylinder`, and a moving `RigidEllipsoid` attached to the end of the spring. The code, excluding include directives, is given below:

```
1 public class CylinderWrapping extends RootModel {
2
3     public void build (String[] args) {
4       MechModel mech = new MechModel ("mech");
```

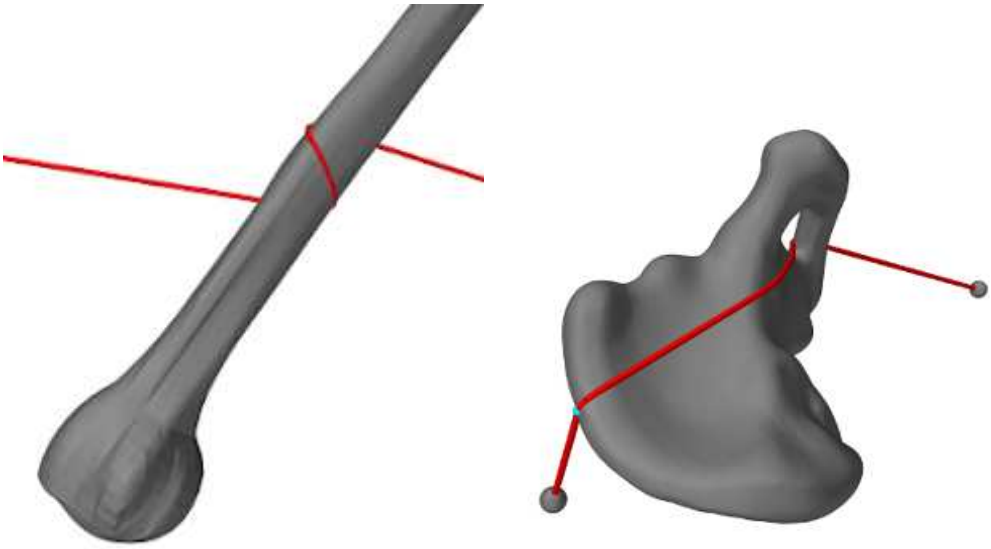


Figure 9.7: Muscle strands wrapped around general bone-shaped meshes: a humerus (left), and a pelvis (right)

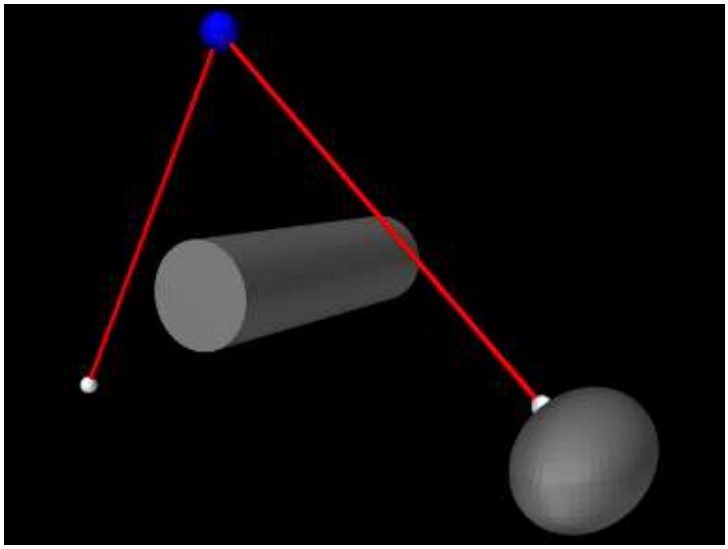


Figure 9.8: CylinderWrapping model loaded into ArtiSynth.

```
5       addModel (mech);
6
7       mech.setFrameDamping (100.0); // set damping parameters
8       mech.setRotaryDamping (10.0);
9
10      double density = 150;
11
12      Particle via0 = new Particle (/*mass=*/0, /*x,y,z=*/-1.0, 0.0, 4.0);
13      via0.setDynamic (false);
14      mech.addParticle (via0);
15      Particle p1 = new Particle (/*mass=*/0, /*x,y,z=*/-3.0, 0.0, 0.0);
16      p1.setDynamic (false);
17      mech.addParticle (p1);
18
19      // create cylindrical wrapping object
20      RigidCylinder cylinder = new RigidCylinder (
21         "cylinder", /*rad=*/0.5, /*height=*/3.5, density, /*nsides=*/50);
22      cylinder.setPose (new RigidTransform3d (0, 0, 1.5, 0, 0, Math.PI/2));
23      cylinder.setDynamic (false);
```

```
24        mech.addRigidBody (cylinder);
25
26        // create ellipsoidal wrapping object
27        double rad = 0.6;
28        RigidEllipsoid ellipsoid = new RigidEllipsoid (
29           "ellipsoid", /*a,b,c=*/rad, 2*rad, rad, density, /*nslices=*/50);
30        ellipsoid.transformGeometry (new RigidTransform3d (3, 0, 0));
31        mech.addRigidBody (ellipsoid);
32
33        // attach a marker to the ellipsoid
34        FrameMarker p0 = new FrameMarker ();
35        double halfRoot2 = Math.sqrt(2)/2;
36        mech.addFrameMarker (
37           p0, ellipsoid, new Point3d (-rad*halfRoot2, 0, rad*halfRoot2));
38
39        // enable collisions between the ellipsoid and cylinder
40        mech.setCollisionBehavior (cylinder, ellipsoid, true);
41
42        // create the spring, making both segments wrappable with 50 knots
43        MultiPointSpring spring = new MultiPointSpring ("spring", 300, 1.0, 0);
44        spring.addPoint (p0);
45        spring.setSegmentWrappable (50);
46        spring.addPoint (via0);
47        spring.setSegmentWrappable (50);
48        spring.addPoint (p1);
49        spring.addWrappable (cylinder);
50        spring.addWrappable (ellipsoid);
51        mech.addMultiPointSpring (spring);
52
53        // set various rendering properties
54        RenderProps.setSphericalPoints (mech, 0.1, Color.WHITE);
55        RenderProps.setSphericalPoints (p1, 0.2, Color.BLUE);
56        RenderProps.setSphericalPoints (spring, 0.1, Color.GRAY);
57        RenderProps.setCylindricalLines (spring, 0.03, Color.RED);
58
59        createControlPanel (spring);
60     }
61
62     private void createControlPanel (MultiPointSpring spring) {
63        ControlPanel panel = new ControlPanel ("options", "");
64        // creates a panel to control knot and A/B point visibility
65        panel.addWidget (spring, "drawKnots");
66        panel.addWidget (spring, "drawABPoints");
67        addControlPanel (panel);
68     }
69  }
```

Lines 4-17 of the `build()` method create a `MechModel` with two fixed particles `via0` and `p1` to be used as via and stop points. Next, two analytic wrappables are created: a `RigidCylinder` and a `RigidEllipsoid`, with the former fixed in place and the latter connected to the start of the spring via the marker `p0` (lines 20-37). Collisions are enabled between these two wrappables at line 40. The spring itself is created (lines 44-52), using `setSegmentWrappable()` to make the segments (`p0`, `via0`) and (`via0`, `p1`) wrappable with 50 knots each, and `addWrappable()` to make it aware of the two wrappables. Finally, render properties at set (lines 55-58), and a control panel (Section 5.1) is added that allows the spring's drawKnots and drawABPoints properties to be interactively set.

To run this example in ArtiSynth, select All demos > tutorial > CylinderWrapping from the Models menu. The model should load and initially appear as in Figure 9.8. Running the model will cause the ellipsoid to fall and the spring to wrap around the cylinder. Using the pull tool (Section "Pull Manipulation" in the ArtiSynth User Interface Guide) on the ellipsoid can cause additional motions and make it also collide with the spring. Selecting drawKnots or drawABPoints in the control panel will cause the spring to render its knots and/or A/B points.

## 9.3 General Surfaces and Distance Grids

As mentioned in Section 9.2, wrapping around general mesh geometries is implemented using a quadratically interpolated signed distance grid. By default, for a rigid body, this grid is generated automatically from the body's collision mesh (as returned by `getCollisionMesh()`; see Section 8.3).

Using a distance grid allows very efficient collision handling between the body and the wrap segment knots. However, it also means that the true wrapping surface is not actually the collision mesh itself, but instead the zero-valued isosurface associated with quadratic grid interpolation. Well-behaved wrapping behavior requires that this isosurface be smooth and free of sharp edges, so that knot motions remain relatively smooth as they move across it. Quadratic interpolation helps with this, which is the reason for employing it. Otherwise, one should try to ensure that (a) the collision mesh from which the grid is generated is itself smooth and free of sharp edges, and (b) the grid has sufficient resolution to not introduce discretization artifacts.

Muscle wrapping is often performed around structures such as bones, for which the representing surface mesh is often insufficiently smooth (especially if segmented from medical image data). In some cases, the distance grid's quadratic interpolation may provide sufficient smoothing on its own; to determine this, one should examine the quadratic isosurface as described below. In other cases, it may be necessary to explicitly smooth the mesh itself, either externally or within ArtiSynth using the LaplacianSmoother class, which can apply iterations of either Laplacian or volume-preserving Taubin smoothing, via the method

```
LaplacianSmoother.smooth (mesh, numi, lam, mu);
```

Here `numi` is the number of iterations and `tau` and `mu` are the Taubin parameters. Setting `lam` $= 1$ and `mu` $= 0$ results in traditional Laplacian smoothing. If this causes the mesh to shrink more than desired, one can counter this by setting `tau` and `mu` to values used for Taubin smoothing, as described in [25].

> If the mesh is large (i.e., has many vertices), then smoothing it may take noticeable computational time. In such cases, it is generally best to simply save and reuse the smoothed mesh.

By default, if a rigid body contains only one polygonal mesh, then its surface and collision meshes (returned by getSurfaceMesh() and getCollisionMesh(), respectively) are the same. However, if it is necessary to significantly smooth or modify the collision mesh, for wrapping or other purposes, it may be desirable to use different meshes for the surface and collision. This can be done by making the surface mesh non-collidable and adding an additional mesh that *is* collidable, as discussed in Section 3.2.9 as illustrated by the following code fragment:

```
PolygonalMesh surfaceMesh;
PolygonalMesh wrappingMesh;

// ... initialize surface and wrapping meshes ...

// create the body from the surface mesh
RigidBody body = RigidBody.createFromMesh (
    "body", mesh, /*density=*/1000, /*scale=*/1.0);

// set the surface mesh to be non-collidable, and add the wrapping mesh as
// collidable but not having mass
body.getSurfaceMeshComp().setIsCollidable (false);
RigidMeshComp wcomp = body.addMesh (
    wrappingMesh, /*hasMass=*/false, /*collidable=*/true);
RenderProps.setVisible (wcomp, false); // hide the wrapping mesh
```

Here, to ensure that the wrapping mesh does *not* to contribute to the body's inertia, its hasMass property is set to false.

Although it is possible to specify a collision mesh that is separate from the surface mesh, there is currently no way to specify *separate* collision meshes for wrapping and collision handling. If this is desired for some reason, then one alternative is to create a separate body for wrapping purposes, and then attach it to the main body, as described in Section 9.5.

To verify that the distance grid's quadratic isosurface is sufficiently smooth for wrapping purposes, it is useful to visualize the both distance grid and its isosurface directly, and if necessary adjust the resolution used to generate the grid. This can be accomplished using the body's DistanceGridComp, which is a subcomponent named `distanceGrid` and which may be obtained using the method

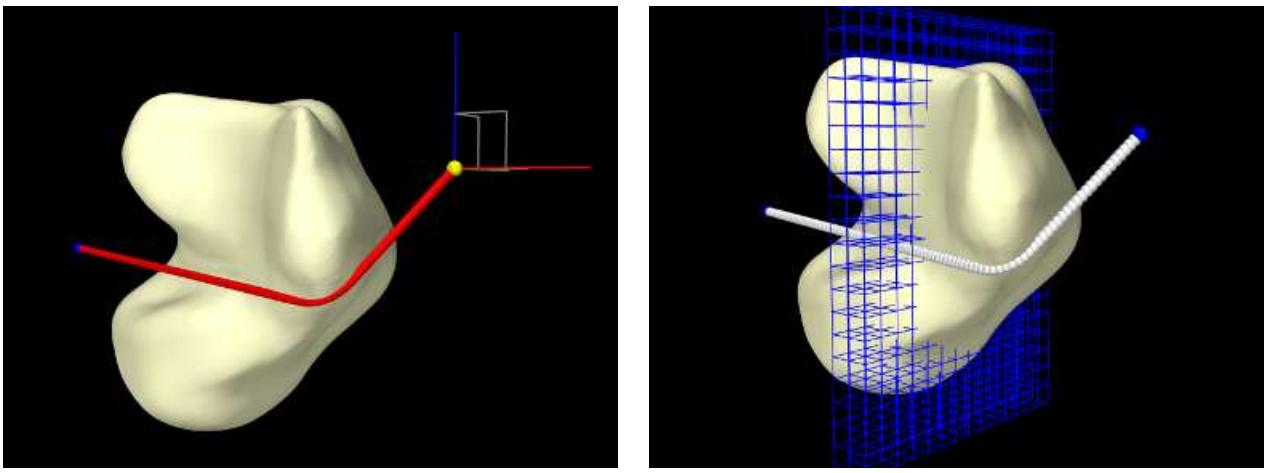


Figure 9.9: TalusWrapping model, with a dragger being used to move `p0` (left), and the knots visible and grid visible with restricted range (right).

```
DistanceGridComp getDistanceGridComp()
```

A `DistanceGridComp` exports a number of properties that can be used to control the grid's visualization, resolution, and fit around the collision mesh. These properties are described in detail in Section 4.6, and can be set either in code using their set/get accessors, or interactively using custom control panels or by selecting the grid component in the GUI and choosing Edit properties ... from the right-click context menu.

> When rendering the mesh isosurface, it is usually desirable to also disable rendering of the collision meshes within the rigid body. For convenience, this can be accomplished by setting the body's gridSurfaceRendering property to `true`, which will cause the grid isosurface to be rendered *instead* of the body's meshes. The isosurface type will be that indicated by the grid component's surfaceType property (which should be `QUADRATIC` for the quadratic isosurface), and the rendering will occur independently of the visibility settings for the meshes or the grid component.

### 9.3.1 Example: wrapping around a bone

An example of wrapping around a general mesh is given by `artisynth.demos.tutorial.TalusWrapping`. It consists of a MultiPointSpring anchored by two via points and wrapped around a rigid body representing a talus bone. The code, with include directives omitted, is given below:

```
1  public class TalusWrapping extends RootModel {
2
3     private static Color BONE = new Color (1f, 1f, 0.8f);
4     private static double DTOR = Math.PI/180.0;
5
6     public void build (String[] args) {
7
8        MechModel mech = new MechModel ("mech");
9        addModel (mech);
10
11       // read in the talus bone mesh
12       PolygonalMesh mesh = null;
13       try {
14          mesh = new PolygonalMesh (
15             PathFinder.findSourceDir(this) + "/data/TalusBone.obj");
16       }
17       catch (Exception e) {
18          System.out.println ("Error reading mesh:" + e);
19       }
```

```
20        // smooth the mesh using 20 iterations of regular Laplacian smoothing
21        LaplacianSmoother.smooth (mesh, /*count=*/20, /*lambda=*/1, /*mu=*/0);
22        // create the talus body from the mesh
23        RigidBody talus = RigidBody.createFromMesh (
24           "talus", mesh, /*density=*/1000, /*scale=*/1.0);
25        mech.addRigidBody (talus);
26        talus.setDynamic (false);
27        RenderProps.setFaceColor (talus, BONE);
28
29        // create start and end points for the spring
30        Particle p0 = new Particle (/*mass=*/0, /*x,y,z=*/2, 0, 0);
31        p0.setDynamic (false);
32        mech.addParticle (p0);
33        Particle p1 = new Particle (/*mass=*/0, /*x,y,z=*/-2, 0, 0);
34        p1.setDynamic (false);
35        mech.addParticle (p1);
36
37        // create a wrappable spring using a SimpleAxialMuscle material
38        MultiPointSpring spring = new MultiPointSpring ("spring");
39        spring.setMaterial (
40           new SimpleAxialMuscle (/*k=*/0.5, /*d=*/0, /*maxf=*/0.04));
41        spring.addPoint (p0);
42        // add an initial point to the wrappable segment to make sure it wraps
43        // around the bone the right way
44        spring.setSegmentWrappable (
45           100, new Point3d[] { new Point3d (0.0, -1.0, 0.0) });
46        spring.addPoint (p1);
47        spring.addWrappable (talus);
48        spring.updateWrapSegments(); // update the wrapping path
49        mech.addMultiPointSpring (spring);
50
51        // set render properties
52        DistanceGridComp gcomp = talus.getDistanceGridComp();
53        RenderProps.setSphericalPoints (mech, 0.05, Color.BLUE); // points
54        RenderProps.setLineWidth (gcomp, 0); // normal rendering off
55        RenderProps.setCylindricalLines (spring, 0.03, Color.RED); // spring
56        RenderProps.setSphericalPoints (spring, 0.05, Color.WHITE); // knots
57
58        // create a control panel for interactive control
59        ControlPanel panel = new ControlPanel();
60        panel.addWidget (talus, "gridSurfaceRendering");
61        panel.addWidget (gcomp, "resolution");
62        panel.addWidget (gcomp, "maxResolution");
63        panel.addWidget (gcomp, "renderGrid");
64        panel.addWidget (gcomp, "renderRanges");
65        panel.addWidget (spring, "drawKnots");
66        panel.addWidget (spring, "wrapDamping");
67        addControlPanel (panel);
68     }
69  }
```

The mesh describing the talus bone is loaded from the file `"data/TalusBone.obj"` located beneath the model's source directory (lines 11-19), with the utility class PathFinder used to determine the file path (Section 2.6). To ensure better wrapping behavior, the mesh is smoothed using Laplacian smoothing (line 21) before being used to create the rigid body (lines 23-27). The spring and its anchor points `p0` and `p1` are created between lines 30-49, with the talus added as a wrappable. The spring contains a single segment which is made wrappable using 100 knots, and initialized with an intermediate point (line 45) to ensure that it wraps around the bone in the correct way. Intermediate points are described in more detail in Section 9.4.

Render properties are set at lines 52-56; this includes turning off rendering for grid normals by zeroing the lineWidth render property for the grid component.

Finally, lines 59-67 create a control panel (Section 5.1) for interactively controlling a variety of properties, including gridSurfaceRendering for the talus (to see the grid isosurface instead of the bone mesh), resolution, maxResolution,

renderGrid, and renderRanges for the grid component (to control its resolution and visibility), and drawKnots and wrapDamping for the spring (to make knots visible and to adjust the wrap damping as described in Section 9.6).

To run this example in ArtiSynth, select All demos > tutorial > TalusWrapping from the Models menu. Since all of the dynamic components are fixed, running the model will not cause any initial motion. However, while simulating, one can use the viewer's graphical dragger fixtures (see the section "Transformer Tools" in the ArtiSynth User Interface Guide) to move `p0` or `p1` and hence pull the spring across the bone surface (Figure 9.9, left). One can also interactively adjust the property settings in the control panel to view the grid, isosurface, and knots, and the adjust the grid's resolution. Figure 9.9, right, shows the model with renderGrid and drawKnots set to `true` and renderRanges set to `"10:12 * *"`.

## 9.4 Initializing the Wrap Path

By default, when a multipoint spring or muscle is initialized (either at the start of the simulation or as a result of calling updateWrapSegments()), each wrappable segment is initialized to a straight line between its via points. This path is then adjusted to avoid and wrap around obstacles, using artificial linear forces as described in Section 9.6. The result is a local shortest path that wraps around obstacles instead of penetrating them. However, in some cases, the initial path may not be the one desired; instead, one may want it to wrap around obstacles some other way. This can be achieved by specifying additional intermediate points to initialize the segment as a piecewise linear path which threads its way around obstacles in the desired manner (Figure 9.10). These are specified using the optional `initialPnts` argument to the `setSegmentWrappable()` methods.

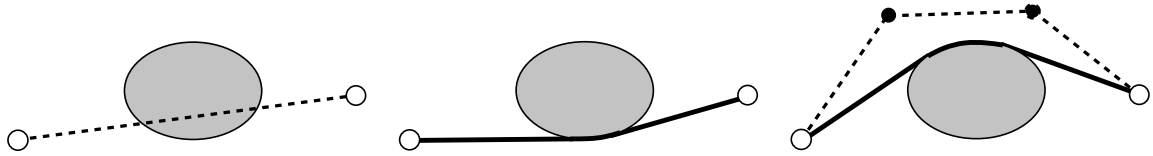


Figure 9.10: By default, the path for each wrappable segment is initialized to a straight line between its via points (dotted line, left), which is then adjusted to wrap around obstacles (solid line, middle). To cause the path to wrap around obstacles in a different way, it can instead be initialized using a piecewise-linear path defined by intermediate initial points (dotted line, right), which will then adjust to an alternate configuration.

When initial points are specified, it is recommended to finish construction of the spring or muscle with a call to updateWrapSegments(). This fits the wrappable segments to their correct path around the obstacles, which can then be seen immediately when the model is first loaded. On the other hand, by *omitting* an initial call to `updateWrapSegments()`, it is possible to see the initial path as specified by the initial points. This may be useful to verify that they are in the correct locations.

`MechModel` also supplies a convenience method, updateWrapSegments()), which calls `updateWrapSegments()` for every multipoint spring contained within it.

In some cases, initial points may also be necessary to help ensure that the initial path does not penetrate obstacles. While obstacle penetration will normally be resolved by the artificial forces described in Section 9.6, this may not always work correctly if the starting path penetrates an obstacle too deeply.

### 9.4.1 Example: wrapping around a torus

An example of using initial points is given by `artisynth.demos.tutorial.TorusWrapping`, in which a spring is wrapped completely around the inner section of a torus. The primary code for the build method is given below:

```
1        MechModel mech = new MechModel ("mech");
2        addModel (mech);
3
4        mech.setFrameDamping (1.0); // set damping parameters
```

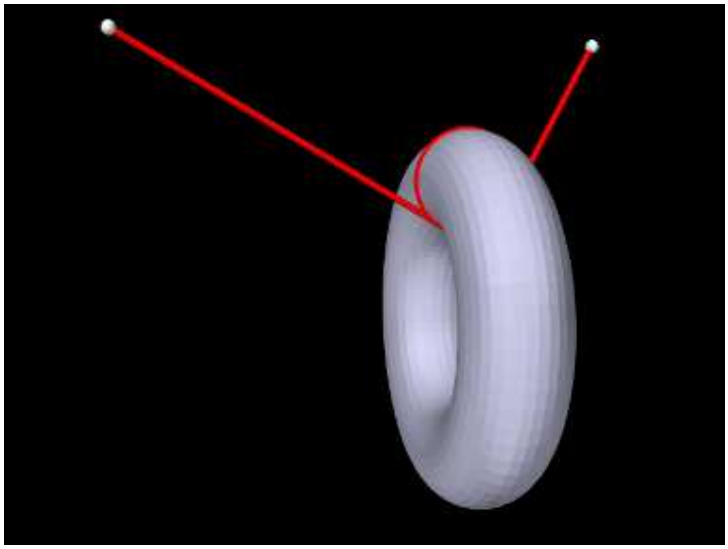


Figure 9.11: `TorusWrapping` model loaded into ArtiSynth.

```
5       mech.setRotaryDamping (10.0);
6
7       // create the torus
8       double DTOR = Math.PI/180;
9       double innerRad = 0.75;
10      double outerRad = 2.0;
11      RigidTorus torus =
12         new RigidTorus ("torus", outerRad, innerRad, /*density=*/1);
13      torus.setPose (new RigidTransform3d (2, 0, -2,  0, DTOR*90, 0));
14      mech.addRigidBody (torus);
15
16      // create start and end points for the spring
17      Particle p0 = new Particle (0, /*x,y,z=*/4, 0.2, 2);
18      p0.setDynamic (false);
19      mech.addParticle (p0);
20      Particle p1 = new Particle (0, /*x,y,z=*/-3, -0.2, 2);
21      p1.setDynamic (false);
22      mech.addParticle (p1);
23
24      // create a wrappable MultiPointSpring between p0 and p1, with initial
25      // points specified so that it wraps around the torus
26      MultiPointSpring spring =
27         new MultiPointSpring (/*k=*/10, /*d=*/0, /*restlen=*/0);
28      spring.addPoint (p0);
29      spring.setSegmentWrappable (
30         100, new Point3d[] {
31            new Point3d (3, 0, 0),
32            new Point3d (2, 0, -1),
33            new Point3d (1, 0, 0),
34            new Point3d (2, 0, 1),
35            new Point3d (3, 0, 0),
36            new Point3d (2, 0, -1),
37         });
38      spring.addPoint (p1);
39      spring.addWrappable (torus);
40      spring.updateWrapSegments(); // ``shrink wrap'' around torus
41      mech.addMultiPointSpring (spring);
```

The mech model is created in the usual way with frame and rotary damping set to 1 and 10 (lines 4-5). The torus is created using the analytic wrappable RigidTorus (lines 8-14). The spring start and end points `p0` and `p1` are created at lines (17-22), and the spring itself is created at lines (26-41), with six initial points being specified to

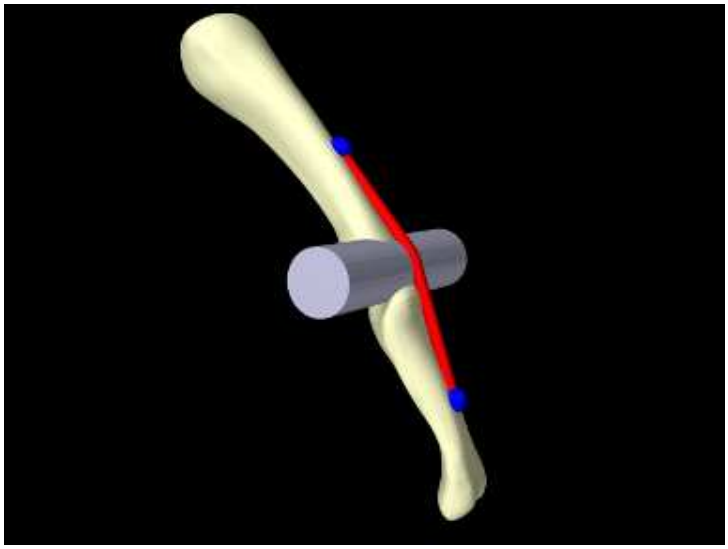


Figure 9.12: PhalanxWrapping model loaded into ArtiSynth.

`setSegmentWrappable()` to wrap the spring completely around the torus inner section.

To run this example in ArtiSynth, select All demos > tutorial > TorusWrapping from the Models menu. The torus will slide along the wrapped spring until it reaches equilibrium.

## 9.5 Alternate Wrapping Surfaces

Although it common to use the general mesh geometry of a `RigidBody` as the wrapping surface, situations may arise where it is desirable to *not* do this. These may include:

- The general mesh geometry is not sufficiently smooth to form a good wrapping surface;
- Wrapping around the default mesh geometry is not stable, in that it is too easy for the wrap strand to "slip off";
- Using one of the simpler analytic geometries (Table 9.1) may result in a more efficient computation.

There are a couple of ways to handle this. One, discussed in Section 9.3, involves creating a collision mesh which is separate from the general mesh geometry. However, that same collision mesh must then also be used for collision handling (Chapter 8). If that is undesirable, or if *multiple* wrapping surfaces are needed, then a different approach may be used. This involves creating the desired wrappable as a separate object and then *attaching* it to the main `RigidBody`. Typically, this wrappable will be created with zero mass (or density), so that it does not alter the effective mass or inertia of the main body. The general procedure then becomes:

1. Create the main `RigidBody` with whatever desired geometry and inertia is needed;
2. Create the additional wrappable object(s), usually with zero density/mass;
3. Attach the wrappables to the main body using one of the `MechModel attachFrame()` methods described in Section 3.7.3.

### 9.5.1 Example: wrapping for a finger joint

An example using an alternate wrapping surface is given by `artisynth.demos.tutorial.PhalanxWrapping`, which shows a a muscle wrapping around a joint between two finger bones. Because the bones themselves are fairly narrow, using them as wrapping surfaces would likely lead to the muscle slipping off. Instead, a RigidCylinder is used for the wrapping and attached to one of the bones. The code, with include directives excluded, is given below:

```
public class PhalanxWrapping extends RootModel {

   private static Color BONE = new Color (1f, 1f, 0.8f);
   private static double DTOR = Math.PI/180.0;

   private RigidBody createBody (MechModel mech, String name, String fileName) {
      // creates a bone from its mesh and adds it to a MechModel
      String filePath = PathFinder.findSourceDir(this) + "/data/" + fileName;
      RigidBody body = RigidBody.createFromMesh (
         name, filePath, /*density=*/1000, /*scale=*/1.0);
      mech.addRigidBody (body);
      RenderProps.setFaceColor (body, BONE);
      return body;
   }

   public void build (String[] args) {

      MechModel mech = new MechModel ("mech");
      addModel (mech);

      // create the two phalanx bones, and offset them
      RigidBody proximal = createBody (mech, "proximal", "HP3ProximalLeft.obj");
      RigidBody distal = createBody (mech, "distal", "HP3MiddleLeft.obj");
      distal.setPose (new RigidTransform3d (0.02500, 0.00094, -0.03979));

      // make the proximal phalanx non dynamic; add damping to the distal
      proximal.setDynamic (false);
      distal.setFrameDamping (0.03);

      // create a revolute joint between the bones
      RigidTransform3d TJW =
         new RigidTransform3d (0.018, 0, -0.022, 0, 0, -DTOR*90);
      HingeJoint joint = new HingeJoint (proximal, distal, TJW);
      joint.setShaftLength (0.02); // render joint as a blue cylinder
      RenderProps.setFaceColor (joint, Color.BLUE);
      mech.addBodyConnector (joint);

      // create markers for muscle origin and insertion points
      FrameMarker origin = mech.addFrameMarkerWorld (
         proximal, new Point3d (0.0098, -0.0001, -0.0037));
      FrameMarker insertion = mech.addFrameMarkerWorld (
         distal, new Point3d (0.0293, 0.0009, -0.0415));

      // create a massless RigidCylinder to use as a wrapping surface and
      // attach it to the distal bone
      RigidCylinder cylinder = new RigidCylinder (
         "wrapSurface", /*rad=*/0.005, /*h=*/0.04, /*density=*/0, /*nsegs=*/32);
      cylinder.setPose (TJW);
      mech.addRigidBody (cylinder);
      mech.attachFrame (cylinder, distal);

      // create a wrappable muscle using a SimpleAxialMuscle material
      MultiPointSpring muscle = new MultiPointMuscle ("muscle");
      muscle.setMaterial (
         new SimpleAxialMuscle (/*k=*/0.5, /*d=*/0, /*maxf=*/0.04));
      muscle.addPoint (origin);
      // add an initial point to the wrappable segment to make sure it wraps
      // around the cylinder the right way
      muscle.setSegmentWrappable (
         50, new Point3d[] { new Point3d (0.025, 0.0, -0.02) });
      muscle.addPoint (insertion);
      muscle.addWrappable (cylinder);
      muscle.updateWrapSegments(); // ``shrink wrap'' around cylinder
      mech.addMultiPointSpring (muscle);
```

```
65
66          // set render properties
67          RenderProps.setSphericalPoints (mech, 0.002, Color.BLUE);
68          RenderProps.setCylindricalLines (muscle, 0.001, Color.RED);
69          RenderProps.setFaceColor (cylinder, new Color (200, 200, 230));
70       }
71    }
```

The method `createBody()` (lines 6-14) creates a rigid body from a geometry mesh stored in a file in the directory "`data`" beneath the source directory, using the utility class PathFinder used to determine the file path (Section 2.6).

Within the `build()` method, a `MechModel` is created containing two rigid bodies representing the bones, `proximal` and `distal`, with `proximal` fixed and `distal` free to move with a frame damping of 0.03 (lines 18-28). A cylindrical joint is then added between the bones, along with markers describing the muscle's origin and insertion points (lines 31-42). A `RigidCylinder` is created to act as a wrapping obstacle and attached to the `distal` bone in the same location as the joint (lines 46-50); since it is created with a density of 0 it has no mass and hence does not affect the bone's inertia. The muscle itself is created at lines 53-64, using a SimpleAxialMuscle as a material and an extra initial point specified to `setSegmentWrappable()` to ensure that it wraps around the cylinder in the correct way (Section 9.4). Finally, some render properties are set at lines 67-69.

To run this example in ArtiSynth, select All demos > tutorial > PhalanxWrapping from the Models menu. The model should load and initially appear as in Figure 9.12. When running the model, one can move the `distal` bone either by using the pull tool (Section "Pull Manipulation" in the ArtiSynth User Interface Guide), or selecting the muscle in the GUI, invoking a property dialog by choosing Edit properties ... from the right-click context menu, and adjusting the excitation property.

### 9.5.2 Example: toy muscle arm with wrapping

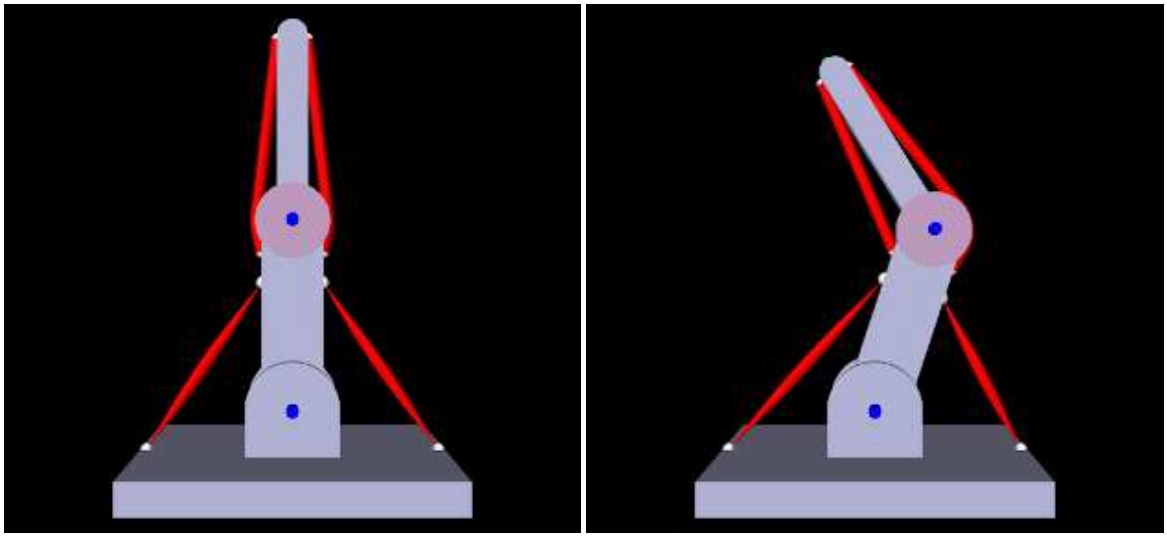


Figure 9.13: Left: ToyMuscleArm loaded into ArtiSynth. Right: demo running with exciter values set from the control panel.

A more complex example of wrapping, combined with multiple joints and point-to-point muscles, is given by `artisynth.demos.tutorial.ToyMuscleArm`, which depicts a two-link "arm", connected by hinge joints, with opposing muscles controlling the poses of each link. The code, with include directives excluded, is given below:

```
1  public class ToyMuscleArm extends RootModel {
2     // geodir is folder in which to locate mesh data:
3     protected String geodir = PathFinder.getSourceRelativePath (this, "data/");
4     protected double density = 1000.0;    // default density
5     protected double stiffness = 1000.0; // passive muscle stiffness
6
7     protected MechModel myMech;      // mech model
```

```
8     protected FrameMarker myTipMkr; // marker at tip of the arm
9     protected HingeJoint myHinge0;  // lower hinge joint
10    protected HingeJoint myHinge1;  // upper hinge joint
11    protected RigidBody myLink0;    // lower link
12    protected RigidBody myLink1;    // upper link
13
14    /**
15     * Add an axial muscle between body0 and body1 using markers attached at
16     * world coords (x0,0,z0) and (x1,0,z1), respectively.
17     */
18    public Muscle addMuscle (
19       RigidBody body0, double x0, double z0,
20       RigidBody body1, double x1, double z1) {
21
22       FrameMarker l0 = myMech.addFrameMarkerWorld (body0, new Point3d(x0, 0, z0));
23       FrameMarker l1 = myMech.addFrameMarkerWorld (body1, new Point3d(x1, 0, z1));
24       Muscle muscle = new Muscle ();
25       muscle.setMaterial (
26          new SimpleAxialMuscle (stiffness, /*damping=*/0, /*fmax=*/1000));
27       myMech.attachAxialSpring (l0, l1, muscle);
28       muscle.setRestLength (muscle.getLength()); // init rest length
29       return muscle;
30    }
31
32    /**
33     * Add an wrapped muscle between body0 and body1 using markers attached at
34     * world coords (x0,0,z0) and (x1,0,z1) and wrapping around wrapBody.
35     */
36    public MultiPointMuscle addWrappedMuscle (
37       RigidBody body0, double x0, double z0,
38       RigidBody body1, double x1, double z1, Wrappable wrapBody) {
39
40       FrameMarker l0 = myMech.addFrameMarkerWorld (body0, new Point3d(x0, 0, z0));
41       FrameMarker l1 = myMech.addFrameMarkerWorld (body1, new Point3d(x1, 0, z1));
42       MultiPointMuscle muscle = new MultiPointMuscle ();
43       muscle.setMaterial (
44          new SimpleAxialMuscle (stiffness, /*damping=*/0, /*fmax=*/500));
45       muscle.addPoint (l0);
46       muscle.setSegmentWrappable (/*numknots=*/50);
47       muscle.addPoint (l1);
48       muscle.addWrappable (wrapBody);
49       muscle.updateWrapSegments(); // shrink wrap to current wrappable
50       muscle.setRestLength (muscle.getLength()); // init rest length
51       myMech.addMultiPointSpring (muscle);
52       return muscle;
53    }
54
55    /**
56     * Adds a hinge joint between body0 and body1, at world coordinates
57     * (x0,0,z0) and with the joint axis parallel to y. Joint limits are set to
58     * minDeg and maxDeg (in degrees).
59     */
60    public HingeJoint addHingeJoint (
61       RigidBody body0, RigidBody body1,
62       double x0, double z0, double minDeg, double maxDeg) {
63
64       HingeJoint hinge = new HingeJoint (
65          body0, body1, new Point3d(x0, 0, z0), new Vector3d (0, 1, 0));
66       myMech.addBodyConnector (hinge);
67       hinge.setThetaRange (minDeg, maxDeg);
68       // set render properties for the hinge
69       hinge.setShaftLength (0.4);
70       RenderProps.setFaceColor (hinge, Color.BLUE);
71       return hinge;
72    }
```

```
73
74    public void build (String[] args) throws IOException {
75       myMech = new MechModel ("mech");
76       myMech.setInertialDamping (1.0);
77       addModel (myMech);
78
79       // create base body
80       PolygonalMesh mesh = new PolygonalMesh (geodir+"flangedBase.obj");
81       RigidBody base = RigidBody.createFromMesh ("base", mesh, density, 1.0);
82       base.setDynamic (false);
83       myMech.addRigidBody (base);
84
85       // create rigid body for link0 and place it above the origin
86       mesh = MeshFactory.createRoundedBox (0.6, 0.2, 0.2, /*nslices=*/20);
87       myLink0 = RigidBody.createFromMesh ("link0", mesh, density, /*scale=*/1.0);
88       myLink0.setPose (new RigidTransform3d (0, 0, 0.3));
89       myMech.addRigidBody (myLink0);
90
91       // create rigid body for link1 and place it after link0
92       mesh = MeshFactory.createRoundedBox (0.6, 0.10, 0.15, /*nslices=*/20);
93       myLink1 = RigidBody.createFromMesh ("link1", mesh, density, /*scale=*/1.0);
94       myLink1.setPose (new RigidTransform3d (0, 0, 0.9));
95       myMech.addRigidBody (myLink1);
96
97       // create massless cylinder for wrapping surface and attach it to link1
98       RigidCylinder cylinder = new RigidCylinder (
99          "wrapSurface", /*rad=*/0.12, /*h=*/0.25, /*density=*/0, /*nsegs=*/32);
100      cylinder.setPose (new RigidTransform3d (0, 0, 0.6,  0, 0, Math.PI/2));
101      myMech.addRigidBody (cylinder);
102      myMech.attachFrame (cylinder, myLink1);
103
104      // add a hinge joints between links
105      myHinge0 = addHingeJoint (myLink0, base, 0, 0, -70, 70);
106      myHinge1 = addHingeJoint (myLink1, myLink0, 0, 0.6, -120, 120);
107
108      Muscle muscleL0 = addMuscle (myLink0, -0.1, 0.4, base, -0.48, -0.15);
109      Muscle muscleR0 = addMuscle (myLink0,  0.1, 0.4, base,  0.48, -0.15);
110      MultiPointMuscle muscleL1 = addWrappedMuscle (
111         myLink1, -0.05, 1.2, myLink0, -0.1, 0.5, cylinder);
112      MultiPointMuscle muscleR1 = addWrappedMuscle (
113         myLink1,  0.05, 1.2, myLink0,  0.1, 0.5, cylinder);
114
115      // add a marker at the tip
116      myTipMkr = myMech.addFrameMarkerWorld (
117         myLink1, new Point3d(0.0, 0.0, 1.25));
118
119      // add a control panel for muscle excitations
120      ControlPanel panel = new ControlPanel ();
121      panel.addWidget ("excitation L0", muscleL0, "excitation");
122      panel.addWidget ("excitation R0", muscleR0, "excitation");
123      panel.addWidget ("excitation L1", muscleL1, "excitation");
124      panel.addWidget ("excitation R1", muscleR1, "excitation");
125      addControlPanel (panel);
126
127      // render properties: muscles as red spindles, points as white spheres,
128      // bodies blue-gray, tip marker green, and wrap cylinder purple-gray
129      RenderProps.setSpindleLines (myMech, 0.02, Color.RED);
130      RenderProps.setSphericalPoints (myMech, 0.02, Color.WHITE);
131      RenderProps.setFaceColor (myMech, new Color (0.71f, 0.71f, 0.85f));
132      RenderProps.setPointColor (myTipMkr, Color.GREEN);
133      RenderProps.setFaceColor (cylinder, new Color (0.75f, 0.61f, 0.75f));
134   }
135 }
```

Within the `build()` method, the mech model is created in the usual way and inertial damping is set to 1.0 (lines 75-77).

Next, the rigid bodies depicting the base and links are created (lines 80-95). All bodies are created from meshes, with the base being made non-dynamic and the poses of the links set so that they line up vertically in their start position (Figure 9.13, left). A massless cylinder is created to provide a wrapping surface around the second hinge joint and attached to link1 (lines 98-102).

Hinge joints (Section 3.4.1) are then added between the base and link0 and link0 and link1 (lines 105-106), using the convenience method `addHingeJoint()` (lines 60-72). This method creates a hinge joint at a prescribed position in the x-z plane, uses it to connect two bodies, sets the range for the joint angle theta, and sets properties to render the shaft in blue.

Next, point-to-point muscles are placed on opposite sides of each link (lines 108-113). For link0, two `Muscle` components are used, added with the method `addMuscle()` (lines 18-30); this method creates a muscle connecting two bodies, using frame markers whose positions are specified in world coordinates in the x-z plane, and a `SimpleMuscleMaterial` (Section 4.5.1.1) with prescribed stiffness and maximum active force. For link1, `MultiPointMuscle` components are used instead so that they can be wrapped around the wrapping cylinder. These muscles are created with the method `addWrappedMuscle()` (lines 36-53), which creates a multipoint muscle connecting two bodies, using a single wrappable segment between two frame markers (also specified in the x-z plane). After the muscle is placed, its `updateWrapSegments()` method is called to "shrink wrap" it around the cylinder. Both the methods `addMuscle()` and `addWrappedMuscle()` set the rest length of each configured muscle to its current length (lines 28 and 50) to ensure that it will not exert passive tension in its initial configuration.

A marker is attached to the tip on link1 at lines 140-141, and then a control panel is created to allow runtime adjustment of the four muscle excitation values (lines 144-149). Finally, render properties are set at lines 153-157.

To run this example in ArtiSynth, select All demos > tutorial > ToyMuscleArm from the Models menu. The loaded model will appear as in Figure 9.13, left. When the model is run, the links can be moved by adjusting the muscle excitation values in the control panel (Figure 9.13, right).

## 9.6 Tuning the Wrapping Behavior

Wrappable segments are implemented internally using artificial linear elastic forces to draw the knots together and keep them from penetrating obstacles. These artificial forces are invisible to the simulation: the wrapping segment has no mass, and the knot forces are used to create what is essentially a first order physics that "shrink wraps" each segment around the obstacles at the beginning of each simulation step, forming a shortest-distance geodesic curve from which the wrapping contact points A and B are calculated. This process is now described in more detail.

Assume that a wrappable segment has $m$ knots, indexed by $k = 1, \ldots, m$, each located at a position $\mathbf{x}_k$. Two types of artificial forces then act on each knot: a *wrapping force* that pulls it closer to other knots, and *contact forces* that push it away from wrappable obstacles. The wrapping force is given by

$$\mathbf{f}_{w,k} = K_w(\mathbf{x}_{k+1} - 2\mathbf{x}_k + \mathbf{x}_{k-1})$$

where $K_w$ is the *wrapping stiffness*. To determine the contact forces, we compute, for each wrappable, the knot's distance to the surface $d_k$ and associated normal direction $\mathbf{n}_k$, where $d_k < 0$ implies that the knot is inside. These quantities are determined either analytically (for analytic wrappables, Table 9.1), or using a signed distance grid (for general wrappables, Section 9.3). The contact forces are then given by

$$\mathbf{f}_{c,k} = \begin{cases} -K_c\, d_k\, \mathbf{n}_k & \text{if } d_k < 0 \\ 0 & \text{otherwise,} \end{cases}$$

where $K_c$ is the *contact stiffness*.

The total force $\mathbf{f}_k$ acting on each knot is then given by

$$\mathbf{f}_k = \mathbf{f}_{w,k} + \sum_c \mathbf{f}_{c,k}$$

where the latter term is the sum of contact forces for all wrappables. If we let $\mathbf{x}$ and $\mathbf{f}$ denote the aggregate position and force vectors for all knots, then computing the wrap path involves finding the equilibrium position such that $\mathbf{f}(\mathbf{x}) = 0$. This is done at the beginning of each simulation step, or whenever updateWrapSegments() is called, and is achieved iteratively using Newton's method. If $\mathbf{x}^j$ and $\mathbf{f}(\mathbf{x}^j)$ denote the positions and forces at iteration $j$, and

$$\mathbf{K} \equiv \frac{\partial \mathbf{f}}{\partial \mathbf{x}}$$

denotes the local force derivative (or "stiffness"), then the basic Newton update is given by

$$\mathbf{x}^{j+1} = \mathbf{x}^j - \mathbf{K}^{-1}\mathbf{f}(\mathbf{x}^j).$$

In practice, to help deal with the nonlinearities associated with contact, we use a damped Newton update,

$$\mathbf{x}^{j+1} = \mathbf{x}^j + \alpha(D\mathbf{I} - \mathbf{K})^{-1}\mathbf{f}(\mathbf{x}^j), \tag{9.1}$$

where $D$ is a constant *wrap damping* parameter, and $\alpha$ is an adaptively computed step size adjustment. The computation of (9.1) can be performed quickly, in $O(m)$ time, since $\mathbf{K}$ is a block-tridiagonal matrix, and the number of iterations required is typically small (on the order of 10 or less), particularly since the iterative procedure continues across simulation steps and so $\mathbf{f}(\mathbf{x})$ does not need to be brought to 0 for any given step. The maximum number of Newton iterations used for each time step is $N_{\max}$.

> Again, it is important to understand the artificial knot forces $\mathbf{f}(\mathbf{x})$ described here are separate from the physical spring/muscle tension forces $f(l,\dot{l},a)$ discussed in Sections 3.1.1 and 4.5.1, and *only* facilitate the computation of each wrappable segment's path around obstacles.

The default values for the wrapping parameters are $K_w = 1$, $K_c = 10$, $D = 10$, and $N_{\max} = 10$, and these often give satisfactory results without the need for modification. However, in some situations the default muscle wrapping may not perform adequately and it is necessary to adjust these parameters. Problems may include:

- The wrapping path does not settle down and tends to "jump around". Solutions include increasing the damping parameter $D$ or the maximum number of wrap iterations $N_i$. For general wrapping surfaces (Section 9.3), one should also ensure that the surface is sufficiently smooth.
- A wrapping surface is too thin and so the wrapping path "jumps through" it. Solutions include increasing the damping parameter $D$, increasing the number of knots in the segment, or decreasing the simulation step size. An alternative approach is to use an alternative wrapping surface (Section 9.5) that is thicker and better behaved.

Wrapping parameters are exported as properties of `MultiPointSpring` and `MultiPointMuscle`, and may be changed in code (using their set/get accessors), or interactively, either by exposing them through a control panel, or by selecting the spring/muscle in the GUI and choosing Edit properties ... from the right-click context menu. Property values include:

**wrapStiffness** Wrapping stiffness $K_w$ between knot points (default value 1). Since the wrapping behavior is determined by the damping to stiffness *ratio*, it is generally not necessary to change this value.

**wrapDamping** Damping factor $D$ (default value 10). Increasing this value relative to $K_w$ results in wrap path motions that are smoother and less likely to penetrate obstacles, but which are also less dynamically responsive. Applications generally work with damping values between 10 and 100 (assuming $K_w = 1$).

**contactStiffness** Contact stiffness $K_c$ used to resolve obstacle penetration (default value 10). It is generally not necessary to change this value. Decreasing it will increase the distance that knots are permitted to penetrate obstacles, which *may* result in a slightly more stable contact behavior.

**maxWrapIterations** Maximum number of Newton iterations $N_{\max}$ per time step (default value 10). If the wrapping simulation exhibits instability, particularly with regard to obstacle contact, increasing the number of iterations (to say 100) may help.

In addition, `MultiPointSpring` and `MultiPointMuscle` also export the following properties to control the rendering of knot and A/B points:

**drawKnots** If true, renders the knot points in each wrappable segment. This can be useful to visualize the knot density. Knots are rendered using the style, size, and color given by the pointStyle, pointRadius, pointSize, and pointColor values of the spring/muscle's render properties.

**drawABPoints** If true, renders the A/B points. These are the first and last points of contact that a wrap segment makes with each wrappable, and correspond to the points where the spring/muscle's tension acts on that wrappable (Section 9 and Figure 9.4). A/B points are rendered using the style and size given by the pointStyle, pointRadius ($\times 1.2$) and pointSize values of the spring/muscle's render properties, and the color given by the ABPointColor property.

# Chapter 10

# Inverse Simulation

## 10.1 Overview

ArtiSynth supports an inverse simulation capability that allows the computation of the excitation signals (for muscles and other actuators) needed to track target trajectories for a set of *source* components within the model (typically points or frames). This is achieved using a specialized controller component known as a TrackingController, defined in the package

```
artisynth.core.inverse
```

An application model creates the tracking controller and configures it with the source components, the *exciter* components needed to effect the tracking, and probes or controllers to specify the target trajectories. Then at each simulation time step the controller computes the excitations (i.e., signals to the exciter components) that allow the trajectories to be followed as closely as possible, in a manner conceptually similar to computed muscle control in OpenSim [7].

> It is currently recommended to run inverse simulation with *hybrid solving* disabled. Hybrid solving combines direct and iterative solution techniques to speed up sparse matrix solves within ArtiSynth's integrators. However, it can occasionally cause stability issues when used with inverse control. Hybrid solves can be disabled in several ways:
>
> 1. By setting hybridSolvesEnabled to `false` under "Settings > Simulation ..." in the GUI; see "Simulation" under "Settings and Preferences" in the ArtiSynth User Interface Guide. This change can be made to persist across ArtiSynth restarts.
> 2. By calling the static method `setHybridSolvesEnabled()` of MechSystemSolver in the model's `build()` method:
>
>    ```
>    MechSystemSolver.setHybridSolvesEnabled(false);
>    ```

### 10.1.1 Tracking controller operation

Let $\mathbf{q}$, $\mathbf{u}$ and $\mathbf{f}$ be composite vectors describing the positions, velocities and forces of all the components in the application model, and let $\mathbf{a}$ be a vector of excitation signals for all the excitation components available to the controller. $\mathbf{f}$ can be decomposed into

$$\mathbf{f} = \mathbf{f}_p(\mathbf{q},\mathbf{u}) + \mathbf{f}_a(\mathbf{q},\mathbf{u},\mathbf{a}) \tag{10.1}$$

where $\mathbf{f}_p$ are the *passive* forces corresponding to zero excitation, and $\mathbf{f}_a$ are the *active* forces. During simulation, the controller computes values for $\mathbf{a}$ to best track the target trajectories.

In the case of motion tracking, let $\mathbf{x}$ and $\mathbf{v}$ be the subvectors of $\mathbf{q}$ and $\mathbf{u}$ containing the positions and velocities of the source components, and let $\mathbf{x}_t$ and $\mathbf{v}_t$ be the corresponding target trajectories. At the beginning of each time step, the controller compares the current source velocity state $(\mathbf{x},\mathbf{v})$ with the desired target state $(\mathbf{x}_t,\mathbf{v}_t)$ and uses this to determine a desired source velocity $\mathbf{v}_*$ for the next time step; this computation is done by the *motion tracker* (Figure 10.1, Section 10.1.2). An *excitation generator* then computes $\mathbf{a}$ in order to try and make $\mathbf{v}$ match $\mathbf{v}_*$ (Section 10.1.3).

The excitation generator accepts a desired velocity $\mathbf{v}_*$ instead of a desired acceleration because ArtiSynth's physics engine computes velocity changes directly.

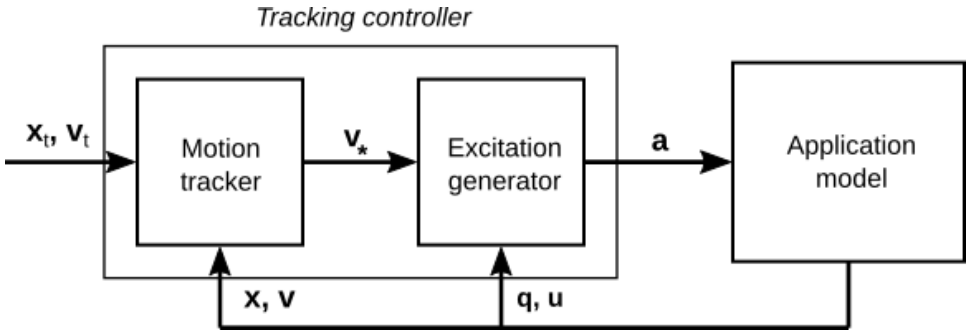



Figure 10.1: Tracking controller operation for motion targets.

### 10.1.2 Motion tracking

As mentioned above, the motion tracker computes a desired source velocity $\mathbf{v}_*$ for the next time step based on the current source state $(\mathbf{x}, \mathbf{v})$ and desired target state $(\mathbf{x}_t, \mathbf{v}_t)$. This can be done in two ways: *chase control* (the default), and *PD control*.

#### 10.1.2.1 Chase control

Chase control simply sets $\mathbf{v}_*$ to $\mathbf{v}_t$, plus an additional velocity that tries to reduce the position error $\mathbf{x}_t - \mathbf{x}$ over a specified time interval $T$, called the *chase time*:

$$\mathbf{v}_* = \mathbf{v}_t + \frac{\mathbf{x}_t - \mathbf{x}}{T}. \tag{10.2}$$

In general, $T$ should be greater than or equal to the simulation step size $h$. If it is greater, then $\mathbf{x}$ will tend to lag behind $\mathbf{x}_t$, but this will also reduce the potential for overshoot due to system nonlinearities. Conversely, if $T$ if *less* than $h$, then $\mathbf{x}$ is much more likely to overshoot. The default value of $T$ is 0.01.

#### 10.1.2.2 PD control

PD control computes a desired source *acceleration* $\dot{\mathbf{v}}_*$ based on

$$\dot{\mathbf{v}}_* = K_p(\mathbf{x}_t - \mathbf{x}) + K_d(\mathbf{v}_t - \mathbf{v}), \tag{10.3}$$

and then integrates this to determine $\mathbf{v}_*$:

$$\mathbf{v}_* = \mathbf{v} + h\dot{\mathbf{v}}_*, \tag{10.4}$$

where $h$ is the simulation step size. PD control offers a greater ability to adjust the tracking behavior than chase control, but it is often necessary to tune the gain parameters $K_p$ and $K_d$. One rule of thumb is to set their initial values such that (10.2) and (10.4) are equal, which leads to

$$K_p = \frac{1}{hT}, \quad K_d = \frac{1}{h}.$$

The default values for $K_p$ and $K_d$ are 10000 and 100, corresponding to $h$ and $T$ both equaling 0.01. Lowering the value of $K_p$ will reduce overshoot but increase tracking lag.

PD control is enabled and adjusted by setting the usePDControl, Kp and Kd properties of the motion target term to `true` (Section 10.3.3).

### 10.1.3 Generating excitations using a quadratic program

Given $\mathbf{v}_*$, the excitation generator computes $\mathbf{a}$ to try and ensure that the velocity at the end of the subsequent time step, $\mathbf{v}^{k+1}$, satisfies

$$\mathbf{v}^{k+1} \approx \mathbf{v}_*. \tag{10.5}$$

This is accomplished using a quadratic program.

First, for a broad class of problems, $\mathbf{f}$ is linearly related to $\mathbf{a}$, so that (10.1) simplifies to

$$\mathbf{f} = \mathbf{f}_p(\mathbf{q},\mathbf{u}) + \Lambda(\mathbf{q},\mathbf{u})\mathbf{a}, \tag{10.6}$$

where $\Lambda$ is an excitation response matrix. Given (10.6), it is possible to show (section "Inverse modeling" in [11]) that

$$\mathbf{v}^{k+1} = \mathbf{v}_0 + \mathbf{H}_m\mathbf{a},$$

where $\mathbf{v}_0$ is the velocity with zero excitations and $\mathbf{H}_m$ is a motion excitation response matrix. To best follow the trajectory, we can compute $\mathbf{a}$ to minimize the quadratic cost function

$$\phi_m(\mathbf{a}) \equiv \frac{1}{2}\|\mathbf{v}_* - \mathbf{v}^{k+1}\| = \frac{1}{2}\|\bar{\mathbf{v}} - \mathbf{H}_m\mathbf{a}\|^2, \quad \bar{\mathbf{v}} \equiv \mathbf{v}_* - \mathbf{v}_0. \tag{10.7}$$

If we add in the constraint that excitations lie in the range $[0,1]$, the problem takes the form of a quadratic program (QP)

$$\begin{aligned} &\min_{\mathbf{a}} \ \phi_m(\mathbf{a}) \\ &\text{subject to} \quad 0 \le \mathbf{a} \le \mathbf{1}. \end{aligned} \tag{10.8}$$

In order to prioritize some target terms over others, $\phi_m(\mathbf{a})$ can be modified to include weights, according to

$$\phi_m(\mathbf{a}) \equiv \frac{1}{2}\|\mathbf{W}_m(\bar{\mathbf{v}} - \mathbf{H}_m\mathbf{a})\|^2, \tag{10.9}$$

where $\mathbf{W}_m$ is a diagonal weighting matrix. To handle excitation redundancy, where the size of $\mathbf{a}$ exceeds the size of $\mathbf{v}$, we can add a regularization term $\frac{1}{2}\mathbf{a}^T\mathbf{W}_a\mathbf{a}$, where $\mathbf{W}_a$ is a diagonal weight matrix that can be used to adjust the relative importance of specific excitations:

$$\begin{aligned} &\min_{\mathbf{a}} \ \phi_m(\mathbf{a}) + \frac{1}{2}\mathbf{a}^T\mathbf{W}_a\mathbf{a} \\ &\text{subject to} \quad 0 \le \mathbf{a} \le \mathbf{1}. \end{aligned} \tag{10.10}$$

> The matrix $\mathbf{W}_a$ described here is the *inverse* of the matrix $\mathbf{W}$ presented in [11].

### 10.1.4 Force tracking

Other cost terms can be added to the tracking controller. For instance, we can request a force trajectory $\mathbf{f}_{et}$ for a selected set of force effectors. As with motions, the forces of these effectors $\mathbf{f}_e$ at the end of step $k+1$ are linearly related to $\mathbf{a}$ via

$$\mathbf{f}_e^{k+1} = \mathbf{f}_{e0} + \mathbf{H}_e\mathbf{a}$$

where $\mathbf{f}_{t0}$ is the force with zero excitations and $\mathbf{H}_e$ is the force excitation response matrix. The force trajectory can then be tracking by minimizing

$$\phi_e(\mathbf{a}) \equiv \frac{1}{2}\|\mathbf{W}_e(\bar{\mathbf{f}}_e - \mathbf{H}_e\mathbf{a})\|^2, \quad \bar{\mathbf{f}}_e \equiv \mathbf{f}_{et} - \mathbf{f}_{e0}, \tag{10.11}$$

where $\mathbf{W}_e$ is a diagonal weighting matrix. When tracking force targets, the target force trajectory $\mathbf{f}_{et}$ is fed directly into the excitation generator (Figure 10.2); there is no equivalent of the motion tracker.

To balance the effect of different cost terms, each is associated with a weight (e.g., $w_m$, $w_e$, and $w_a$ for the motion, force and regularization terms), so that the QP takes a form such as

$$\begin{aligned} &\min_{\mathbf{a}} \ w_m\phi_m(\mathbf{a}) + w_e\phi_f(\mathbf{a}) + \frac{w_a}{2}\mathbf{a}^T\mathbf{W}_a\mathbf{a} \\ &\text{subject to} \quad 0 \le \mathbf{a} \le \mathbf{1}. \end{aligned} \tag{10.12}$$

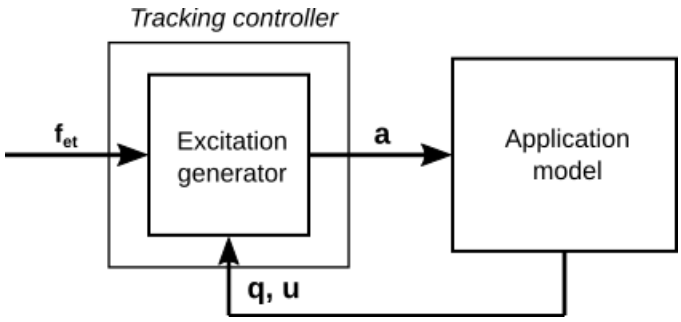



Figure 10.2: Tracking controller operation for force targets.

### 10.1.5 Incremental computation

In some cases, the system forces are *not* linear with respect to the excitations (i.e., equation (10.6) is not valid). One situation where this occurs is when equilibrium muscle models are used (Section 4.5.3).

When forces are not linear in $\mathbf{a}$, the force relationship can be linearized and the quadratic program can be reformulated in terms of excitation *changes* $\Delta\mathbf{a}$; for example,

$$\min_{\Delta\mathbf{a}} \; w_m\phi_m(\Delta\mathbf{a}) + w_e\phi_f(\Delta\mathbf{a}) + \frac{w_a}{2}\Delta\mathbf{a}^T\mathbf{W}_a\Delta\mathbf{a} + \mathbf{a}_0^T\mathbf{W}_a\Delta\mathbf{a}$$
$$\text{subject to} \quad -\mathbf{a}_0 \le \Delta\mathbf{a} \le \mathbf{1} - \mathbf{a}_0. \tag{10.13}$$

where $\mathbf{a}_0$ denotes the excitation values at the beginning of the time step. Excitations are then updated according to

$$\mathbf{a} = \mathbf{a}_0 + \Delta\mathbf{a}.$$

Incremental computation can be enabled, at the expense of slower computation, by setting the tracking controller property computeIncrementally to `true` (Section 10.3.2).

### 10.1.6 Setting up the tracking controller

Applications will generally set up a tracking controller using the following steps inside the application's `build()` method:

1. Create an instance of TrackingController and add it to the application using the root model's addController() method.
2. Configure the controller with the available excitation components using its addExciter(ex) and addExciter(weight,ex) methods.
3. Configure the controller to track position or force trajectories for selected source components, which may include Point, Frame, or ForceEffector components. These are specified to the controller using methods such as addPointTarget(point), addFrameTarget(frame), and addForceEffectorTarget(forceEffector). These methods return a *target* object, which is allocated and maintained by the controller, and which is used for specifying the target positions or forces.
4. Create input probes or controllers to specify the desired trajectories for the sources added in Step 3. Trajectories are specified by using probes or controllers to set the appropriate target properties of the target objects returned by the `addXXXTarget()` methods. Likewise, output probes or monitors can be created to record the excitations and the tracked positions or forces of the sources.
5. Add other cost terms as needed. These may include an L2 regularization term, added using the controller's addL2RegularizationTerm() method. Regularization attempts to minimize the norm of the excitation values and is needed to resolve redundancy if the excitation set has more degrees of freedom that the target space.
6. Set controller configuration parameters.

Once the tracking controller has been set up, during subsequent simulation it will compute excitation values, using the quadratic program described in Sections 10.1.3 and 10.1.4, so as to best follow the prescribed target trajectories.

### 10.1.7 Example: moving a point with multiple Muscles

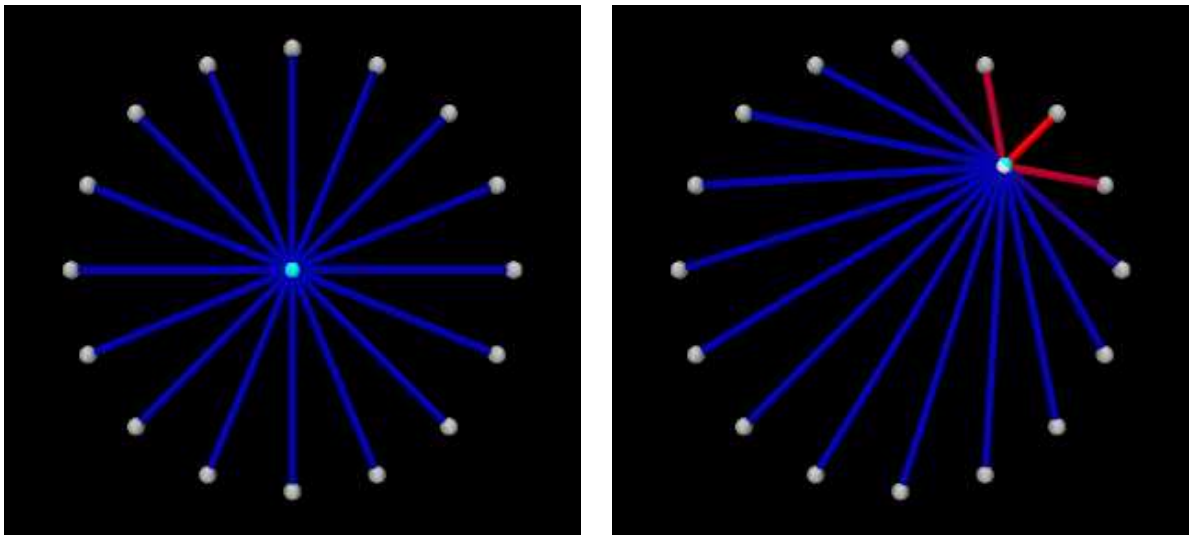


Figure 10.3: `InverseParticle` when first loaded (left), and during simulation with the muscles activated to move the center particle (right).

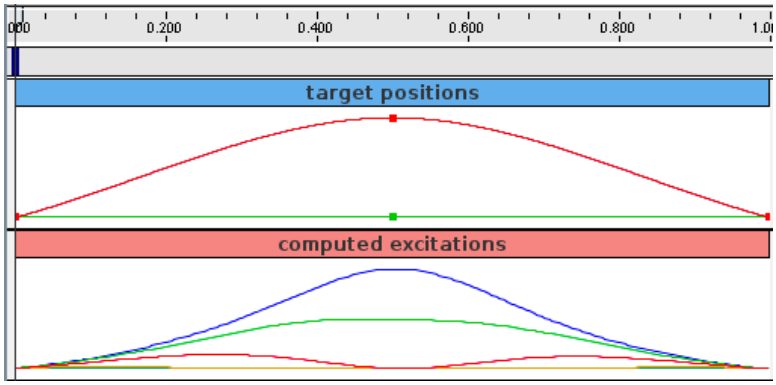



Figure 10.4: Probes showing the target position and computed excitations for `InverseParticle`.

An simple example illustrating the steps of Section 10.1.6 is given by

```
artisynth.demos.tutorial.InverseParticle
```

which uses a set of point-to-point muscles to drive the position of a single particle. The model consists of a single dynamic particle, initially located at the origin, attached to a set of 16 point-to-point muscles arranged radially around it. A tracking controller is created to control the excitations of these muscles so as to enable the particle to follow a trajectory specified by a probe. The code for the model, without the package and include directives, is listed below:

```
1  public class InverseParticle extends RootModel {
2
3     int numMuscles = 16; // num radial muscles surrounding the dynamic particle
4     double muscleStiffness = 200; // passive muscle stiffness
5     double muscleDamping = 1.0; // passive muscle damping
6     double muscleFmax = 1000; // max active force at excitation = 1
7     double dist = 1.0; // distance of anchor point from world origin
8
9     /**
10     * Create a muscle excitable spring extending out from the origin
11     * {@code part} at an angle {@code ang}.
12     */
```

```
13    void createMuscle (MechModel mech, Particle part, double ang) {
14       // create and add non-dynamic particle as fixed end point for muscle
15       Particle fixed = new Particle(
16          /*mass=*/1d, new Point3d(dist*Math.sin(ang),0.0,dist*Math.cos(ang)));
17       fixed.setDynamic(false);
18       mech.addParticle(fixed);
19       // create muscle and set its material
20       Muscle muscle = new Muscle (fixed, part);
21       muscle.setName (  // name the muscle using its angle
22          "muscle_"+Integer.toString((int)Math.round(Math.toDegrees(ang))));
23       muscle.setMaterial (
24          new SimpleAxialMuscle (muscleStiffness, muscleDamping, muscleFmax));
25       mech.addAxialSpring (muscle);
26
27       // make muscles red when activated
28       muscle.setExcitationColor (Color.RED);
29       muscle.setMaxColoredExcitation (0.5);
30       muscle.setRestLength (muscle.getLength());
31    }
32
33    public void build (String[] args) {
34       // create MechModel and add to RootModel
35       MechModel mech = new MechModel ("mech");
36       addModel (mech);
37       mech.setGravity (Vector3d.ZERO); // disable gravity
38
39       // create and add particle whose position is to be controlled
40       Particle part = new Particle ("center", /*mass=*/0.1, /*x,y,z=*/0, 0, 0);
41       part.setPointDamping (0.1); // add damping
42       mech.addParticle(part);
43
44       // create radial muscles connected to center particle
45       for (int i = 0; i < numMuscles; i++) {
46          double angle = 2*Math.PI*((double)i/numMuscles);
47          createMuscle (mech, part, angle);
48       }
49
50       // create the tracking controller and add it to the root model
51       TrackingController tcon = new TrackingController(mech, "tcon");
52       addController(tcon);
53       // set all muscles to be "exciters" for the controller to control
54       for (AxialSpring s : mech.axialSprings()) {
55          if (s instanceof Muscle) {
56             tcon.addExciter((Muscle)s);
57          }
58       }
59       // set the center dynamic particle to be the component that is tracked
60       TargetPoint target = tcon.addPointTarget(part);
61       // add an L-2 regularization term, since there are more muscles than
62       // target degrees-of-freedom
63       tcon.setL2Regularization(/*weight=*/0.1);
64
65       // Render properties: make points gray spheres, central particle white,
66       // and muscles as blue cylinders.
67       RenderProps.setSphericalPoints (this, dist/25, Color.LIGHT_GRAY);
68       RenderProps.setPointColor (part, Color.WHITE);
69       RenderProps.setCylindricalLines (mech, dist/50, Color.BLUE.darker ());
70
71       // add an input probe to control the position of the target:
72       NumericInputProbe targetprobe = new NumericInputProbe (
73          target, "position", /*startTime=*/0, /*stopTime=*/1);
74       targetprobe.setName ("target positions");
75       targetprobe.addData (
76          new double[] {0d,0d,0d, 0.5,0d,0.5, 0d,0d,0d}, // three knot points
77          /*timestep=*/0.5);
```

```
78        targetprobe.setInterpolationOrder (Interpolation.Order.Cubic);
79        addInputProbe (targetprobe);
80
81        // add an output probe to record the excitations:
82        NumericOutputProbe exprobe = InverseManager.createOutputProbe (
83           tcon, ProbeID.COMPUTED_EXCITATIONS, /*fileName=*/null,
84           /*startTime=*/0, /*stopTime=*/1, /*interval=*/-1);
85        addOutputProbe (exprobe);
86     }
87  }
```

The `build()` method begins by creating a `MechModel` in the usual way and disabling gravity (lines 35-37). The particle to be controlled is then created and placed at the origin and with a damping factor of 0.1 (lines 40-42). Next, the particle is attached to a set of point-to-point muscles that are arranged radially around it (lines 45-48). These are created using the method `createMuscle()` (lines 13-31), which attaches each to a fixed non-dynamic particle located a distance of `dist` from the origin, and sets its material to a `SimpleAxialMuscle` (Section 4.5.1.1) with a passive stiffness, damping and maximum force (at excitation 1) defined by `muscleStiffness`, `muscleDamping`, and `muscleFmax` (lines 4-6). Each muscle's excitationColor and maxColoredExcitation property is set so that its color transitions to red as its excitation value varies from 0 to 0.5 (lines 28-29, Section 10.2.1.1)), and its rest length is initialized to its current length (line 30).

After the model components have been built, a `TrackingController` is created and added to the root model's controller set (lines 51-52). All of the muscles are then added to the controller as exciters to be controlled (lines 54-58). The particle is added to the controller as a motion source using the `addPointTarget()` method (line 60), which returns a `target` object in the form of a `TargetPoint`. Since the number of exciters exceeds the degrees-of-freedom of the target space, an L2-regularization term is added (line 63) to resolve the redundancy.

Rendering properties are set at lines 67-69: points in the model are rendered as gray spheres, except for the dynamic particle which is colored white, and the muscles are drawn as blue cylinders. (The `target` particle, which is contained within the controller, is displayed using its default color of cyan.)

Probes are created to specify the target trajectory and record the computed excitations. The first, `targetprobe`, is an input probe attached to the position property of the `target` component, running from time 0 to 1 (lines 72-79). Its data is specified in code using `addData()`, which specifies three knot points with a time step of 0.5. Interpolation is set to cubic (line 78) for greater smoothness. The second, `exprobe`, is a probe that records all the excitations computed by the controller (lines 82-85). It is created by the utility method `createOutputProbe()` supplied by the InverseManager class (Section 10.4.1).

To run this example, select All demos > tutorial > InverseParticle from the Models menu. The model should load and initially appear as in Figure 10.3 (left). When run, the controller computes excitations to move the particle along the trajectory specified by the probe, which is upward and to the right (Figure 10.3, right) and back again.

The target point created by the controller is rendered separately from the source point as a cyan-colored sphere (see Section 10.2.2.2). As the simulation proceeds and certain muscles are excited, their coloring changes from blue to red, in proportion to their excitation, in accordance with the properties excitationColor and maxColoredExcitation. Recorded data for both the trajectory and the computed excitation probes are shown in Figure 10.4.

## 10.2 Tracking controller components

This section describes in greater detail how exciters, motion and force sources, and other cost terms can be specified for the controller.

### 10.2.1 Exciters

An exciter can be any model component that implements ExcitationComponent. These include point-to-point `Muscles` (Section 4.5), muscle bundles in FEM muscle models (Section 6.9.1.1), and `MuscleExciters` (Section 6.9.1.2). Each exciter exports an excitation property which accepts a scalar input signal (usually in the range $[0, 1]$) that drives the exciter's active force (or fans it out to other exciters in the case of a `MuscleExciter`).

The set of excitation components used by a tracking controller is managed by the following methods:

| | |
|---|---|
| void addExciter(ExcitationComponent ex) | Adds an exciter with default weight 1.0. |
| void addExciter(double w, ExcitationComponent ex) | Adds an exciter with weight `w`. |
| void addExciters(Collection<ExcitationComponent> exciter) | Adds a collection of exciters with weights 1.0. |
| int numExciters() | Returns the number of exciters. |
| ExcitationComponent getExciter (int idx) | Returns the idx-th exciter. |
| ListView<ExcitationComponent> getExciters() | Returns a list of all exciters. |
| void clearExciters() | Remove all exciters. |

An exciter's weight forms its corresponding entry in the diagonal matrix $\mathbf{W}_a$ used by the quadratic program ((10.10 and (10.12)), with a smaller weight allowing the exciter to assume greater prominence in the solution.

By default, the computed excitations are bounded by the interval $[0, 1]$, as indicated in Section 10.1.3. However, these bounds can be altered, either collectively using the tracking controller property excitationBounds, or individually for specific exciters:

| | |
|---|---|
| void setExcitationBounds(DoubleInterval range) | Sets the excitationBounds property. |
| DoubleInterval getExcitationBounds() | Queries the excitationBounds property. |
| void setExcitationBounds(ExcitationComponent ex, double low, double high) | Sets excitation bounds for exciter `ex`. |
| DoubleInterval getExcitationBounds(ExcitationComponent ex) | Queries excitation bounds for exciter `ex`. |

The excitation values themselves can also be queried and set with the following methods:

| | |
|---|---|
| void getExcitations(VectorNd values) | Returns excitations in `values`. |
| double getExcitation(int idx) | Returns the excitation of the idx-th exciter. |

By default, the controller initializes excitations to zero and then updates them at the beginning of each time step. However, when excitations are being computed incrementally (Section 10.1.5), it may be desirable to start with non-zero excitation values. In that case, the following methods may be useful:

| | |
|---|---|
| void initializeExcitations() | Sets excitations to the current exciter values. |
| void setExcitations(VectorNd values) | Sets excitations to `values`. |

The first sets the controller's internal excitation values to those stored in the exciters, while the second sets both the internal values and the values in the exciters.

#### 10.2.1.1 Excitation coloring

Some exciters (such as `Muscle` and `MultiPointMuscle`) support the ability to change color in proportion to their excitation value. This makes it easier to visualize the extent to which exciters are being activated within the model. Exciters which support this capability export the properties excitationColor, which is the color the exciter should transition to as excitation is increased, and maxColoredExcitation, which the excitation value at which the color transition is complete. In other words, the exciter's color will vary from its default color to excitationColor as its excitation varies from 0 to maxColoredExcitation.

Excitation coloring does not occur if the exciter's excitationColor is set to `null`.

The tracking controller exports a property, `configExcitationColoring`, which if true enables the automatic configuration of excitation coloring for exciters that support this capability. If enabled, then as these exciters are added to the controller, their excitationColor is inspected. If it has not been set (i.e., if it is `null`), then it is set to red and the nominal color for the exciter is set to white, enabling a white-to-red color transition for the exciter as it is activated.

### 10.2.2 Motion targets

As indicated above, motion source components can be specified to the controller using the `addPointTarget()` and `addFrameTarget()` methods. Source components may be instances of Point or Frame, and thereby may include both FEM nodes and rigid bodies. The methods for managing motion targets include:

| | |
|---|---|
| TargetPoint addPointTarget(Point source) | Adds point `source` with weight 1.0. |
| TargetPoint addPointTarget(Point source, double w) | Adds point `source` with weight `w`. |
| TargetFrame addFrameTarget(Frame source) | Adds frame `source` with weight 1.0. |
| TargetFrame addFrameTarget(Frame source, double w) | Adds frame `source` with weight `w`. |
| int numMotionTargets() | Number of motion targets. |
| void removeMotionTarget(MotionTargetComponent source) | Removes motion source. |

The `addPointTarget()` and `addFrameTarget()` methods each create and return a *target* component, allocated and contained within the controller, that mirrors the type of source component. These target components are either TargetPoint for point sources or TargetFrame for frame sources. Both `TargetPoint` and `TargetFrame`, together with the source components `Point` and `Frame`, are instances of MotionTargetComponent.

As simulation proceeds, the desired trajectory position $\mathbf{x}_t$ for each source component is specified by setting the target component's position property (or position, orientation and/or pose properties for frame targets). As described elsewhere, this can be done using either probes or other controller objects.

Likewise, a corresponding velocity trajectory $\mathbf{v}_t$ can also be specified by setting the velocity property of the target object. If this is not set, $\mathbf{v}_t$ defaults to 0, which will introduce a small lag into the tracking behavior. If $\mathbf{v}_t$ *is* set, care should be taken the ensure that it is consistent with the actual time derivative of $\mathbf{x}_t$.

> Applications typically do not need to specify $\mathbf{v}_t$. However, doing so may improve tracking performance, particularly when using PD control (Section 10.1.2).

Each target component also implements the interface TrackingTarget, described in Section 10.2.8, which supplies methods to specify the weights in the weighting matrix $\mathbf{W}_m$ of the motion tracking cost term $\phi_m(\mathbf{a})$ (10.9).

Lists of all the motion source components and their associated target components (i.e., those returned by the `addXXXTarget()` methods) can be obtained using

| | |
|---|---|
| ArrayList<MotionTargetComponent> getMotionSources() | Returns motion source components. |
| ArrayList<MotionTargetComponent> getMotionTargets() | Returns motion target components. |

#### 10.2.2.1 Motion target term

The cost term $\phi_m(\mathbf{a})$ that is responsible for tracking motion targets is contained within a tracking controller subcomponent that is named `"motionTerm"` and which is an instance of MotionTargetTerm. Users may access this component directly using

| | |
|---|---|
| MotionTargetTerm getMotionTargetTerm() | Returns the controller's motion target term. |

and then use it to set motion tracking properties such as those described in Section 10.3.3.

The motion tracking weight $w_m$ in (10.12) is given by the weight property of the motion target term, which can also be accessed directly by the controller methods

| | |
|---|---|
| double getMotionTargetTermWeight() | Returns the motion target term weight. |
| void setMotionTargetTermWeight(double w) | Sets the motion target term weight. |

#### 10.2.2.2 Motion target rendering

The motion target components, which are allocated and maintained by the controller, are rendered by default, which makes it easy to visualize significant errors between the target positions and the tracked source positions. Target points are drawn as cyan colored spheres. For target frames, if the source component corresponds to a `RigidBody`, the target is rendered using a cyan colored wireframe copy of the source's surface mesh.

Rendering of targets can be enabled or disabled by setting the tracking controller's targetsVisible property. Otherwise, the application is free to set the render properties of individual target components, or their containing lists within the `MotionTargetTerm`, which can be accessed by the methods

| | |
|---|---|
| PointList<TargetPoint> getTargetPoints() | Return all point motion target components. |
| RenderableComponentList<TargetFrame> getTargetFrames() | Return all frame motion target components. |

Other methods of `MotionTargetTerm` allow the render properties for both the point and frame target lists to be set together:

| | |
|---|---|
| RenderProps getTargetRenderProps() | Return render properties for the motion target lists. |
| void setTargetRenderProps (RenderProps props) | Set render properties for the motion target lists. |
| void setTargetsPointRadius (double rad) | Set pointRadius target render property to `rad`. |

### 10.2.3 Regularization

#### 10.2.3.1 L2 Regularization

L2 regularization attempts to minimize the weighted square of the excitation values, as described by

$$\frac{w_a}{2}\mathbf{a}^T\mathbf{W}_a\mathbf{a} \tag{10.14}$$

in (10.12), and so provides a way to resolve redundancy when the number of exciters is greater than needed to perform the required tracking. It can be enabled or disabled by adding or removing an L2 regularization term from the controller, as managed by the following methods:

| | |
|---|---|
| void setL2Regularization() | Enables L2 regularization with default weight 0.0001. |
| void setL2Regularization(double w) | Enables L2 regularization with weight `w`. |
| double getL2RegularizationWeight() | Returns regularization weight, or 0 if not enabled. |
| boolean removeL2Regularization() | Removes L2 regularization. |

The weight associated with the above methods corresponds to $w_a$ in (10.14) and (10.12). The entries in the diagonal matrix $\mathbf{W}_a$ are given by the weights associated with the exciters themselves, as described in Section 10.2.1.

L2 regularization is implemented using a controller subcomponent of type L2RegularizationTerm, which is added or removed from the controller as required and can be accessed using

| | |
|---|---|
| L2RegularizationTerm getL2RegularizationTerm() | Returns the controller's L2 regularization term |

which will return `null` if regularization is not being applied.

Because the L2 regularizer tries to reduce the excitations $\mathbf{a}$, its use will reduce the controller's tracking accuracy. Therefore, the regularizer is often employed with a weight value $w_a$ well below 1.0, with values around 0.1 or 0.01 being common. The best weight choice will of coarse depend on the application.

#### 10.2.3.2 Excitation damping

The controller also provides a damping term that serves to minimize the time derivative of $\mathbf{a}$, or more precisely,

$$\frac{1}{2}\dot{\mathbf{a}}^T\mathbf{W}_a\dot{\mathbf{a}},$$

where $\mathbf{W}_a$ is the diagonal excitation weight matrix used for L2 regularization (10.14). Letting $\mathbf{a}_0$ denote the excitations at the beginning of the time step, and approximating $\dot{\mathbf{a}}$ by

$$\dot{\mathbf{a}} \approx \frac{\mathbf{a}-\mathbf{a}_0}{h},$$

where $h$ is the time step size, the damping cost term $\phi_d(\mathbf{a})$ becomes

$$\phi_d(\mathbf{a}) = \frac{1}{2h^2}\left(\mathbf{a}^T\mathbf{W}_a\mathbf{a} - 2\mathbf{a}_0^T\mathbf{W}_a\mathbf{a}\right).$$

Excitation damping can be managed by the following methods:

| | |
|---|---|
| void setExcitationDamping() | Enable excitation damping with default weight $10^{-5}$. |
| void setExcitationDamping(double w) | Enable excitation damping with weight `w`. |
| double getExcitationDampingWeight() | Returns damping weight, or 0 if not enabled. |
| void removeExcitationDamping() | Removes excitation damping. |

Excitation damping is implemented using a controller subcomponent of type DampingTerm, which is added or removed from the controller as required and can be accessed using

| | |
|---|---|
| DampingTerm getDampingTerm() | Returns the controller's excitation damping. |

which will return `null` if damping is not being applied.

### 10.2.4 Example: controlling ToyMuscleArm

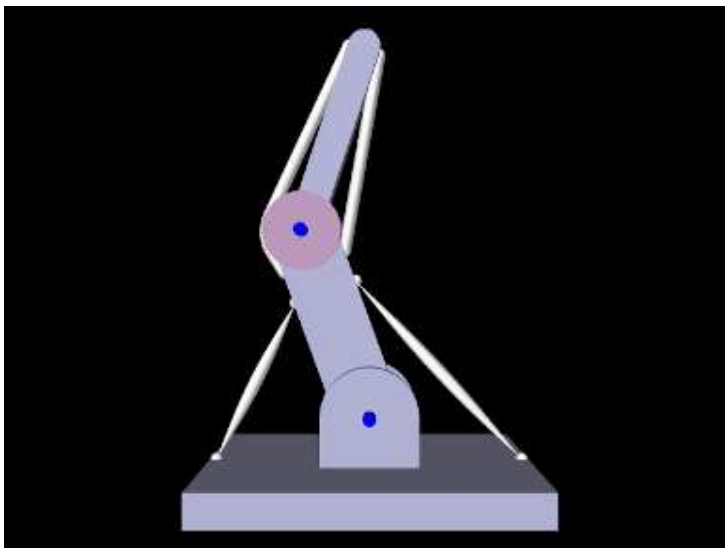


Figure 10.5: `InverseMuscleArm` when first loaded into ArtiSynth.

A good tracking controller example is given by the model `artisynth.demos.tutorial.InverseMuscleArm`, which computes the muscle excitations needed to make the tip marker of the `ToyMuscleArm` demo (described in Section 9.5.2) follow a prescribed trajectory. The model extends `artisynth.demos.tutorial.ToyMuscleArm`, and then adds the tracking controller and probes needed to move the tip marker, as shown in the code below:

```
1  public class InverseMuscleArm extends ToyMuscleArm {
2
3     public void build (String[] args) throws IOException {
4        super.build(args); // create ToyMuscleArm
5
6        // move the model into a non-singular position so it can track a target
7        // trajectory more easily
8        myHinge0.setTheta (-20);
9        myHinge1.setTheta (38.4);
10       myMech.updateWrapSegments(); // update muscle wrapping for new config
11
12       // Create a tracking controller
13       TrackingController tcon = new TrackingController (myMech, "tcon");
14       addController (tcon);
15       // For each muscle, reinitialize its rest length for the new
16       // configuration and add it to the controller as an exciter
17       for (AxialSpring spr : myMech.axialSprings()) {
```

```
18            spr.setRestLength (spr.getLength());
19            tcon.addExciter ((Muscle)spr);
20         }
21         for (MultiPointSpring spr : myMech.multiPointSprings()) {
22            spr.setRestLength (spr.getLength());
23            tcon.addExciter ((MultiPointMuscle)spr);
24         }
25
26         // Add the tip marker to the controller as a motion target
27         TargetPoint target = tcon.addPointTarget (myTipMkr);
28         // add an L-2 regularization term to handle exciter redundancy
29         tcon.setL2Regularization(/*weight=*/0.1);
30
31         double startTime = 0; // probe start times
32         double stopTime = 5;  // probe stop times
33         // Specify a target trajectory for the tip marker using an input probe.
34         NumericInputProbe targetprobe = new NumericInputProbe (
35            target, "position", startTime, stopTime);
36         targetprobe.setName ("target positions");
37         double x0 = 0;      // initial x coordinate of the marker
38         double z0 = 1.1806; // initial z coordinate of the marker
39         double xmax = 0.6;  // max x coordinate of the trajectory
40         // Trajectory data: five cubically interpolated knot points, running for
41         // 5 seconds, giving a closed loop shape:
42         targetprobe.addData (new double[] {
43            x0,0,z0,  xmax,0,z0-0.2,  x0,0,z0-0.4,  -xmax,0,z0-0.2,  x0,0,z0},
44            /*timestep=*/stopTime/4);
45         targetprobe.setInterpolationOrder (Interpolation.Order.Cubic);
46         addInputProbe (targetprobe);
47
48         // add an output probe to record the excitations:
49         NumericOutputProbe exprobe = InverseManager.createOutputProbe (
50            tcon, ProbeID.COMPUTED_EXCITATIONS, /*fileName=*/null,
51            startTime, stopTime, /*interval=*/-1);
52         addOutputProbe (exprobe);
53
54         // add tracing probes to view both the tracking target (in cyan) and the
55         // actual tracked position (in red).
56         TracingProbe tprobe;
57         tprobe = addTracingProbe (target, "position", startTime, stopTime);
58         tprobe.setName ("target tracing");
59         RenderProps.setLineColor (tprobe, Color.CYAN);
60         tprobe = addTracingProbe (myTipMkr, "position", startTime, stopTime);
61         tprobe.setName ("source tracing");
62         RenderProps.setLineColor (tprobe, Color.RED);
63
64         // add inverse control panel
65         InverseManager.addInversePanel (this, tcon);
66         // settings to allow probe management by InvereManager:
67         tcon.setProbeDuration (stopTime); // default probe duration
68         // set working folder for probe files
69         ArtisynthPath.setWorkingFolder (
70            new File (PathFinder.getSourceRelativePath (this, "inverseMuscleArm")));
71      }
72   }
```

As mentioned above, this model extends `ToyMuscleArm` (line 1), and so calls `super.build(args)` in the `build()` method to create all the components of `ToyMuscleArm`. Once this is done, the link positions are adjusted by setting the hinge joint angles (lines 8-9); this is done to move the arm away from the kinematic singularity at full extension and thus make it easier for the tip to follow a prescribed path. After the links are moved, the all wrap paths are updated by calling the `MechModel` method `updateWrapPath()` (line 10).

A tracking controller is then created (lines 13-14), and every `Muscle` and `MultiPointMuscle` is added to it as an exciter (lines 17-24). When iterating through the muscles, their rest lengths are reinitialized to their current lengths (which changed when the links were repositioned) to ensure that passive muscle forces are zero in the initial position.

Next, the marker at the tip of link1 (referenced by the inherited attribute `myTipMkr`) is added to the controller as a motion source (line 27) and the returned target object is stored in `target`. An L2 regularization term is added to the controller with weight 0.01 (line 29), and an input probe is created to provide the marker trajectory by setting the target's `position` property (lines 31-46). The probe has a duration of 5 seconds, and the trajectory is a closed path, specified by 5 cubically interpolated knot points (line 43), that lie in the x-z plane and start at and return to the marker's initial position `x0`, `z0`.

Probes are created to record the computed excitations and trace the desired and actual trajectories in the viewer. The first, `exprobe`, is created by the utility method `createOutputProbe()` supplied by the InverseManager class (lines 49-52, Section 10.4.1). The tracing probes are created by the `RootModel` method addTracingProbe() (lines 56-62), and two are created: one to trace the desired target by recording the position property of `target`, and another to trace the actual tracked (source) position by recording the position property of `myTipMkr`. The line colors of these are set to cyan and red, respectively, which specifies their display color in the viewer.

Lastly, an *inverse control panel* (Section 10.4.3) is created to manage controller properties (line 65), and some settings are made to allow for probe management by the InverseManager class, as discussed in Section 10.4.1 (lines 67-70); this includes setting the default probe duration to `stopTime` and the working folder to `inverseMuscleArm` located under the source folder.

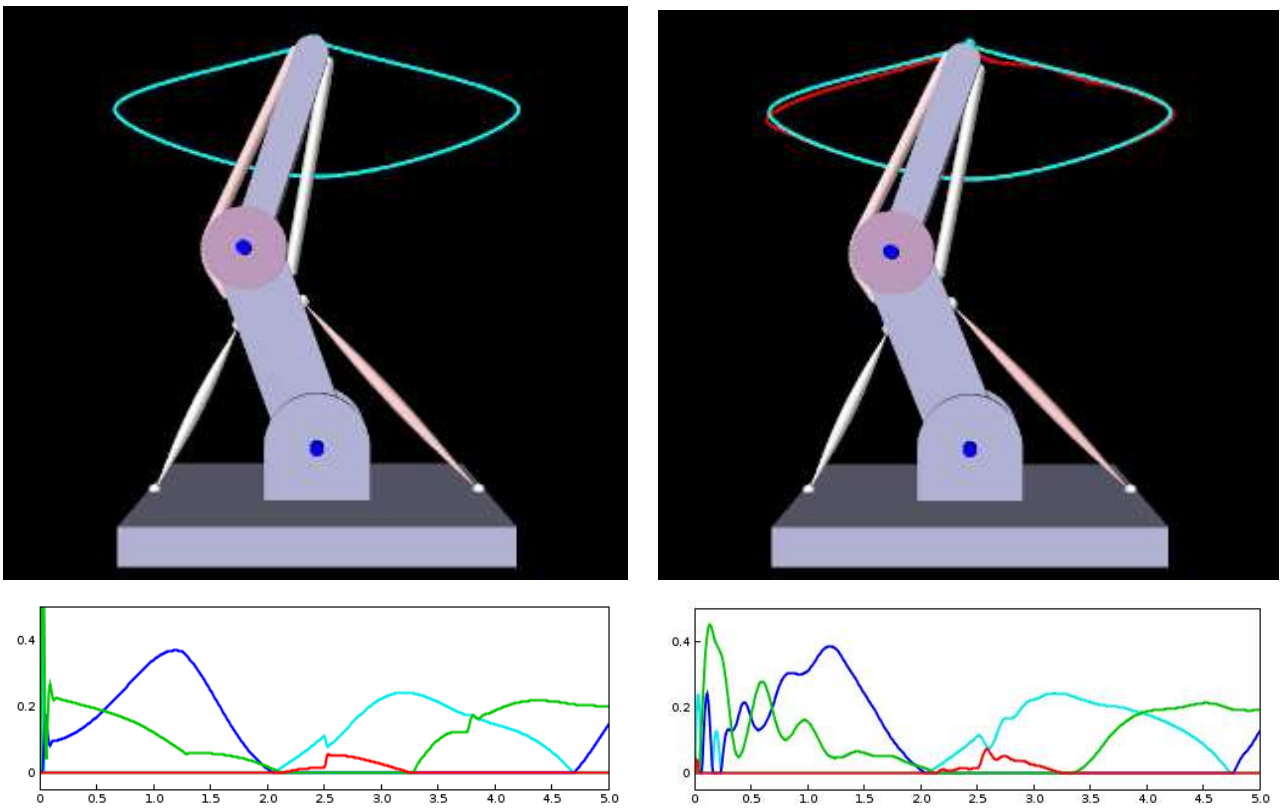



Figure 10.6: `InverseMuscleArm` run with L2 regularization weights of 0.1 (left) and 10 (right). Traces of the desired and actual trajectories are shown in cyan and red, respectively, and computed excitation values are shown in graphs below. A regularization weight of 10results in a larger tracking error.

To run this example, select All demos > tutorial > InverseMuscleArm from the Models menu. The model should load and initially appear as in Figure 10.5. When run, the controller will compute excitations to make the tip marker trace the desired trajectory, as shown in Figure 10.6 (left), where the tracing probe shows the target trajectory in cyan; the source trajectory appears beneath it in red but is largely invisible because it follows the target quite accurately. Computed excitation values are shown below. The target component created for the tip marker is rendered as a cyan-colored sphere (Section 10.2.2.2).

> The muscles in `InverseMuscleArm` are initially colored white instead of red (as they are for `ToyMuscleArm`). That is because if a muscle's excitationColor property is `null`, the controller automatically configures the muscle to vary

in color from white to red as the muscle is activated, as described in Section 10.2.1.1. This behavior can be disabled by setting the controller property configExcitationColoring to `false`.

To illustrate how the L2 regularization weight can affect the tracking error, 10.6 (right) shows the model run with a regularization weight of 10 instead of 0.1. The tracking error is now visible, with the red source trace probe clearly distinct from the cyan target trace. The computed muscle excitations are also noticeably different.

### 10.2.5 Example: controlling an FEM muscle model

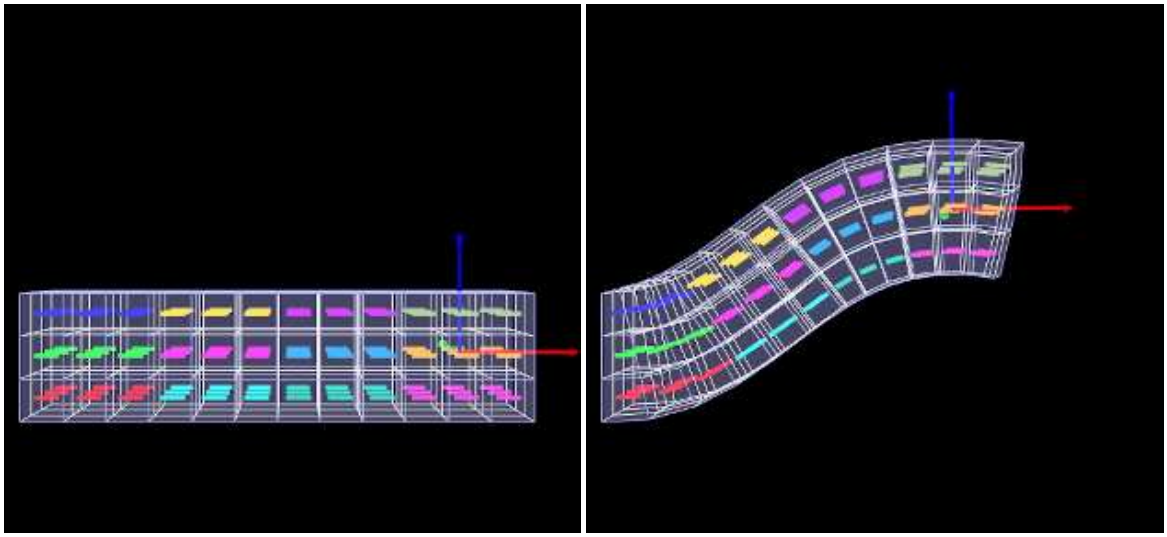


Figure 10.7: `InverseMuscleFem` when first loaded into ArtiSynth (left), and after executing in the inverse simulation (right).

Another example involving an FEM muscle model is given by `artisynth.demos.tutorial.InverseMuscleFem`, which computes the muscle excitations needed to make an attached frame of the `ToyMuscleFem` demo (described in Section 6.9.4) follow a prescribed trajectory.

The model extends `artisynth.demos.tutorial.ToyMuscleFem`, and then adds the tracking controller and probes needed to move the tip marker, as shown in the code below:

```
public class InverseMuscleFem extends ToyMuscleFem {

   protected String dataDir = PathFinder.getSourceRelativePath (this, "data/");

   public void build (String[] args) throws IOException {
      super.build (args);  // build the underlying ToyMuscleFem model

      // create a tracking controller
      TrackingController tcon = new TrackingController (myMech, "tcon");
      addController (tcon);
      // add each FEM muscle bundle to it as an exciter
      for (MuscleBundle b : myFem.getMuscleBundles()) {
         tcon.addExciter (b);
      }
      // add the frame attached to the FEM as a motion target
      TargetFrame target = tcon.addFrameTarget (myFrame);

      // add an L-2 regularization term to handle exciter redundancy
      tcon.setL2Regularization(/*weight=*/0.1);
      // set the controller motion term to use PD control
      tcon.getMotionTargetTerm().setUsePDControl (true);
      tcon.getMotionTargetTerm().setKp (1000);
      tcon.getMotionTargetTerm().setKd (100);
```

```
24
25       // add input probes specifying the desired position and orientation
26       // trajectory of the target:
27       double startTime = 0;
28       double stopTime = 5;
29       NumericInputProbe tprobePos =
30          new NumericInputProbe (
31             target, "position", dataDir+"inverseFemFramePos.txt");
32       tprobePos.setName ("target frame position");
33       addInputProbe (tprobePos);
34       NumericInputProbe tprobeRot =
35          new NumericInputProbe (
36             target, "orientation", dataDir+"inverseFemFrameRot.txt");
37       tprobeRot.setName ("target frame orientation");
38       addInputProbe (tprobeRot);
39
40       // add output probes showing the tracked position and orientation of the
41       // target frame source:
42       NumericOutputProbe sprobePos =
43          new NumericOutputProbe (
44             myFrame,"position", startTime, stopTime, /*interval*/-1);
45       sprobePos.setName ("source frame position");
46       addOutputProbe (sprobePos);
47       NumericOutputProbe sprobeRot =
48          new NumericOutputProbe (
49             myFrame,"orientation", startTime, stopTime, /*interval*/-1);
50       sprobeRot.setName ("source frame orientation");
51       addOutputProbe (sprobeRot);
52
53       // add an output probe to record the excitations:
54       NumericOutputProbe exprobe = InverseManager.createOutputProbe (
55          tcon, ProbeID.COMPUTED_EXCITATIONS, /*fileName=*/null,
56          startTime, stopTime, /*interval=*/-1);
57       addOutputProbe (exprobe);
58       // create a control panel for the controller
59       InverseManager.addInversePanel (this, tcon);
60    }
61 }
```

Since the model extends `ToyMuscleFem` (line 1), the `build()` method begins by calling `super.build(args)` to create the model defined by the superclass. A tracking controller is then created and added to the root model's controller set, all the muscle bundles in the FEM muscle model are added to it as exciters, and the attached frame is added to it as a motion source using `addFrameTarget` (lines 9-16).

An L2 regularization term is added, and then the motion tracking is set to use PD control (Section 10.1.2), using gains of $K_p = 1000$ and $K_d = 100$ (line 19-23).

To control position and orientation of the frame, two input probes are created (lines 27-38), one attached to the target's position and the other to its orientation. (While it is possible to use a single probe to control both of these properties, or the single pose property, separate probes are used here to provide easier visualization.) Their data and settings are read from the probe files `inverseFemFramePos.txt` and `inverseFemFrameRot.txt`, located in the folder `data` beneath the application model source folder. Each specifies a probe in the time range from 0 to 5, with 26 knot points and cubic interpolation.

To monitor the controller's tracking performance, two output probes are created and attached to the position and orientation properties of the source component `myFrame` (lines 58-67). Setting the `interval` argument to -1 in the probes' constructors causes them to use the model's current step size as the update interval. Finally, the utility class InverseManager (Section 10.4) is used to create an output probe for the computed excitations, as well as a control panel giving access to the various controller properties (lines 70-75).

To run this example, select All demos > tutorial > InverseMuscleFem from the Models menu. The model should load and initially appear as in Figure 10.7 (left). When run, the controller will compute excitations to move the attached frame along the specified position/orientation trajectory, reaching the final pose shown in Figure 10.7 (right). The target and actual source trajectories for the frame's position (i.e., its origin) is shown in Figure 10.8, together with the computed excitations.

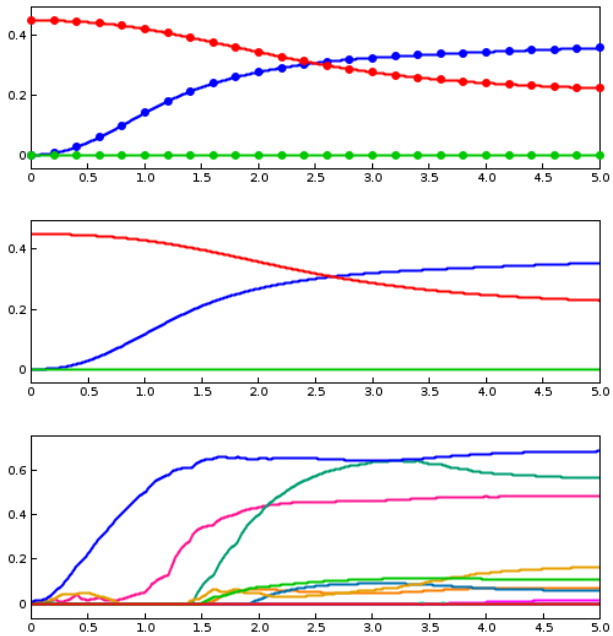



Figure 10.8: Probe data generated by `InverseMuscleFem`. Top: target trajectory for the frame position (26 knot points, cubically interpolated). Middle: actual position tracked by the controller. Bottom: computed muscle excitations.

### 10.2.6 Force effector targets

The controller can also be asked to track a force trajectory for a set of force effector components; the excitations will then be computed to try to generate the prescribed forces. Any force component that implements the interface ForceTargetComponent can be used as a force effector source; this interface extends ForceEffector to supply several additional methods including:

```
interface ForceTargetComponent extends ForceEffector, ModelComponent {

   // gets the dimension of the generated force
   public int getForceSize();

   // returns the current force value
   public void getForce (VectorNd minf, boolean staticOnly);

   ...
}
```

At the time of this writing, `ForceTargetComponents` include:

| Component | Force size | Force description |
|---|---|---|
| AxialSpring | 1 | Tension in the spring |
| Muscle | 1 | Tension in the muscle |
| FrameSpring | 6 | Spring wrench as seen in body A (world coordinates) |

Controller methods for adding and managing force effector targets include:

| | |
|---|---|
| ForceEffectorTarget addForceEffectorTarget ( ForceTargetComponent source) | Adds static-only force source with weight 1.0 |
| ForceEffectorTarget addForceEffectorTarget ( ForceTargetComponent source, double w) | Adds static-only force source with weight `w`. |
| ForceEffectorTarget addForceEffectorTarget ( ForceTargetComponent source, double w, boolean staticOnly) | Adds force source with weight `w` and static-only specified. |
| int numForceEffectorTargets() | Number of force effector targets. |
| boolean removeForceEffectorTarget( ForceTargetComponent source) | Removes force source. |

The `addForceEffectorTarget()` methods each create and return a ForceEffectorTarget component. As simulation proceeds, the desired target force for the source component is specified by setting the target component's targetForce property. As described elsewhere, this can be done using either probes or other controller objects. The target component also implements the interface TrackingTarget, described in Section 10.2.8, which supplies methods to specify the weighting matrix $\mathbf{W}_e$ in the force effector cost term $\phi_e(\mathbf{a})$ (10.11).

By default, force effector tracking is *static-only*, meaning that only static forces (i.e., forces that are not velocity dependent, such as damping) are considered. However, the third `addForceEffectorTarget()` method allows this to be explicitly specified.

Lists of all the force effector source components and their associated targets can be obtained using the methods:

| | |
|---|---|
| ArrayList<ForceEffectorTarget> getForceEffectorSources() | Returns force effector source components. |
| ArrayList<ForceEffectorTarget> getForceEffectorTargets() | Returns force effector target components. |

> Specifying a force effector targetForce of 0 will have the same effect as *minimizing* the force associated with that force effector.

#### 10.2.6.1 Force effector term

The cost term $\phi_e(\mathbf{a})$ that is responsible for tracking force effector targets is contained within a tracking controller subcomponent named `"forceEffectorTerm"` and which is an instance of ForceEffectorTerm. This is added to the controller automatically whenever force effector tracking is requested, and may accessed directly using

| | |
|---|---|
| ForceEffectorTerm getForceEffectorTerm() | Returns the force effector term. |

The method will return `null` if the force effector term is not present.

The force effector tracking weight $w_e$ in (10.12) is given by the weight property of the force effector term, which can also be accessed directly by the controller methods

| | |
|---|---|
| double getForceEffectorTermWeight() | Returns the force effector term weight. |
| void setForceEffectorTermWeight(double w) | Sets the force effector term weight. |

### 10.2.7 Example: controlling tension in a spring

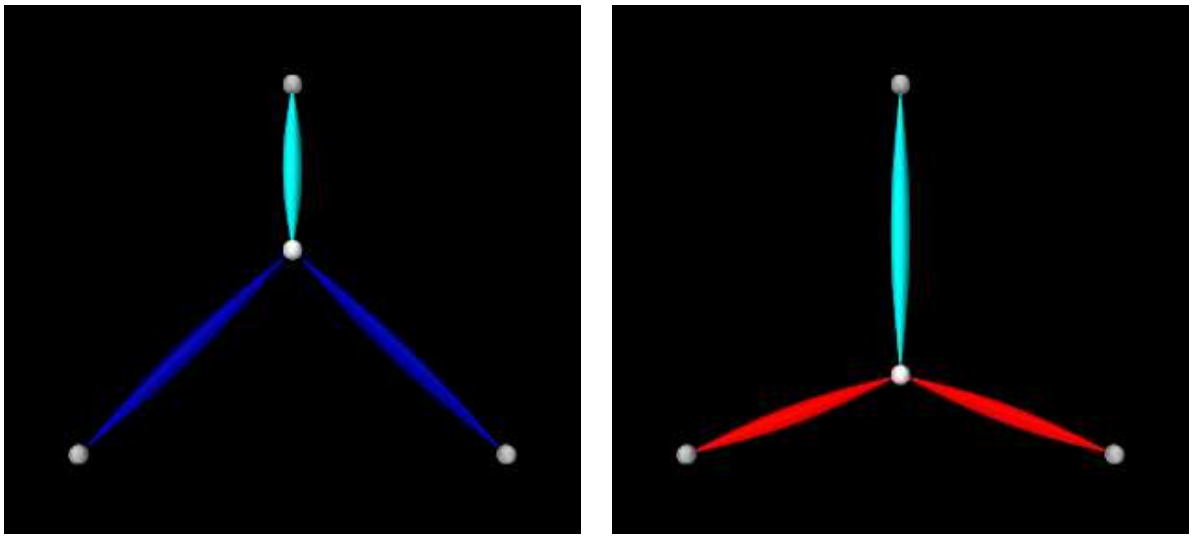


Figure 10.9: `InverseSpringForce` when first loaded (left), and during simulation with the lower muscles fully activated to control the tension in the upper passive spring (cyan).

A simple example of force effector tracking is given by

```
artisynth.demos.tutorial.InverseSpringForce
```

which uses two point-to-point muscles to control the tension in a passive spring. The initial part of the code is the same as that for `InverseParticle` (Section 10.1.7), except for different parameter definitions,

```
   int numMuscles = 3; // num radial muscles surrounding the dynamic particle
   double muscleStiffness = 200; // passive muscle stiffness
   double muscleDamping = 0.1; // passive muscle damping
   double muscleFmax = 200; // max active force at excitation = 1
   double dist = 1.0; // distance of anchor point from world origin
```

which give the muscles different strengths and cause only 3 to be created instead of 16, and the fact that the `"center"` particle is initially placed at $(0, 0, 0.33)$ instead of the origin. The rest of the code diverges after the tracking controller is created, as shown below:

```
17      // create the tracking controller and add it to the root model
18      TrackingController tcon = new TrackingController(mech, "tcon");
19      addController(tcon);
20      // set all muscles but the first to be "exciters" for the controller
21      for (int i=1; i<numMuscles; i++) {
22         tcon.addExciter((Muscle)mech.axialSprings().get(i));
23      }
24      // set the first muscle to be the force effector target. This
25      // will be unactivated and will simple serve as a passive spring
26      AxialSpring passiveSpring = mech.axialSprings().get(0);
27      ForceEffectorTarget target =
28         tcon.addForceEffectorTarget(passiveSpring);
29      // add an L-2 regularization term, since there are more muscles than
30      // target degrees-of-freedom
31      tcon.setL2Regularization(/*weight=*/0.1);
32
33      // Render properties: make points gray spheres, central particle white,
34      // muscles as blue spindles, and passive spring as a cyan spindle.
35      RenderProps.setSphericalPoints (this, dist/25, Color.LIGHT_GRAY);
36      RenderProps.setPointColor (part, Color.WHITE);
37      RenderProps.setSpindleLines (mech, dist/25, Color.BLUE.darker ());
38      RenderProps.setLineColor (passiveSpring, Color.CYAN);
39
40      // add an input probe to control the desired target tension:
41      NumericInputProbe targetprobe = new NumericInputProbe (
42         target, "targetForce", /*startTime=*/0, /*stopTime=*/1);
43      targetprobe.setName ("target tension");
44      targetprobe.addData (
45         new double[] {0d, 120d, 0d}, // three knot points
46         /*timestep=*/0.5);
47      targetprobe.setInterpolationOrder (Interpolation.Order.Cubic);
48      addInputProbe (targetprobe);
49
50      // add an output probe to show both the target tension ("targetForce"
51      // of target) and actual tension ("forceNorm" of passiveSpring)
52      Property[] props = new Property[] {
53         target.getProperty ("targetForce"),
54         passiveSpring.getProperty ("forceNorm"),
55      };
56      NumericOutputProbe trackingProbe =
57         new NumericOutputProbe (props, /*interval=*/-1);
58      trackingProbe.setName ("target and source tension");
59      trackingProbe.setStartTime (0);
60      trackingProbe.setStopTime (1);
61      addOutputProbe (trackingProbe);
62
63      // add an output probe to record the excitations:
64      NumericOutputProbe exprobe = InverseManager.createOutputProbe (
65         tcon, ProbeID.COMPUTED_EXCITATIONS, /*fileName*/null,
66         /*startTime*/0, /*stopTime*/1, /*interval*/-1);
67      addOutputProbe (exprobe);
```

After the tracking controller is created (lines 18-19), the last two muscles are added to it as exciters (lines 21-23). The first muscle is then added as the force effector sources using `addForceEffectorTarget()` (lines 26-28), which returns

a `target` object. (The first muscle will be unactuated and so will behave as a passive spring.) Since the number of exciters exceeds the degrees-of-freedom of the target space, an L2-regularization term is added (line 31).

Rendering properties are set at lines 35-38: points are rendered as gray spheres, except for the center particle which is white, and muscles are drawn as spindles, with the exciter muscles blue and the passive target cyan.

Lastly, probes are created: an input probe to specify the target trajectory, an output probe to record target and tracked tensions, and an output probe to record the computed excitations. The first, `targetprobe`, is attached to the targetForce property of the `target` component and runs from time 0 to 1 (lines 41-48). Its data is specified in code using `addData()`, which specifies three knot points with a time step of 0.5. Interpolation set to cubic (line 47) for greater smoothness. The second probe, `trackingProbe`, records both the target and source tension from the targetForce property of `target` and the forceNorm property of the passive spring (lines 52-61). The excitation probe is created by the utility method `createOutputProbe()` supplied by the InverseManager class (lines 64-67, Section 10.4).

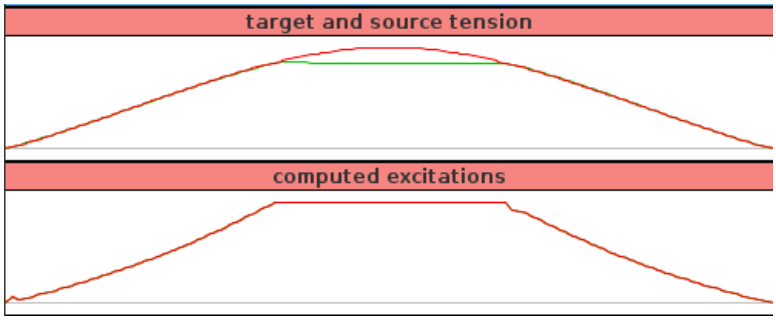



Figure 10.10: Probes showing the combined target and source tension (top) and the computed excitations (bottom) for `InverseSpringForce`.

To run this example, select All demos > tutorial > InverseSpringForce from the Models menu. The model should load and initially appear as in Figure 10.9 (left). When run, the controller computes excitations to generate the requested tension in the passive spring; this causes the center particle to be pulled down (Figure 10.3, right). Both the target-source and computed excitation probes are shown in Figure 10.10.

For the middle third of the trajectory, the excitation values have reached their threshold value of 1 and so are unable to deliver more force. This in turn causes the source tension (green line) to deviate from the target tension (red line) in the target-source probe.

### 10.2.8 Target components

As described above, when a motion or force source is added to the controller (using methods such as `addPointTarget()` or `addForceEffectorTarget()`), the controller creates and returns a target component that is used for specifying the desired position or force trajectory. Each target term also contains a scalar weight and a vector of subweights, described by the properties weight and subWeights, all of which have default values of 1.0. These weights are used to manage the priority of the target within the tracking computation; the subWeights vector has a size equal to the number of degrees of freedom (DOF) in the target (3 for points and 6 for frames), and permits fine-grained weighting for each of the targets DOFs. The product of the weight with each subweight produces the entries for the diagonal weighting matrix that appears in the various cost terms for motion and force tracking (e.g., $\mathbf{W}_m$ for the motion tracking cost function $\phi_m(\mathbf{a})$ in (10.9) and $\mathbf{W}_e$ for the force effector cost function $\phi_e(\mathbf{a})$ in (10.11)). Targets (or specific DOFs) with higher weights will be tracked more accurately, while weights of 0 effectively disable tracking.

Target components vary depending on the source component whose position or force is being tracking, but each implements the interface TrackingTarget, which supplies the following methods for controlling the weights and subweights and querying the target's source component:

| | |
|---|---|
| ModelComponent getSourceComp() | Returns the target's source component. |
| int getTargetSize() | Returns number of target DOFs, which equals the number of subweights. |
| double getWeight() | Returns the target component weight property. |
| void setWeight(double w) | Sets the target component weight property. |
| Vector getSubWeights() | Returns the target component subweights property. |
| void setSubWeights(VectorNd subw) | Sets the target component subweights property. |

### 10.2.9 Point and frame exciters

In addition to muscle components, exciters may also include PointExciters and FrameExciters, which can be used to apply forces directly to Point or Frame components. This effectively gives the inverse controller the ability to do direct inverse simulation. These exciter components can also assume a role similar to that of the "reserve actuators" used in OpenSim [7], augmenting a model's tracking capabilities to account for force sources that are not explicitly supplied by the model.

Each exciter applies a force along (or about) a single degree-of-freedom, as specified by the enumerated types PointExciter.ForceDof or FrameExciter.WrenchDof and described in the following table:

| Enum field | Description |
|---|---|
| `ForceDof.FX` | force along the point x axis |
| `ForceDof.FY` | force along the point y axis |
| `ForceDof.FZ` | force along the point z axis |
| `WrenchDof.FX` | force along the frame x axis (world coordinates) |
| `WrenchDof.FY` | force along the frame y axis (world coordinates) |
| `WrenchDof.FZ` | force along the frame z axis (world coordinates) |
| `WrenchDof.MX` | moment about the frame x axis (world coordinates) |
| `WrenchDof.MY` | moment about the frame y axis (world coordinates) |
| `WrenchDof.MZ` | moment about the frame z axis (world coordinates) |

Point or frame exciters may be created with the following constructors:

| | |
|---|---|
| PointExciter(Point point, FrameDof dof, double maxForce) | Exciter for `point` with `dof` and `maxForce`. |
| PointExciter(String name, Point point, FrameDof dof, double maxForce) | Named exciter for `point` with `dof` and `maxForce`. |
| FrameExciter(Frame frame, WrenchDof dof, double maxForce) | Exciter for `frame` with `dof` and `maxForce`. |
| FrameExciter(String name, Frame frame, WrenchDof dof, double maxForce) | Named exciter for `frame` with `dof` and `maxForce`. |

The `maxForce` argument specifies the maximum force (or moment) that the exciter supplies at an excitation value of 1.0.

> If an exciter does not have sufficient strength to facilitate tracking, its excitation value is likely to saturate. This can often be solved by simply increasing the `maxForce` value.

To allow it to produce negative forces or moments along/about its specified degree of freedom, the excitation bounds for a point or frame exciter must be set to $[-1, 1]$. The `TrackingController` does this automatically whenever a point or frame exciter is added to it.

For convenience, `PointExciter` and `FrameExciter` provide static methods for creating sets of exciters to control all the translational forces and/or moments on a given point or frame:

| | |
|---|---|
| ArrayList<PointExciter> createPointExciters (MechModel mech, Point point, double maxForce, boolean createNames) | Create three exciters to control all forces on a point. |
| ArrayList<FrameExciter> createFrameExciters ( MechModel mech, Frame frame, double maxForce, double maxMoment, boolean createNames) | Create six exciters to control all forces & and moments on a frame. |
| ArrayList<FrameExciter> createForceExciters (MechModel mech, Frame frame, double maxForce, boolean createNames) | Create three exciters to control all forces on a frame. |
| ArrayList<FrameExciter> createMomentExciters ( MechModel mech, Frame frame, double maxMoment, boolean createNames) | Create three exciters to control all moments on a frame. |

If the (optional) `mech` argument in these methods is is non-`null`, the exciters are added to the `MechModel`'s `forceEffectors` list. If the argument `createNames` is `true`, and the point or frame itself has a non-`null` name, then the exciter is assigned a name based on the point/frame name and the degree of freedom.

### 10.2.10 Example: controlling ToyMuscleArm with FrameExciters

The `InverseMuscleArm` example of Section 10.2.4 can be modified to use frame exciters in place of its muscles, allowing link forces to be controlled directly to make the marker follow the specified path. The modified model is `artisynth.demos.tutorial.InverseFrameExcitereArm`, and the sections of code where it differs from `InverseMuscleArm:sec` are listed below:

```
27     /**
28      * Creates a frame exciter for a rigid body, controlling the force DOF
29      * described by 'dof' with a maximum activation for of 'maxf', and adds it
30      * to both the mech model and the tracking controller 'tcon'.
31      */
32     void addFrameExciter (
33        TrackingController tcon, RigidBody body, WrenchDof dof, double maxf) {
34        FrameExciter fex = new FrameExciter (null, body, dof, maxf);
35        myMech.addForceEffector (fex);
36        tcon.addExciter (fex);
37     }
```

```
48        TrackingController tcon = new TrackingController (myMech, "tcon");
49        addController (tcon);
50        // For each muscle, reinitialize its rest length for the new
51        // configuration
52        for (AxialSpring spr : myMech.axialSprings()) {
53           spr.setRestLength (spr.getLength());
54        }
55        for (MultiPointSpring spr : myMech.multiPointSprings()) {
56           spr.setRestLength (spr.getLength());
57        }
58
59        // For each link, add two frame exciters to give the controller access to
60        // translational forces in the x-z plane
61        addFrameExciter (tcon, myLink0, WrenchDof.FX, 200);
62        addFrameExciter (tcon, myLink0, WrenchDof.FZ, 200);
63        addFrameExciter (tcon, myLink1, WrenchDof.FX, 200);
64        addFrameExciter (tcon, myLink1, WrenchDof.FZ, 200);
```

After the controller is created (lines 48-49), the muscle rest lengths are reset, as they are for `InverseMuscleArm`, but they are *not* added to controller as exciters. Instead, four frame exciters are created to generate translational forces on each link in the x-z plane, up to a maximum force of 200 (lines 61-64). These exciters are created using the method `addFrameExciter()` (lines 32-37), which creates the exciter and adds it to the `MechModel` and to the controller's exciter list.

Instead of calling the method `addFrameExciter()`, the frame exciters could have been generated using the following code fragment:

```
tcon.addExciters (
   FrameExciter.createForceExciters (
      myMech, myLink0, 200, /*createNames*/false));
tcon.addExciters (
   FrameExciter.createForceExciters (
      myMech, myLink1, 200, /*createNames*/false));
```

This would create six exciters instead of four (three forces per link, instead of limiting forces to x-z plane), but the additional forces would simply not be recruited by the controller.

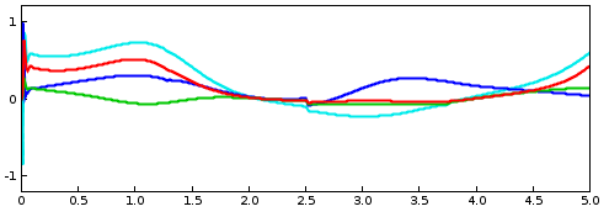



Figure 10.11: Excitations computed for `InverseFrameExciterArm`.

To run this example, select All demos > tutorial > InverseFrameExciterArm from the Models menu. The excitations generated by running the model are shown in Figure 10.11.

## 10.3 Tracking controller structure and settings

### 10.3.1 Controller structure

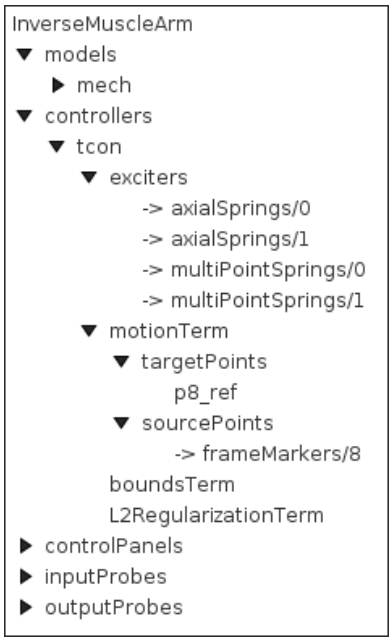



Figure 10.12: Expanded navigation panel view of the tracking controller (named `"tcon"`) and all its subcomponents, for the example `InverseMuscleArm`.

The tracking controller is a composite component, which contains the exciter list and the various cost and constraint terms as subcomponents. Some of these components are added as needed. They include the following, as described by their names:

`exciters`

A list of references to the exciters which have been added to the controller. Note that this is a list of *references*; the exciters themselves exist elsewhere in the model. Always present.

`motionTerm`

The `MotionTargetTerm` detailed in Section 10.2.2.1. Implements the motion cost term $\phi_m(\mathbf{a})$ defined in (10.7), and the motion tracker of Figure 10.1. Allocates and maintains target components for motion sources, storing the targets in the subcomponent lists `targetPoints` and `targetFrames`, and references to the sources in the reference lists `sourcePoints` and `sourceFrames`. Always present.

`forceEffectorTerm`

The `ForceEffectorTerm` detailed in Section 10.2.6.1. Implements the force effector cost term $\phi_e(\mathbf{a})$ defined in (10.11). Allocates and maintains target components for force effector sources, storing then in the subcomponent list `forceTargets`. Added on demand.

`boundsTerm`

Implements the actuator bounds $\mathbf{a}_{\min} \leq \mathbf{a} \leq \mathbf{a}_{\max}$, where $\mathbf{a}_{\min}$ and $\mathbf{a}_{\max}$ are nominally $\mathbf{0}$ and $\mathbf{1}$. Always present.

`L2RegularizationTerm`

Implements the cost term for L2 regularization (Section 10.2.3.1). Added on demand.

`dampingTerm`

Implements the cost term for excitation damping (Section 10.2.3.2). Added on demand.

### 10.3.2 Controller properties

The controller and its cost terms have a number of properties that can be used to adjust the controller's behavior. They can be set interactively, either through the inverse controller panel created by the `InverseManager` (Section 10.4.3), or by selecting the relevant component in the navigation panel and choosing "Edit properties ..." from the context menu. The properties can also be set in code, using the accessor methods supplied by their host component. These methods will usually be named `getXxx()` and `setXxx()` for property xxx.Properties of the tracking controller itself include:

**active**

Controls whether or not the controller is active. Setting this to `false` turns the controller off. Default value is `true`.

**normalizeCostTerms**

Controls whether or not the cost terms are normalized. Normalizing the terms helps ensure that their weights (e.g., $w_m$, $w_e$, and $w_a$ in (10.12) more accurately control the tradeoffs between terms. Default value is `true`.

**computeIncrementally**

Controls whether excitation values should be computed incrementally, as described in Section 10.1.5. Default value is `false`.

**excitationBounds**

Specifies the default excitation bounds for exciter components. Default value is $[0, 1]$.

**targetsVisible**

Controls the visibility of the motion target terms contained within the controller, as described in Section 10.2.2.2. Default value is `true`.

**configExcitationColoring**

If enabled, automatically configures exciters which support excitation coloring to vary from white to red as their excitation increases (Section 10.2.1.1). Default value is `true`.

**probeDuration**

Default duration of probes managed by the InverseManager class (Section 10.4). Default value is 1.0.

**probeUpdateInterval**

Default update interval of output probes managed by the InverseManager class (Section 10.4). Default value is -1, which means that the interval will default to the simulation step size.

**useKKTFactorization**

Uses a more computationally expensive but possibly more accurate method to compute the excitation response matrices matrices such as $\mathbf{H}_m$ and $\mathbf{H}_e$ (Section 10.1.3). Default value is `false`.

**maxExcitationJump**

Experimental property that limits the change in excitation during any time step. Default value is `false`.

**useTimestepScaling**

Experimental property which enables scaling of all the cost terms by the simulation time step. Default value is `false`.

**debug**

Enabled debugging messages. Default value is `false`.

### 10.3.3 Motion term properties

The motion target term (Section 10.2.2.1) exports a number of properties that control motion tracking:

**enabled**

Controls whether or not this term is enabled. Default value is `true`.

**weight**

Specifies the weight $w_m$ of this term relative to the other cost terms. Default value is 1.0.

**chaseTime**

Specifies the *chase time* $T$ used for chase control (Section 10.1.2.1). Default value is 0.01.

**usePDControl**

Controls whether or not PD control is used (Section 10.1.2.2). Default value is `false`.

**Kp**

Specifies the proportional terms for PD control (Section 10.1.2.2). Default value is 10000.

**Kd**

Specifies the derivative terms for PD control (Section 10.1.2.2). Default value is 100.

**legacyControl**

Controls whether or not legacy control is used. Default value is `false`.

### 10.3.4 Properties for other cost terms

All other cost terms, including the L2 regularization (Section 10.2.3.1), export the following properties:

**enabled**

Controls whether or not the term is enabled. Default value is `true`.

**weight**

Specifies the weight of this term relative to the other cost terms. Default value is 1.0.

## 10.4 Managing probes and control panels

The tracking controller package provides the helper class InverseManager to aid with the creation of probes and control panels that are useful in for inverse simulation applications. The functionality provided by `InverseManager` is described in this section, and additional documentation may be found in its API documentation.

### 10.4.1 Inverse simulation probes

| ProbeID | I/O type | Probe name | Default filename |
|---|---|---|---|
| `TARGET_POSITIONS` | input | `"target positions"` | `target_positions.txt` |
| `INPUT_EXCITATIONS` | input | `"input excitations"` | `input_excitations.txt` |
| `TRACKED_POSITIONS` | output | `"tracked positions"` | `tracked_positions.txt` |
| `SOURCE_POSITIONS` | output | `"source positions"` | `source_positions.txt` |
| `COMPUTED_EXCITATIONS` | output | `"computed excitations"` | `computed_excitations.txt` |

Table 10.1: Probe types, names, and default file names associated with `ProbeID`.

`InverseManager` contains static methods that support the creation and management of a variety of probes related to inverse simulation. These probes are defined by the enumerated type InverseManager.ProbeID, which includes the following identifiers:

`TARGET_POSITIONS`

: An input probe which provides a trajectory $\mathbf{x}_t(t)$ for all motion targets. Its input values are attached to the position property (3 values) of each point target and the position and orientation properties of each frame target (7 values), in the order the targets appear in the controller.

`INPUT_EXCITATIONS`

: An input probe which provides input excitation values for all the controller's exciters, in the order they appear in the controller. This probe can be used for testing purposes, in order to see what trajectory can be expected to arise from a given excitation input.

`TRACKED_POSITIONS`

: An output probe which records the positions of all motion targets as they are set by the controller. Its output values are extracted from the same component properties used by `TARGET_POSITIONS`.

`SOURCE_POSITIONS`

: An output probe which records that actual position trajectory $\mathbf{x}(t)$ followed by all motion sources. Its output values are extracted from the same component properties used by `TARGET_POSITIONS`.

`INPUT_EXCITATIONS`

: An output probe which records the computed excitation values for all the controller's exciters, in the order they appear in the controller.

Each of these probes is associated with both a name and a default file name (Table 10.1), where the default file name may be assigned if another file path is not explicitly specified.

> When a default file name is used, it is assumed to be relative to the ArtiSynth working folder, as described in Section 5.4.

These probes may be created, for a given tracking controller, with the following (static) `InverseManager` methods:

| | |
|---|---|
| NumericInputProbe createInputProbe (TrackingController tcon ProbeID pid, String filePath, double start, double stop) | Create input probe specified by `pid`. |
| NumericOutputProbe createOutputProbe (TrackingController tcon ProbeID pid, String filePath, double start, double stop, double interval) | Create output probe specified by `pid`. |

Here pid specifies the probe type, filePath an (optional) file path, start and stop the start and stop times, and interval the output probe update interval.

Created probes may be added to the application root model as needed.

Most of the examples above use createOutputProbe() to create a computed excitations probe. In InverseMuscleFem (Section 10.2.5), the two output probes used to display the tracked position and orientation of myFrame, created at lines (42-51), *could* be replaced by a single probe of type SOURCE_POSITIONS, using a code fragment like this:

```
NumericOutputProbe sourceProbe = InverseManager.createOutputProbe (
   tcon, ProbeID.SOURCE_POSITIONS, /*fileName=*/null,
   startTime, stopTime, /*interval=*/-1);
addOutputProbe (sourceProbe);
```

Likewise, a probe showing the target position and orientation, for purposes of comparing it against SOURCE_POSITIONS and evaluating the tracking error, could be created using

```
NumericOutputProbe trackedProbe = InverseManager.createOutputProbe (
   tcon, ProbeID.TRACKED_POSITIONS, /*fileName=*/null,
   startTime, stopTime, /*interval=*/-1);
addOutputProbe (trackedProbe);
```

For both these method calls, the source and target components (myFrame and target in the example) are automatically located within the tracking controller.

> One reason to create a TRACKED_POSITIONS probe, instead of comparing SOURCE_POSITIONS directly against the trajectory input probe, is that the former is guaranteed to have the same number of data points as SOURCE_POSITIONS, whereas a trajectory probe, being an input probe, may not. Moreover, if the trajectory is being produced using a controller, a trajectory probe may not even exist.

For convenience, InverseManager also provides methods to add a complete set of default probes to the root model:

| | |
|---|---|
| void addProbes (RootModel root, TrackingController tcon, double duration, double interval) | Add default probes to root. |
| void resetProbes (RootModel root, TrackingController tcon, double duration, double interval) | Clear probes and add default probes to root. |

addProbes() creates a COMPUTED_EXCITATIONS probe and an INPUT_EXCITATIONS probe, with the latter deactivated by default. If the controller has any motion targets, it will also create TARGET_POSITIONS, TRACKED_POSITIONS and SOURCE_POSITIONS probes. Probe start times are set to 0, stop times are set to duration, and the output probe update interval is set to interval, with -1 causing the interval to default to the simulation step size. Probe file names are set to the default file names described in Table 10.1. Since these file names are relative, the files themselves are assumed to exist in the ArtiSynth working folder, as described in Section 5.4. If the file for an input probe actually exists, it is used to initialize the probe's data; otherwise, the probe data is initialized to 0 and the probe is deactivated.

resetProbes() removes all existing probes and then behaves identically to addProbes().

InverseManager also supplies methods to locate and adjust the probes that it has created:

| | |
|---|---|
| NumericInputProbe findInputProbe (RootModel root, ProbeID pid) | Find specified input probe. |
| NumericOutputProbe findOutputProbe (RootModel root, ProbeID pid) | Find specified output probe. |
| Probe findProbe (RootModel root, ProbeID pid) | Find specified input or output probe. |
| boolean setInputProbeData (RootModel root, ProbeID pid, double[] data, double timeStep) | Set input probe data. |
| boolean setProbeFileName (RootModel root, ProbeID pid, String filePath) | Set probe file path. |

The findProbe() methods search for the probe specified by pid within a root model, using the probe's file name (Table 10.1) as a search key. The method setInputProbeData() searches for a specified probe, and if it is found, sets its data using the probe's setData(double[],double) method and returns true. Likewise, setProbeFileName() searches for a specified probe and sets its file path.

Since probes maintained by `InverseManager` use probe names as a search key, care should be taken to ensure that these names do not conflict with those of other application probes.

### 10.4.2 Example: using `InverseManager` probes

The example `InverseMuscleArm` (Section 10.2.4) has been configured for use with the `InverseManager` by setting its working folder to `inverseMuscleArm` located beneath its source folder. If instead of creating the target and output excitations probes (lines 34-52), `addProbes()` is simply called at the end of the `build()` method (after the working folder has been set), as in

```
// set working folder for probe files
ArtisynthPath.setWorkingFolder (
   new File (PathFinder.getSourceRelativePath (this, "inverseMuscleArm")));
InverseManager.addProbes (this, tcon, 5.0, -1);
```

then a set of probes will be created as shown in Figure 10.13. The tracing probes (`"target tracing"` and `"source tracing"`) are still present, in addition to five default probes. `TARGET_POSITIONS` and `INPUT_EXCITATIONS` are expanded to show the data that has been automatically loaded from the files `target_positions.txt` and `input_excitations.txt` located in the working folder. By default, the former probe is enabled and the latter is disabled. If the model is run, inverse simulation will proceed as before.

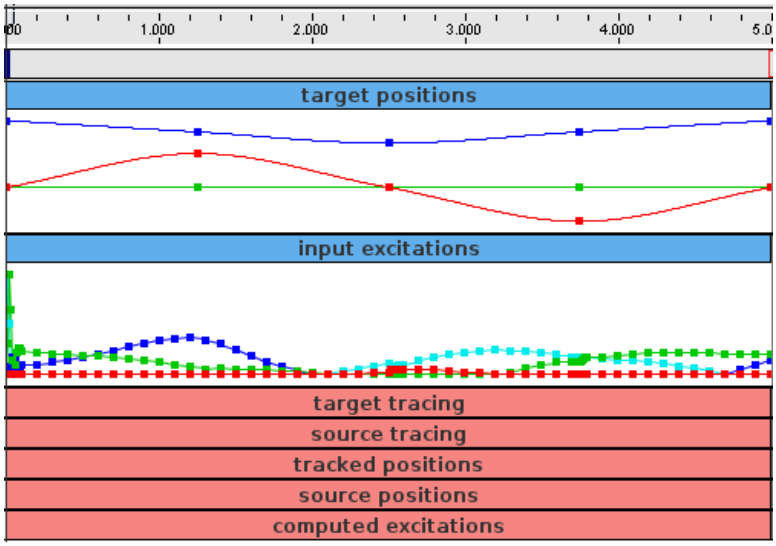



Figure 10.13: `InverseMuscleArm` probes with original tracing probes and default probes.

After loading the probes, the application *could* disable the tracking controller and enable the input excitations:

```
// set working folder for probe files
ArtisynthPath.setWorkingFolder (
   new File (PathFinder.getSourceRelativePath (this, "inverseMuscleArm")));
InverseManager.addProbes (this, tcon, 5.0, -1);
tcon.setActive (false);
InverseManager.findProbe (this, ProbeID.INPUT_EXCITATIONS).setActive (true);
```

This will cause the simulation to run "open loop" in response to the excitations. However, because the input excitations are slightly different than those that were computed by the controller, there is a now a considerable tracking error (Figure 10.14, left).

After the open loop run, one can save the resulting `SOURCE_POSITIONS` probe, which will write the file `"target_positions.txt"` into the working folder. This can then be used to generate a new target trajectory, by copying it to `"target_positions.txt"`. If the model is reloaded, the new trajectory will now appear in the `TARGET_POSITIONS` probe (Figure 10.15). Deactivating the `INPUT_EXCITATIONS` probe and reactivating the tracking controller will cause this new trajectory to be tracked properly (Figure 10.14, right).

In ways such as this, the probes may be used to examine the behavior and performance of the tracking controller.

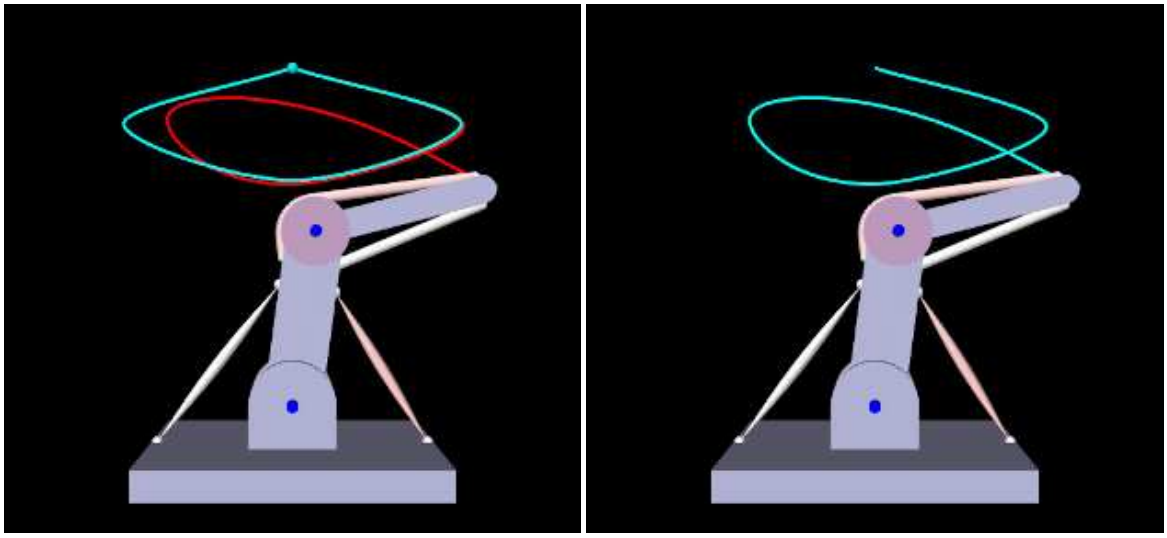


Figure 10.14: Left: `InverseMuscleFem` when run open loop with excitations loaded from `"input_excitations.txt"`. Right: model run with inverse simulation, using new targets generated by the open loop run.

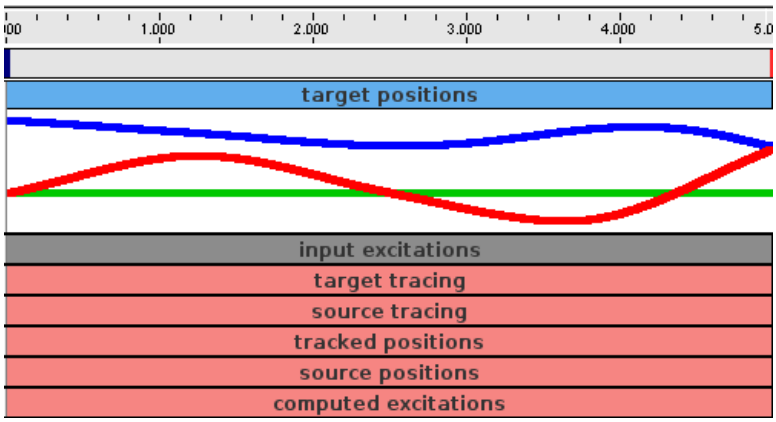



Figure 10.15: `InverseMuscleArm` probes with new target trajectory copied from the open loop run.

### 10.4.3 Inverse control panel

`InverseManager` also provide methods to create and manage an *inverse control panel*, which allows interactive editing of the properties of the tracking controller and its various cost terms, as described in Section 10.3. Applications can create an instance of this panel in their `build()` method, as the `InverseMuscleArm` example (Section 10.2.4) does at line 65:

```
InverseManager.addInversePanel (this, tcon);
```

The panel is automatically configured to display the properties of the cost terms with which the controller is currently configured. Figure 10.16 shows the panel for `InverseManager`.

The following methods are supplied by `InverseManager()` to manage inverse control panels:

| | |
|---|---|
| ControlPanel createInversePanel (TrackingController tcon) | Create inverse control panel for `tcon`. |
| ControlPanel createInversePanel (RootModel root, TrackingController tcon) | Create and add inverse control panel to `root`. |
| ControlPanel findInversePanel (RootModel root) | Find inverse control panel in `root`. |

At the top of the inverse panel are three buttons: "reset probes", "sync excitations", and "sync targets". The first uses `InverseManager.resetProbes()` to clear all probes and recreate the default probes, using the duration and update interval specified by the tracking controller properties probeDuration and probeUpdateInterval. The second button copies

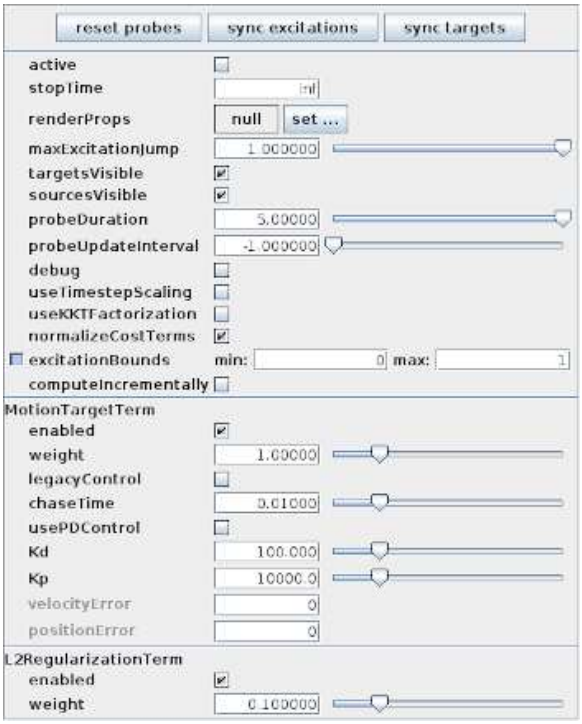



Figure 10.16: Inverse control panel created for the `InverseMuscleArm` demo.

the contents of the `COMPUTED_EXCITATIONS` probe into the `INPUT_EXCITATIONS` probe. If this is done after excitations have been computed, then running the simulation again with the controller deactivated and `INPUT_EXCITATIONS` activated should produce an identical motion. The third button copies the contents of the `SOURCE_POSITIONS` probe into the `TARGET_POSITIONS`. This can be used to verify that a source trajectory created open loop with a prescribed set of input excitations can in fact be tracked by the controller.

The probe copy methods used by the "sync" buttons are also available to the application code:

| | |
|---|---|
| boolean syncTargetProbes (RootModel root) | Copy `SOURCE_POSITIONS` into `TARGET_POSITIONS`. |
| boolean syncExcitationProbes (RootModel root) | Copy `COMPUTED_EXCITATIONS` into `INPUT_EXCITATIONS`. |

## 10.5 Caveats and limitations

Use of the inverse controller is subject to some caveats.

- Tracked components must be dynamic, in that their motion is controlled by forces. This means that their dynamic property must be set to `true` or they must be attached to other bodies that are dynamic.
- The algorithm which computes the excitations currently has a complexity $O(n^3)$ in the number of excitations $n$. This is due both to the use of numeric differentiation for determining excitation response matrices such as $\mathbf{H}_m$ and $\mathbf{H}_e$ (Section 10.1.3), and also the fact that the QP solver is at present a dense solver. This means that applications with large numbers of excitations may require considerably longer to simulate than those with fewer excitations.
- The controller is not designed to work with unilateral constraints (the most common examples of which are constraint-based contact and joint limits). For models containing these behaviors, inverse simulation may produce excitations that jump around erratically. In the case of collisions, one solution may be to use force-based collisions, implemented with either PointPlaneForce or PointMeshForce, as described in Section 3.6. Setting these components to use a quadratic force function may improve performance even more. Another possible solution, for any kind of unilateral constraint, may be to make it compliant (Sections 3.3.8 and 8.7), since this has the same effect as making the constraint force-based.

- Prescribed motion trajectories may need to be sufficiently smooth the avoid large transient excitations, particularly at the start and end of motions when velocities transition from and to 0. Transient effects are often more pronounced for models where inertial effects are high. Cubic interpolation is useful for creating smoother trajectories, particularly with a limited number of data points. It may also be necessary to explicitly smooth an input trajectory with additional points.

  In the `InverseMuscleArm` example, there is a noticeably transient at the start (Figure 10.6, left). Increasing the L2 regularization weight reduces the transient amplitude but spreads it out over a longer time (Figure 10.6, right), at the expense of tracking accuracy.

# Chapter 11

# Skinning

A useful technique for creating anatomical and biomechanical models is to attach a passive mesh to an underlying set of dynamically active bodies so that it deforms in accordance with the motion of those bodies. ArtiSynth allows meshes to be attached, or "skinned", to collections of both rigid bodies and FEM models, facilitating the creation of structures that are either embedded in, or connect or envelope a set of underlying components. Such approaches are well known in the computer animation community, where they are widely used to represent the deformable tissue surrounding a "skeleton" of articulated rigid bodies, and have more recently been applied to biomechanics [17].

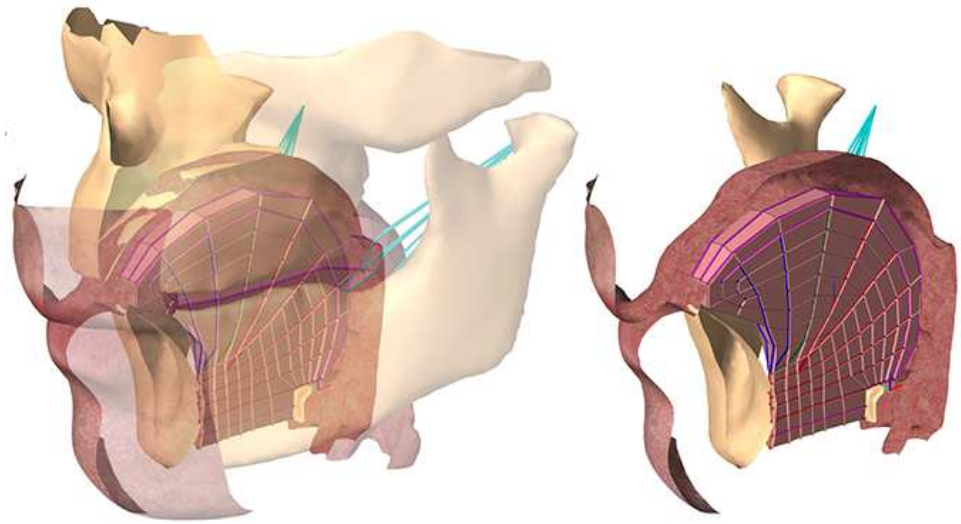


Figure 11.1: A skin mesh used to delimit the boundary of the human upper airway, connected to various surrounding structures including the palate, tongue, and jaw [24].

One application of skinning is to create a continuous skin surrounding an underlying set of anatomical components. For example, for modeling the human airway, a disparate set of models describing the tongue, jaw, palate and pharynx can be connected together with a surface skin to form a seamless airtight mesh (Figure 11.1), as described in [24]. This then provides a uniform boundary for handling air or fluid interactions associated with tasks such as speech or swallowing.

ArtiSynth provides support for "skinning" a mesh over an underlying set of *master bodies*, consisting of rigid bodies and/or FEM models, such that the mesh vertices deform in response to changes in the position, orientation and shape of the master bodies.

## 11.1 Implementation

This section describes the technical details of the ArtiSynth skinning mechanism. A skin mesh is implemented using a SkinMeshBody, which contains a base mesh and references to a set of underlying dynamic *master bodies*. A master body can be either a Frame (of which RigidBody is a subclass), or a FemModel3d. The positions of the mesh vertices

(along with markers and other points that can be attached to the skin mesh) are determined by a weighted sum of influences from each of the $m$ master bodies, such that as the latter move and/or deform, the vertices and attached points deform as well. More precisely, for each master body $k, k \in \{0,\ldots,m-1\}$, let $w_k$ be the weighting factor and $f_k(\mathbf{q}_k)$ the *connection function* that describes the contribution of body $k$ to the position of the vertices (or attached points) as a function of its generalized coordinates $\mathbf{q}_k$. Then if the position of a vertex (or attached point) is denoted by $\mathbf{p}$ and its initial (or *base*) position is $\mathbf{p}_0$, we have

$$\mathbf{p} = \sum_{k=0}^{m-1} w_k f_k(\mathbf{q}_k) + w_m \mathbf{p}_0. \tag{11.1}$$

The weight $w_m$ in the last term is known as the *base weight* and describes an optional contribution from the base position $\mathbf{p}_0$. Usually $w_m = 0$, unless the vertex is not connected to `any` master bodies, in which case $w_m = 1$, so that the vertex is anchored to its initial position.

In general, connection weights $w_k$ are computed based on the distances $d_k$ between the vertex (or attached point) and each master body $k$. More details on this are given in Sections 11.2 and 11.3.

For `Frame` master bodies, the connection function is one associated with various rigid body skinning techniques known in the literature. These include linear, linear dual quaternion, and iterative dual quaternion skinning. Which technique is used is determined by the frameBlending property of the `SkinMeshBody`, which can be queried or set in code using the methods

```
FrameBlending getFrameBlending()

void setFrameBlending (FrameBlending blending)
```

where FrameBlending is an enumerated type defined by `SkinMeshBody` with the following values:

**LINEAR**

Linear blending, in which the connection function $f_k()$ implements a standard rigid connection between the vertex and the frame coordinates. Let the frame's generalized coordinates $\mathbf{q}_k$ be given by the $3 \times 3$ rotation matrix $\mathbf{R}$ and translation vector $\mathbf{p}_F$ describing its pose, with its initial pose given by $\mathbf{R}_0$ and $\mathbf{p}_{F0}$. The connection function $f_k()$ then takes the form

$$f_k(\mathbf{R}, \mathbf{p}_F) = \mathbf{R}\mathbf{R}_0^T(\mathbf{p}_0 - \mathbf{p}_{F0}) + \mathbf{p}_F. \tag{11.2}$$

Linear blending is faster than other blending techniques but is more prone to pinching and creasing artifacts in the presence of large rotations between frames.

**DUAL_QUATERNION_LINEAR**

Linear dual quaternion blending, which is more computationally expensive but typically gives better results than linear blending, and is described in detail as DLB in [8]. Let the frame's generalized coordinates $\mathbf{q}_k$ be given by the dual-quaternion $\hat{\mathbf{q}}_k$ (describing both rotation and translation), with the initial pose given by the dual-quaternion $\hat{\mathbf{q}}_{k0}$. Then define the relative dual-quaternion $\tilde{\mathbf{q}}_k$ as

$$\tilde{\mathbf{q}}_k = \frac{\hat{\mathbf{q}}_k \hat{\mathbf{q}}_{k0}^{-1}}{\| \sum_j w_j \hat{\mathbf{q}}_j \hat{\mathbf{q}}_{j0}^{-1} \|}, \tag{11.3}$$

where the denominator is formed by summing over *all* master bodies $j$ which are frames. The connection function $f_k()$ is then given by

$$\mathbf{f}_k(\hat{\mathbf{q}}_k) = \tilde{\mathbf{q}}_k \mathbf{p}_0 \tilde{\mathbf{q}}_k^{-1} - \mathbf{p}_0, \tag{11.4}$$

where we note that a dual quaternion multiplied by a position vector yields a position vector.

**DUAL_QUATERNION_ITERATIVE**

Dual quaternion iterative blending, which is a more complex dual quaternion technique described in detail as DIB in [8]. The connection function for iterative dual quaternion blending involves an iterative process and is not described here. It also does not conform to (11.1), because the connection functions $f_k()$ for the `Frame` master bodies do not combine linearly. Instead, if there are $r$ `Frame` master bodies, there is a *single* connection function

$$f(w_0,\ldots,w_{r-1},\hat{\mathbf{q}}_0,\ldots,\hat{\mathbf{q}}_{r-1}) \tag{11.5}$$

that determines the connection for *all* of them, given their weighting factors $w_j$ and generalized coordinates $\hat{\mathbf{q}}_j$. Iterative blending relies on two parameters: a blend tolerance, and a maximum number of blend steps, both of which are controlled by the SkinMeshBody properties DQBlendTolerance and DQMaxBlendSteps, which have default values of $1^{-8}$ and 3.

> Iterative dual quaternion blending is not completely supported in ArtiSynth. In particular, because of its complexity, the associated force and velocity mappings are computed using the simpler computations employed for linear dual quaternion blending. For the examples shown in this chapter, iterative dual quaternion gives results that are quite close to those of linear dual quaternion blending.

For FEM master bodies, the connection works by tying each vertex (or attached point) to a specific FEM element using a fixed-length offset vector $\mathbf{d}$ that rotates in conjunction with the element. This is illustrated in Figure 11.2 for the case of a single FEM master body. Starting with the initial vertex position $\mathbf{p}_0$, we find the nearest point $\mathbf{p}_{e0}$ on the nearest FEM element, along with the offset vector $\mathbf{d}_0 \equiv \mathbf{p}_0 - \mathbf{p}_{e0}$. The point $\mathbf{p}_{e0}$ can be expressed as the weighted sum of the initial element nodal positions $\mathbf{x}_{j0}$,

$$\mathbf{p}_{e0} = \sum_{j=0}^{n-1} \alpha_j \mathbf{x}_{j0}, \tag{11.6}$$

where $n$ is the number of nodes and $\alpha_j$ represent the (constant) nodal coordinates. As the element moves and deforms, the element point $\mathbf{p}_e$ moves with the nodal positions $\mathbf{x}_j$ according to the same relationship, while the offset vector $\mathbf{d}$ rotates according to $\mathbf{d} = \mathbf{R}_E d_0$, where $\mathbf{R}_E$ is the rotation of the element's coordinate frame $E$ with respect to its initial orientation. The connection function $f_k()$ then takes the form

$$f_k(\mathbf{x}_0, \ldots, \mathbf{x}_{n-1}) = \sum_{j=0}^{n-1} \alpha_j \mathbf{x}_j + \mathbf{R}_E d_0. \tag{11.7}$$

$\mathbf{R}_E$ is determined by computing a polar decomposition $\mathbf{F} = \mathbf{R}_E \mathbf{P}$ on the deformation gradient $\mathbf{F}$ at the element's center. We note that the displacement $\mathbf{d}$ is only rotated and so the distance $\|\mathbf{d}\| = \|\mathbf{d}_0\|$ of the vertex from the element remains constant. If the vertex is initially on or inside the element, then $\mathbf{d}_0 = 0$ and (11.7) takes the form of a standard point/element attachment as described in 6.4.3.

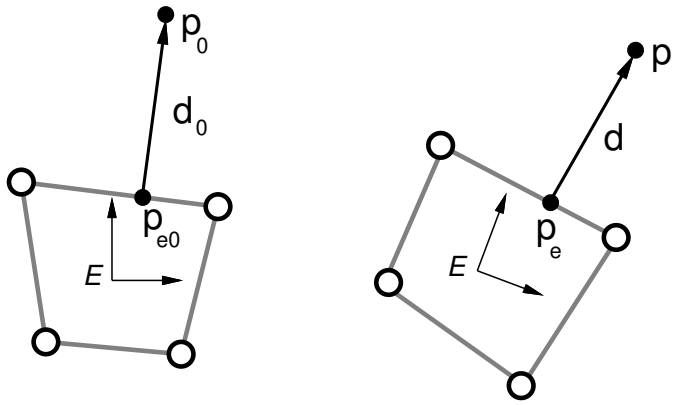



Figure 11.2: Illustration of FEM skinning, showing how a position $\mathbf{p}$ is tied to an FEM element. Given the initial position $\mathbf{p}_0$, we find the nearest point $\mathbf{p}_{e0}$ on the element, along with the offset vector $\mathbf{d}_0 = \mathbf{p}_0 - \mathbf{p}_{e0}$ (left). As the element moves and deforms, the updated position is obtained from $\mathbf{p} = \mathbf{p}_e + \mathbf{d}$, where $\mathbf{p}_e$ deforms with the element, and $\mathbf{d}$ rotates in tandem with its coordinate frame $E$.

> While it is sometimes possible to determine weights $\alpha_j$ that control a vertex position *outside* an element, without the need for an offset vector $\mathbf{d}$, the resulting vertex positions tend to be very sensitive to element distortions, particularly when the vertex is located at some distance. Keeping the element-vertex distance constant via an offset vector usually results in more plausible skinning behavior.

## 11.2 Creating a skin mesh

As mentioned above, skin meshes within ArtiSynth are implemented using the SkinMeshBody component. Applications typically create a skin mesh in code according to the following steps:

1. Create an instance of `SkinMeshBody` and assign its underlying mesh, usually within the constructor;
2. Add references to the required master bodies;
3. Compute the master body connections

This is illustrated by the following example:

```
MechModel mech;
PolygonalMesh mesh;

// ... initialize mesh ...

// create the body with an underlying mesh:
SkinMeshBody skinMesh = new SkinMeshBody(mesh);

// add references to the master bodies:
skinMesh.addMasterBody (rigidBody1);
skinMesh.addMasterBody (rigidBody2);
skinMesh.addMasterBody (femModel1);

// compute the weighted connections for each vertex:
skinMesh.computeAllVertexConnections();

// add to the MechModel
mech.addMeshBody (skinMesh)
```

Master body references are added using addMasterBody(). When all the master bodies have been added, the method computeAllVertexConnections() computes the weighted connections to each vertex. The connection weights $w_k$ for each vertex are determined by a *weighting function*, based on the distances $d_k$ between the vertex and each master body. The default weighting function is *inverse-square weighting*, which first computes a set of raw weights $w_k^*$ according to

$$w_k^* = \frac{d_{\min}^2}{d_k^2}, \tag{11.8}$$

where $d_{\min} \equiv \min(d_j)$ is the minimum master body distance, and then normalizes these to determine $w_k$:

$$w_k = \frac{w_k^*}{\sum_j w_j^*}. \tag{11.9}$$

Other weighting functions can be specified, as described in Section 11.3.

`SkinMeshBody` provides the following set of methods to set and query its master body configuration:

```
void addMasterBody (ModelComponent body)     // add a master body
int numMasterBodies()                        // query number of master bodies
boolean hasMasterBody (ModelComponent body) // query master body presence
ModelComponent getMasterBody (int idx)       // get a master body by index

RigidTransform3d getBasePose (Frame frame)  // query base pose for frame
void setBasePose (Frame frame, RigidTransform3d T) // set base pose for frame
```

When a `Frame` master body is added using `addMasterBody()`, its initial, or *base*, pose (corresponding to $\mathbf{R}_0$ and $\mathbf{p}_{F0}$ in Section 11.1) is set from its current pose. If necessary, applications can later query and reset the base pose using the methods `getBasePose()` and `setBasePose()`.

Internally, each vertex is connected to the master bodies by a PointSkinAttachment, which contains a list of PointSkinAttachment.SkinConnection components describing each master connection. Applications can obtain the `PointSkinAttachment` for each vertex using the method

```
PointSkinAttachment getVertexAttachment (int vidx)
```

where `vidx` is the vertex index, which must be in the range 0 to `numv`-1, with `numv` the number of vertices as returned by `numVertices()`. Methods also exist to query and set each vertex's base (i.e., initial) position $\mathbf{p}_0$:

```
Point3d getVertexBasePosition (int vidx)

void setVertexBasePosition (int vidx, Point3d pos)
```

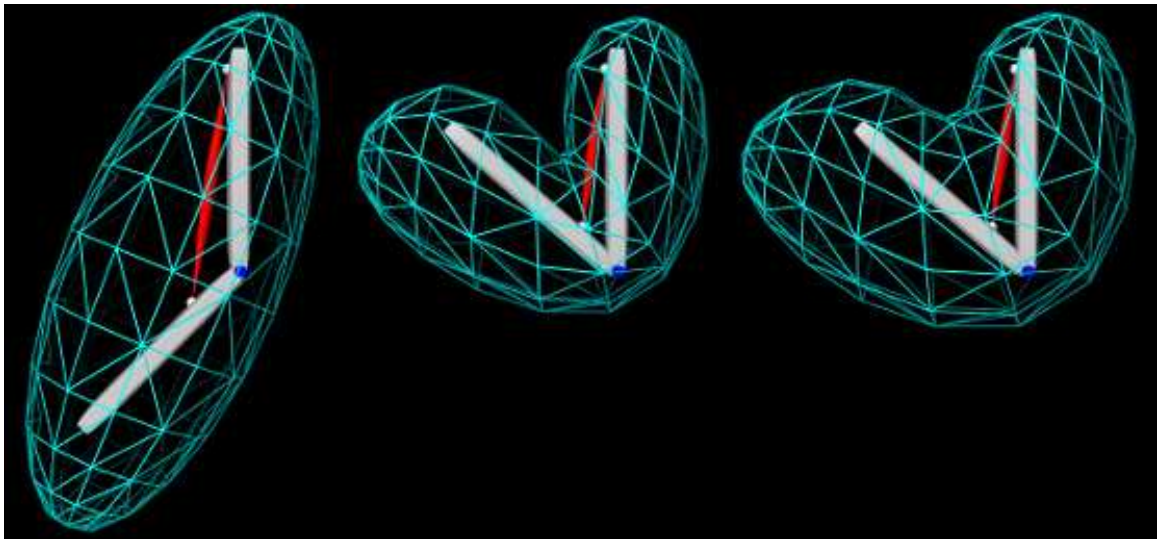


Figure 11.3: RigidBodySkinning model loaded into ArtiSynth (left), then run with excitation set to 0.7 (middle). The right image shows the result of changing the frameBlending property from `LINEAR` to `DUAL_QUATERNION_LINEAR`
.

### 11.2.1 Example: skinning over rigid bodies

An example of skinning a mesh over two rigid bodies is given by `artisynth.demos.tutorial.RigidBodySkinning`. It consists of a `SkinMeshBody` placed around two rigid bodies connected by a hinge joint to form a toy "arm", with a Muscle added to move the lower body with respect to the upper. The code for the `build()` method is given below:

```
1     public void build (String[] args) throws IOException {
2        MechModel mech = new MechModel ("mech");
3        addModel (mech);
4
5        // set damping parameters for rigid bodies
6        mech.setFrameDamping (10);
7        mech.setRotaryDamping (100.0);
8
9        // create a toy "arm" conisting of upper and lower rigid bodies connected
10       // by a revolute joint:
11       double len = 2.0;
12       RigidBody upper = addBody (mech, "upper");
13       upper.setPose (new RigidTransform3d (0, 0, len/2));
14       upper.setDynamic (false);  // upper body is fixed
15
16       RigidBody lower = addBody (mech, "lower");
17       // reposition the lower body"
18       double angle = Math.toRadians(225);
19       double sin = Math.sin(angle);
20       double cos = Math.cos(angle);
21       lower.setPose (new RigidTransform3d (sin*len/2, 0, cos*len/2, 0, angle, 0));
22
23       // add the revolute joint between the upper and lower bodies:
24       HingeJoint joint =
25          new HingeJoint (lower, upper, new Point3d(), Vector3d.Y_UNIT);
26       joint.setName ("elbow");
27       mech.addBodyConnector (joint);
28
29       // add two frame markers and a "muscle" to move the lower body
30       FrameMarker mku = mech.addFrameMarker (
31          upper, new Point3d(-len/20, 0, len/2.4));
32       FrameMarker mkl = mech.addFrameMarker (
33          lower, new Point3d(len/20, 0, -len/4));
34       Muscle muscle = new Muscle("muscle");
```

```
35        muscle.setMaterial (new SimpleAxialMuscle (1000.0, 0, 2000.0));
36        mech.attachAxialSpring (mku, mkl, muscle);
37
38        // create an ellipsoidal base mesh for the SkinMeshBody by scaling a
39        // spherical mesh
40        PolygonalMesh mesh = MeshFactory.createSphere (1.0, 12, 12);
41        mesh.scale (1, 1, 2.5);
42        mesh.transform (
43           new RigidTransform3d (-0.6, 0, 0, 0, Math.toRadians(22.5),0));
44
45        // create the skinMesh, with the upper and lower bodies as master bodies
46        SkinMeshBody skinMesh = new SkinMeshBody (mesh);
47        skinMesh.addMasterBody (upper);
48        skinMesh.addMasterBody (lower);
49        skinMesh.computeAllVertexConnections();
50        mech.addMeshBody (skinMesh);
51
52        // add a control panel to adjust the muscle excitation and frameBlending
53        ControlPanel panel = new ControlPanel();
54        panel.addWidget (muscle, "excitation");
55        panel.addWidget (skinMesh, "frameBlending");
56        addControlPanel (panel);
57
58        // set up render properties
59        RenderProps.setFaceStyle (skinMesh, Renderer.FaceStyle.NONE);
60        RenderProps.setDrawEdges (skinMesh, true);
61        RenderProps.setLineColor (skinMesh, Color.CYAN);
62        RenderProps.setSpindleLines (muscle, 0.06, Color.RED);
63        RenderProps.setSphericalPoints (mech, 0.05, Color.WHITE);
64        RenderProps.setFaceColor (joint, Color.BLUE);
65        joint.setShaftLength(len/3);
66        joint.setShaftRadius(0.05);
67        RenderProps.setFaceColor (mech, new Color (0.8f, 0.8f, 0.8f));
68     }
```

A `MechModel` is created in the usual way (lines 2-7). To this is added a very simple toy "arm" consisting of an upper and lower body connected by a hinge joint (lines 9-27), with a simple point-to-point muscle attached between frame markers on the upper and lower bodies to provide a means of moving the arm (lines 29-36). Creation of the arm bodies uses an `addBody()` method which is not shown.

The mesh to be skinned is an ellipsoid, created using the FemFactory method `createSphere()` to produce a spherical mesh which is then scaled and repositioned (lines 38-43). The skin body itself is then created around this mesh, with the upper and lower bodies assigned as master bodies and the connections computed using `computeAllVertexConnections()` (lines 45-50).

A control panel is added to allow control over the muscle's excitation as well as the skin body's frameBlending property (lines 52-56). Finally, render properties are set (lines 58-67): the skin mesh is made transparent by setting its faceStyle and drawEdges properties to `NONE` and `true`, respectively, with cyan colored edges; the muscle is rendered as a red spindle; the joint is drawn as a blue cylinder and the bodies are colored light gray.

To run this example in ArtiSynth, select All demos > tutorial > RigidBodySkinning from the Models menu. The model should load and initially appear as in Figure 11.3 (left). When running the simulation, the arm can be flexed by adjusting the muscle excitation property in the control panel, causing the skin mesh to deform (Figure 11.3, middle). Changing the frameBlending property from its default value of `LINEAR` to `DUAL_QUATERNION_LINEAR` causes the mesh deformation to become fatter and less prone to creasing (Figure 11.3, right).

## 11.3 Computing weights

As described above, the default method for computing skin connection weights is inverse-square weighting (equations 11.8) and 11.9). However, applications can specify alternatives to this. The method

```
    void setGaussianWeighting (double sigma)
```

causes weights to be computed according to a *Gaussian weighting* scheme, with `sigma` specifying the standard deviation $\sigma$. Raw weights $w_k^*$ are then computed according to

$$w_i = \exp\left(-\frac{(d_k - d_{\min})^2}{2\sigma^2}\right),$$

and then normalized to form $w_k$.

The method

```
void setInverseSquareWeighting()
```

reverts the weighting function back to inverse-square weighting.

It is also possible to specify a custom weighting function by implementing a subclass of SkinWeightingFunction. Subclasses must implement the function

```
void computeWeights (
    double[] weights, Point3d pos, NearestPoint[] nearestPnts);
```

in which the weights for each master body are computed and returned in `weights`. `pos` gives the initial position of the vertex (or attached point) being skinning, while `nearestPnts` provides information about the distance from `pos` to each of the master bodies, using an array of SkinMeshBody.NearestPoint objects:

```
class NearestPoint {
   public Point3d nearPoint;   // nearest point on the body
   public double distance;     // distance to the body
   public ModelComponent body; // master body (either Frame or FemModel3d)
}
```

Once an instance of `SkinWeightingFunction` has been created, it can be set as the skin mesh weighting function by calling

```
void setWeightingFunction (SkinWeightingFunction fxn)
```

Subsequent calls to `computeAllVertexConnections()`, or the `addMarker` or `computeAttachment` methods described in Section 11.4, will then employ the specified weighting.

As an example, imagine an application wishes to compute weights according to an inverse-cubic weighting function, such that to

$$w_k^* = \frac{d_{\min}^3}{d_k^3}.$$

A subclass of `SkinWeightingFunction` implementing this could then be defined as

```
class MyWeighting extends SkinWeightingFunction {

   // implements inverse-cubic weighting
   public void computeWeights (
      double[] weights, Point3d pos, NearestPoint[] nearestPnts) {

      // find minimum distance to all the master bodies
      double dmin = Double.POSITIVE_INFINITY;
      for (int i=0; i<nearestPnts.length; i++) {
         if (nearestPnts[i].distance < dmin) {
            dmin = nearestPnts[i].distance;
         }
      }
      double sumw = 0; // sum of all weights (for normalizing)
      // compute raw weights:
      for (int i=0; i<nearestPnts.length; i++) {
         double d = nearestPnts[i].distance;
         double w;
         if (d == dmin) {
            w = 1;  // handles case where dmin = d = 0
         }
```

```
            else {
               w = dmin*dmin*dmin/(d*d*d);
            }
            weights[i] = w;
            sumw += w;
         }
         // normalize the weights:
         for (int i=0; i<nearestPnts.length; i++) {
            weights[i] /= sumw;
         }
      }
   }
```

and then set as the weighting function using the code fragment:

```
      SkinMeshBody skinMesh;
      // ...
      skinMesh.setWeightingFunction (new MyWeighting());
```

The current weighting function for a skin mesh can be queried using

```
      SkinWeightingFunction getWeightingFunction()
```

The inverse-square and Gaussian weighting methods described above are implemented using the system-provided `SkinWeightingFunction` subclasses InverseSquareWeighting and GaussianWeighting, respectively.

### 11.3.1 Setting weights explicitly

As an alternative to the weighting function, applications can also create connections to vertices or points in which the weights are explicitly specified. This allows for situations in which a weighting function is unable to properly specify all the weights correctly.

When a mesh is initially added to a skin body, via either the constructor SkinMeshBody(mesh), or by a call to setMesh(mesh), all master body connections are cleared and the vertex position is "fixed" to its initial position, also known as its *base position*. After the master bodies have been added, vertex connections can be created by calling computeAllVertexConnections(), as described above. However, connections can also be created on a per-vertex basis, using the method

```
    void computeVertexConnections (int vidx, VectorNd weights)
```

where `vidx` is the index of the desired vertex and `weights` is an optional argument which if non-`null` explicitly specifies the connection weights. A sketch example of how this can be used is given in the following code fragment:

```
      VectorNd weights = new VectorNd (skinMesh.numMasterBodies());
      // compute connections for each vertex
      for (int i=0; i<skinMesh.numVertices(); i++) {
         // ... compute connections weights as required ...
         skinMesh.computeVertexConnections (i, weights);
      }
```

For comparison, it should be noted that the code fragment

```
      for (int i=0; i<skinMesh.numVertices(); i++) {
         skinMesh.computeVertexConnections (i, null);
      }
```

in which weights are *not* explicitly specified, is is equivalent to calling `computeAllVertexConnections()`.

If necessary, after vertex connections have been computed, they can also be cleared, using the method

```
    void clearVertexConnections (int vidx)
```

This will disconnect the vertex with index `vidx` from the master bodies, and set its base weighting $w_m$ (equation 11.1) to 1, so that it will remain fixed to its initial position.

In some special cases, it may be desirable for an application to set attachment base weights to some value other than 0 when connections are present. Base weights for vertex attachments can be queried and set using the methods

```
double getVertexBaseWeight (int vidx)

void setVertexBaseWeight (int vidx, double weight, boolean normalize)
```

In the second method, the argument `normalize`, if `true`, causes the weights of the other connections to be scaled so that the total weight sum remains the same. For skin markers and point attachments (Section 11.4), base weights can be set by calling the equivalent `PointSkinAttachment` methods

```
double getBaseWeight ()

void setBaseWeight (double weight, boolean normalize)
```

(If needed, the attachment for a skin marker can be obtained by calling its `getAttachment()` method.) In addition, base weights can also be specified in the `weights` argument to the method `computeVertexConnections(vidx,weights)`, as well as the methods `addMarker(name,pos,weights)` and `createPointAttachment(pnt,weights)` described in Section 11.4. This is done by giving `weights` a size equal to $m+1$, where $m$ is the number of master bodies, and specifying the base weight in the last location.

## 11.4 Markers and point attachments

In addition to controlling the positions of mesh vertices, a `SkinMeshBody` can also be used to control the positions of dynamic point components, including markers and other points which can be attached to the skin body. For both markers and attached points, any applied forces are propagated back onto the skin body's master bodies, using the principle of virtual work. This allows skin bodies to be fully incorporated into a dynamic model.

Markers and point attachments can be created even if the `SkinMeshBody` does not have a mesh, a fact that can be used in situations where a mesh is unnecessary, such as when employing skinning techniques for muscle wrapping (Section 11.7).

### 11.4.1 Markers

Markers attached to a skin body are instances of SkinMarker, and are contained in the body's subcomponent list `markers` (analogous to the `markers` list for FEM models). Markers can be created and maintained using the following `SkinMeshBody` methods:

```
// create markers:
SkinMarker addMarker (Point3d pos)
SkinMarker addMarker (String name, Point3d pos)
SkinMarker addMarker (
   String name, Point3d pos, VectorNd weights)

// remove markers:
boolean removeMarker (SkinMarker mkr)
void clearMarkers();

// access the marker list:
PointList<SkinMarker> markers()
```

The `addMarker` methods each create and return a `SkinMarker` which is added to the skin body's list of markers. The marker's initial position is specified by `pos`, while the second and third methods also allow a name to be specified. Connections between the marker and the master bodies are created in the same way as for mesh vertices, with the connection weights either being determined by the skin body's weighting function (as returned by getWeightingFunction()), or explicitly specified by the argument `weights` (third method).

Once created, markers can be removed individually or all together by the `removeMarker()` and `clearMarkers()` methods. The entire marker list can be accessed on a read-only basis by the method `markers()`.

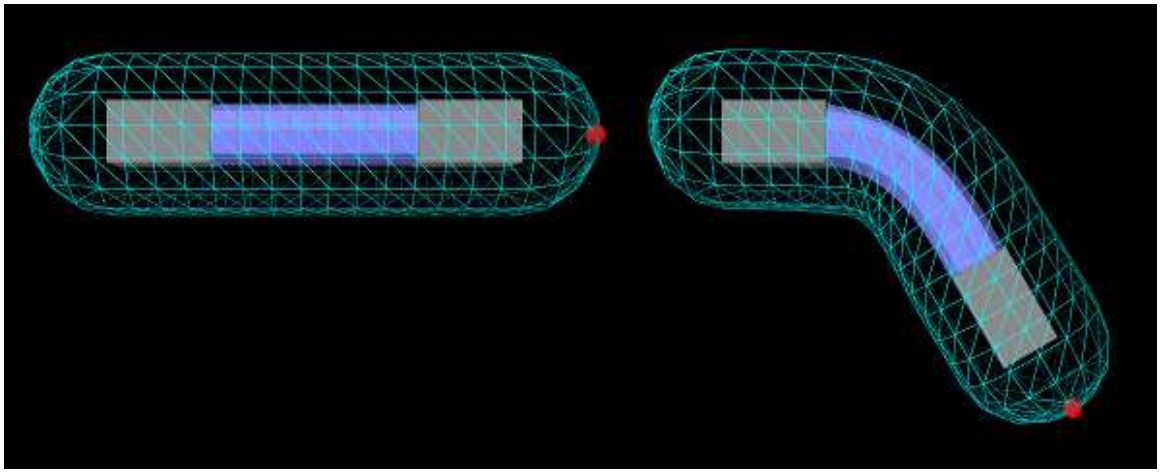


Figure 11.4: AllBodySkinning model as first loaded into ArtiSynth (left), and after the simulation is run and the model has fallen under gravity (right). The skin mesh is rendered in cyan using only its edges.

### 11.4.2 Point attachments

In addition to markers, applications can also attach any regular Point component (including particles and FEM nodes) to a skin body by using one of its `createPointAttachment` methods:

```
PointSkinAttachment createPointAttachment (Point pnt)
PointSkinAttachment createPointAttachment (Point pnt, VectorNd weights)
```

Both of these create a PointSkinAttachment that connects the point `pnt` to the master bodies in the same way as for mesh vertices and markers, with the connection weights either being determined by the skin body's weighting function or explicitly specified by the argument `weights` in the second method.

Once created, the point attachment must also be added to the underlying `MechModel`, as illustrated by the following code fragment:

```
MechModel mech;
SkinMeshBody skinBody;
Point pnt;

// ... initialize ...

PointSkinAttacment a = skinBody.createPointAttachment(pnt);
mech.addAttachment (a);
```

### 11.4.3 Example: skinning rigid bodies and FEM models

An example of skinning a mesh over both rigid bodies and FEM models is given by the demo model `artisynth.demos.tutorial.AllBodySkinning`. It consists of a skin mesh placed around a tubular FEM model connected to rigid bodies connected at each end, and a marker attached to the mesh tip. The code for the `build()` method is given below:

```
1    public void build (String[] args) {
2       MechModel mech = new MechModel ("mech");
3       addModel (mech);
4
5       // size and density parameters
6       double len = 1.0;
7       double rad = 0.15;
8       double density = 1000.0;
9
10      // create a tubular FEM model, and rotate it so it lies along the x axis
```

```
11        FemModel3d fem = FemFactory.createHexTube (
12           null, len, rad/3, rad, 8, 8, 2);
13        fem.transformGeometry (new RigidTransform3d (0, 0, 0, 0, Math.PI/2, 0));
14        mech.addModel (fem);
15
16        // create two rigid body blocks
17        RigidBody block0 =
18           RigidBody.createBox ("block0", len/2, 2*rad, 2*rad, density);
19        block0.setPose (new RigidTransform3d (-3*len/4, 0, 0));
20        mech.addRigidBody (block0);
21        block0.setDynamic (false);
22
23        RigidBody block1 =
24           RigidBody.createBox ("block1", len/2, 2*rad, 2*rad, density);
25        block1.setPose (new RigidTransform3d (3*len/4, 0, 0));
26        mech.addRigidBody (block1);
27
28        // attach the blocks to each end of the FEM model
29        for (FemNode3d n : fem.getNodes()) {
30           if (Math.abs(n.getPosition().x-len/2) < EPS) {
31              mech.attachPoint (n, block1);
32           }
33           if (Math.abs(n.getPosition().x+len/2) < EPS) {
34              mech.attachPoint (n, block0);
35           }
36        }
37        fem.setMaterial (new LinearMaterial (500000.0, 0.49));
38
39        // create base mesh to be skinned
40        PolygonalMesh mesh =
41           MeshFactory.createRoundedCylinder (
42              /*r=*/0.4, 2*len, /*nslices=*/16, /*nsegs=*/15, /*flatbotton=*/false);
43        // rotate mesh so its long axis lies along the x axis
44        mesh.transform (new RigidTransform3d (0, 0, 0, 0, Math.PI/2, 0));
45
46        // create the skinBody, with the FEM model and blocks as master bodies
47        SkinMeshBody skinBody = new SkinMeshBody ("skin", mesh);
48        skinBody.addMasterBody (fem);
49        skinBody.addMasterBody (block0);
50        skinBody.addMasterBody (block1);
51        skinBody.computeAllVertexConnections();
52        mech.addMeshBody (skinBody);
53
54        // add a marker point to the end of the skin mesh
55        SkinMarker marker =
56           skinBody.addMarker ("marker", new Point3d(1.4, 0.000, 0.000));
57
58        // set up rendering properties
59        RenderProps.setFaceStyle (skinBody, FaceStyle.NONE);
60        RenderProps.setDrawEdges (skinBody, true);
61        RenderProps.setLineColor (skinBody, Color.CYAN);
62        fem.setSurfaceRendering (FemModel.SurfaceRender.Shaded);
63        RenderProps.setFaceColor (fem, new Color (0.5f, 0.5f, 1f));
64        RenderProps.setSphericalPoints (marker, 0.05, Color.RED);
65     }
```

A `MechModel` is first created and length and density parameters are defined (lines 2-8). Then a tubular FEM model is created, by using the FemFactory method `createHexTube()` and transforming the result by rotating it by 90 degrees about the *y* axis (lines 10-14). Two rigid body blocks are then created (lines 16-26) and attached to the ends of the FEM model by finding and attaching the left and rightmost nodes (lines 28-24). The model is anchored to ground by setting the left block to be non-dynamic (line 21).

The mesh to be skinned is a rounded cylinder, created using the MeshFactory method `createRoundedCylinder()` and rotating the result by 90 degrees about the *y* axis (lines 39-44). This is then used to create the skin body itself, to which both rigid bodies and the FEM model are added as master bodies and the vertex connections are computed using a

call to `computeAllVertexConnections()` (lines 46-52). A marker is then added to the tip of the skin body, using the `addMarker()` method (lines 54-56). Finally render properties are set (lines 58-64): The mesh is made transparent by drawing only its edges in cyan; the FEM model surface mesh is rendered in blue-gray, and the tip marker is drawn as a red sphere.

To run this example in ArtiSynth, select All demos > tutorial > AllBodySkinning from the Models menu. The model should load and initially appear as in Figure 11.4 (left). When running the simulation, the FEM and the rightmost rigid body fall under gravity, causing the skin mesh to deform. The pull tool can then be used to move things around by applying forces to the master bodies or the skin mesh itself.

### 11.4.4 Mesh-based markers and attachments

For the markers and point attachments described above, the connections to the underlying master bodies are created in the same manner as connections for individual mesh vertices. This means that the resulting markers and attached points move *independently* of the mesh vertices, as though they were vertices in their own right.

An advantage to this is that such markers and attachments can be created even if the `SkinMeshBody` does not even have a mesh, as noted above. However, a disadvantage is that such markers will not remain tightly connected to vertex-based features (such as the faces of a PolygonalMesh or the line segments of a PolylineMesh). For example, consider a marker defined by

```
SkinMarker mkr = skinBody.addMarker (pos);
```

where `pos` is a point that is initially located on a face of the body's mesh. As the master bodies move and the mesh deforms, the resulting marker may not remain strictly on the face. In many cases, this may not be problematic or the deviation may be too small to matter. However, *if* it is desirable for markers or point attachments to be tightly bound to mesh features, they can instead be created with the following methods:

```
// create mesh-based markers:
SkinMarker addMarkerToMesh (Point3d pos)
SkinMarker addMarkerToMesh (String name, Point3d pos)

// create mesh-based attachments:
PointSkinAttachment createPointMeshAttachment (Point pnt)
```

The requested position `pos` will then be projected onto the nearest mesh feature (e.g., a face for a `PolygonalMesh` or a line segment for a `PolylineMesh`), and the resulting position $\mathbf{p}$ will be defined as a linear combination of the vertex positions $\mathbf{p}_i$ for this feature,

$$\mathbf{p} = \sum_i \beta_i \mathbf{p}_i, \tag{11.10}$$

where $\beta_i$ are the barycentric coordinates of $\mathbf{p}$ with respect to the feature. The master body connections are then defined by the same linear combination of the connections for each vertex. When the master bodies move, the marker or attached point will move with the feature and remain in the same relative position.

Since `SkinMeshBody` implements the interface PointAttachable, it provides the general point attachment method

```
PointSkinAttachment createPointAttachment (Point pnt)
```

which allows it to be acted on by agents such as the ArtiSynth marker tool (see the section "Marker tool" in the ArtiSynth User Interface Guide). Whether or not the resulting attachment is a regular attachment or mesh-based is controlled by the skin body's attachPointsToMesh property, which can be adjusted in the GUI or in code using the property's accessor methods:

```
boolean getAttachPointsToMesh()
void setAttachPointsToMesh (boolean enable)
```

## 11.5 Resolution and Limitations

Skinning techniques do have limitations, which are common to all methodologies.

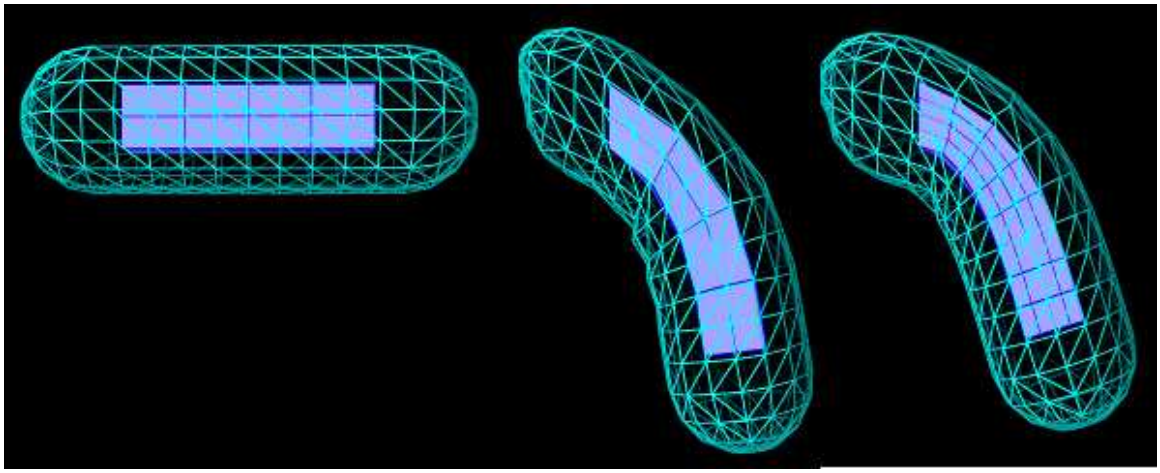


Figure 11.5: A skin mesh enveloping a single FEM body whose element resolution is lower than that of the mesh (left). When the FEM undergoes a large deformation, this disparity in resolution can cause artifacts such as crimping, as seen on the underside of the mesh in the middle image. Using an FEM model with a higher resolution can mitigate this (right).

- The passive nature of the connection between skinned vertices and the master bodies means that it can be easy for the skin mesh to self intersect and/or fold and crease in ways that are not physically realistic. These effects are often more pronounced when mesh vertices are relatively far away from the master bodies, or in places where the mesh undergoes a concave deformation. When the master bodies include frames, these effects can sometimes be reduced by setting the frameBlending property to `DUAL_QUATERNION_LINEAR` instead of the default `LINEAR`.
- When the master bodies include FEM models which undergo large deformations, crimping artifacts may arise if the skin mesh has a higher resolution that the FEM model (Figure 11.5). This is because each mesh vertex is connected to the coordinate frame of a single FEM element, and for reasons of computational efficiency the influence of these coordinate frames is not blended as it is for frame-based master bodies. If crimping artifacts occur, one solution may be to adjust the mesh and/or the FEM model so that their resolutions are more compatible (Figure 11.5, right).

## 11.6 Collisions

It is possible to make skin bodies collide with other ArtiSynth bodies, such as `RigidBody` and `FemModel3d`, which implement the Collidable interface (Chapter 8). `SkinMeshBody` itself implements CollidableBody, and is considered a deformable body, so that collisions can be activated either by setting one of the default collision behaviors involving deformable bodies, or by setting an explicit collision behavior between it and another body. Self collisions involving `SkinMeshBody` are not currently supported.

As described in Section 8.4, collisions work by computing the intersection between the meshes of the skin body and other collidables. The vertices, faces, and (possibly) edges of the resulting intersection region are then used to compute contact constraints, which propagate the effect of the contact back onto the collidable bodies' dynamic components. For a skin mesh, the dynamic components are the Frame master bodies and the nodes of the FEM master bodies.

Collisions involving `SkinMeshBody` frequently suffer from the problem of being overconstrained (Section 8.6), whereby the the number of contacts exceeds the number of master body DOFs available to handle the collision. This may occur if the skin body contains only rigid bodies, or if the mesh resolution exceeds the resolution of the FEM master bodies. Managing overconstrained collisions is discussed in Section 8.6, with the easiest method being constraint reduction, which can be activated by setting to `true` the reduceConstraints property for either the collision manager *or* a specific CollisionBehavior involving the skin body.

> Caveats: The distance between the vertices of a skinned mesh and its master bodies can sometimes cause odd or counter-intuitive collision behavior. Collision handling may also be noticeably slower if frameBlending is set to `DUAL_QUATERNION_LINEAR` or `DUAL_QUATERNION_ITERATIVE`. If any of the master bodies are FEM models, collisions resulting in large friction forces may result in unstable behavior.

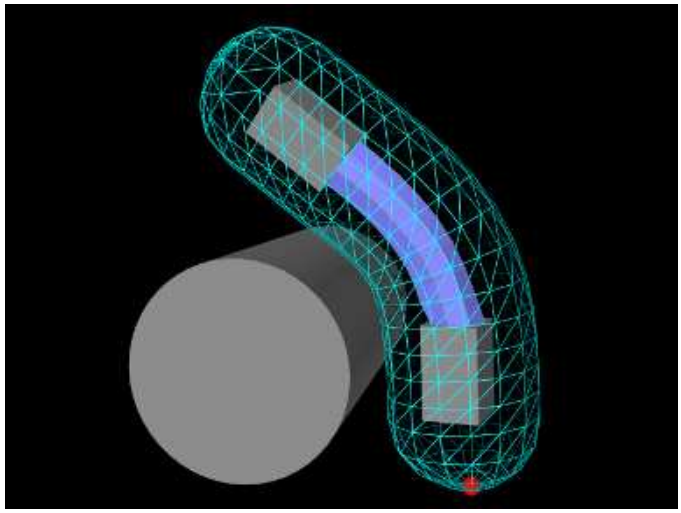


Figure 11.6: `SkinBodyCollide` demo, showing a skin mesh colliding with a cylinder about 0.6 seconds into the simulation.

### 11.6.1 Example: collision with a cylinder

Collisions involving `SkinMeshBody` are illustrated by the demo model `artisynth.demos.tutorial.SkinBodyCollide`, which extends the demo `AllBodySkinning` to add a cylinder with which the skin body can collide. The code for the demo is given below:

```
1  package artisynth.demos.tutorial;
2
3  import artisynth.core.femmodels.SkinMeshBody;
4  import artisynth.core.mechmodels.CollisionBehavior;
5  import artisynth.core.mechmodels.MechModel;
6  import artisynth.core.mechmodels.RigidBody;
7  import maspack.matrix.RigidTransform3d;
8
9  public class SkinBodyCollide extends AllBodySkinning {
10
11    public void build (String[] args) {
12       super.build (args);
13
14       // get components from the super class
15       MechModel mech = (MechModel)models().get("mech");
16       SkinMeshBody skinBody = (SkinMeshBody)mech.meshBodies().get("skin");
17       RigidBody block0 = mech.rigidBodies().get("block0");
18
19       // set block0 dynamic so the skin body and its masters can
20       // fall under gravity
21       block0.setDynamic (true);
22
23       // create a cylinder for the skin body to collide with
24       RigidBody cylinder =
25          RigidBody.createCylinder (
26             "cylinder", 0.5, 2.0, /*density=*/1000.0, /*nsides=*/50);
27       cylinder.setDynamic (false);
28       cylinder.setPose (
29          new RigidTransform3d (-0.5, 0, -1.5, 0, 0, Math.PI/2));
30       mech.addRigidBody (cylinder);
31
32       // enable collisions between the cylinder and the skin body
33       CollisionBehavior cb = new CollisionBehavior (true, 0);
34       mech.setCollisionBehavior (cylinder, skinBody, cb);
35       mech.getCollisionManager().setReduceConstraints (true);
36    }
```

```
37 }
```

The model subclasses `AllBodySkinning`, using the superclass `build()` method within its own `build` method to create the original model (line 12). It then obtains references to the `MechModel`, `SkinMeshBody`, and leftmost block, using their names to find them in various component lists (lines 14-17). (If the original model had stored references to these components as accessible member attributes, this step would not be needed.)

These component references are then used to make changes to the model: the left block is made dynamic so that the skin mesh can fall freely (line 21), a cylinder is created and added (lines 23-30), collisions are enabled between the skin body and the cylinder (lines 32-34), and the collision manager is asked to use constraint reduction to minimize the chance of overconstrained contact (line 35).

To run this example in ArtiSynth, select All demos > tutorial > SkinBodyCollide from the Models menu. When run, the skin body should fall and collide with the cylinder as shown in Figure 11.6.

## 11.7 Application to muscle wrapping

It is sometimes possible to use skinning as a computationally cheaper way to implement muscle wrapping (Chapter 9).

Typically, the end points (i.e., origin and insertion points) of a point-to-point muscle are attached to different bodies. As these bodies move with respect to each other, the path of the muscle may *wrap* around portions of these bodies and perhaps other intermediate bodies as well. The wrapping mechanism of Chapter 9 manages this by performing the computations necessary to allow one or more *wrappable* segments of a MultiPointSpring to wrap around a prescribed set of rigid bodies. However, if the induced path deformation is not too great, it may be possible to achieve a similar effect at much lower computational cost by simply "skinning" the via points of the multipoint spring to the underlying rigid bodies.

The general approach involves:

1. Creating a SkinMeshBody which references as master bodies the bodies containing the origin and insertion points, and possibly other bodies as well;
2. Creating the wrapped muscle using a MultiPointSpring with via points that are attached to the skin body.

It should be noted that for this application, the skin body does not need to contain a mesh. Instead, "skinning" connections can be made solely between the master bodies and the via points. An easy way to do this is to simply use skin body markers as via points. Another way is to create the via points as separate particles, and then attach them to the skin body using one of its `createPointAttachment` methods.

> It should also be noted that unlike with the wrapping methods of Chapter 9, skin-based wrapping can be applied around FEM models as well as rigid bodies.

> Generally, we observe better wrapping behavior if the frameBlending property of the `SkinMeshBody` is set to `DUAL_QUATERNION_LINEAR` instead of the default value of `LINEAR`.

### 11.7.1 Example: wrapping for a finger joint

An example of skinning-based muscle wrapping is given by `artisynth.demos.tutorial.PhalanxSkinWrapping`, which is identical to the demo `artisynth.demos.tutorial.PhalanxWrapping` except for using skinning to achieve the wrapping effect. The portion of the code which differs is shown below:

```
1       // create a SkinMeshBody and use it to create "skinned" muscle via points
2       SkinMeshBody skinBody = new SkinMeshBody();
3       skinBody.addMasterBody (proximal);
4       skinBody.addMasterBody (distal);
5       skinBody.setFrameBlending (FrameBlending.DUAL_QUATERNION_LINEAR);
6       mech.addMeshBody (skinBody);
7       SkinMarker via1 = skinBody.addMarker (new Point3d (0.0215, 0, -0.015));
```

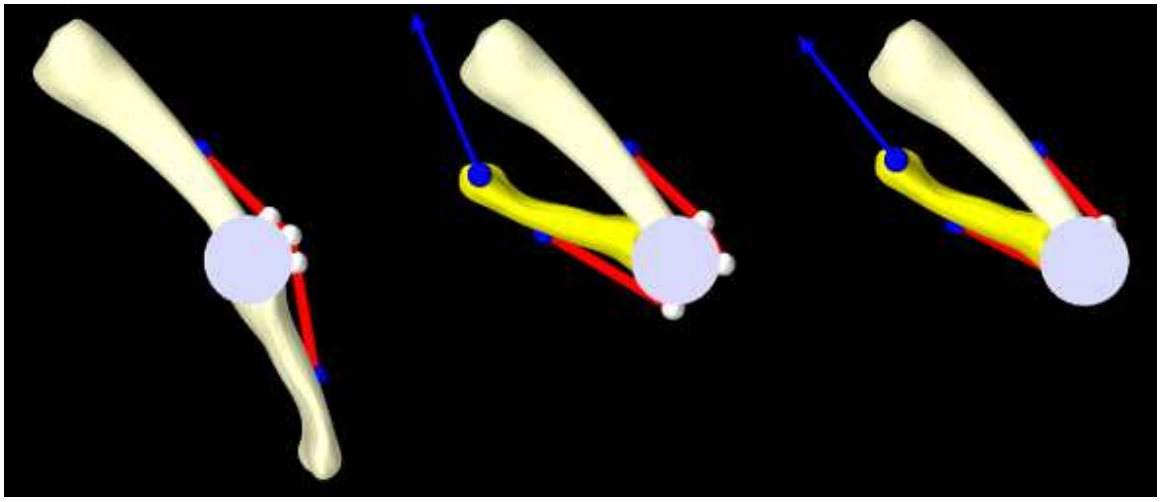


Figure 11.7: PhalanxSkinWrapping model loaded into ArtiSynth and running with no load on the distal bone (left). The pull tool is then used to exert forces on the distal bone and pull it around the joint (middle). The viewer has been set to orthographic projection to enable better visualization of the wrapping behavior of the muscle via points (shown in white). The results appear better when the SkinMeshBody's frameBlending property is set to `DUAL_QUATERNION_LINEAR` (middle) instead of `LINEAR` (right).

```
8         SkinMarker via2 = skinBody.addMarker (new Point3d (0.025, 0, -0.018));
9         SkinMarker via3 = skinBody.addMarker (new Point3d (0.026, 0, -0.0225));
10
11        // create a cylindrical mesh around the joint as a visualization aid to
12        // see how well the via points "wrap" as the lower bone moves
13        PolygonalMesh mesh = MeshFactory.createCylinder (
14           /*rad=*/0.0075, /*h=*/0.04, /*nsegs=*/32);
15        FixedMeshBody meshBody = new FixedMeshBody (
16           MeshFactory.createCylinder (/*rad=*/0.0075, /*h=*/0.04, /*nsegs=*/32));
17        meshBody.setPose (TJW);
18        mech.addMeshBody (meshBody);
19
20        // create a wrappable muscle using a SimpleAxialMuscle material
21        MultiPointSpring muscle = new MultiPointMuscle ("muscle");
22        muscle.setMaterial (
23           new SimpleAxialMuscle (/*k=*/0.5, /*d=*/0, /*maxf=*/0.04));
24        muscle.addPoint (origin);
25        // add via points to the muscle
26        muscle.addPoint (via1);
27        muscle.addPoint (via2);
28        muscle.addPoint (via3);
29        muscle.addPoint (insertion);
30        mech.addMultiPointSpring (muscle);
31
32        // create control panel to allow frameBlending to be set
33        ControlPanel panel = new ControlPanel();
34        panel.addWidget (skinBody, "frameBlending");
35        addControlPanel (panel);
36
37        // set render properties
38        RenderProps.setSphericalPoints (mech, 0.002, Color.BLUE);
39        RenderProps.setSphericalPoints (skinBody, 0.002, Color.WHITE);
40        RenderProps.setCylindricalLines (muscle, 0.001, Color.RED);
41        RenderProps.setFaceColor (meshBody, new Color (200, 200, 230));
```

First, a `SkinMeshBody` is created referencing the proximal and distal bones as master bodies, with the frameBlending property set to `DUAL_QUATERNION_LINEAR` (lines 1-6). Next, we create a set of three via points that will be attached to the skin body to guide the muscle around the joint in lieu of making it actually wrap around a cylinder (lines 7-9).

In the original `PhalanxWrapping` demo, a RigidCylinder was used as a muscle wrap surface. In this demo, we replace this with a simple cylindrical mesh which has no dynamic function but allows us to visualize the wrapping behavior of the via points (lines 11-18). The muscle itself is created using the three via points but with no wrappable segments or bodies (lines 20-30).

A control panel is added to allow for the adjustment of the skin body's frameBlending property (lines 32-35). Finally, render properties are set as for the original demo, only with the skin body markers rendered as white spheres to make them more visible (lines 37-41).

To run this example in ArtiSynth, select All demos > tutorial > PhalanxSkinWrapping from the Models menu. The model should load and initially appear as in Figure 11.7 (left). The pull tool can then be used to move the distal joint while simulating, to illustrate how well the via points "wrap" around the joint, using the cylindrical mesh as a visual reference (Figure 11.7, middle). Changing the frameBlending property to `LINEAR` results in a less satisfactory behavior (Figure 11.7, right).

# Chapter 12

# DICOM Images

Some models are derived from image data, and it may be useful to show the model and image in the same space. For this purpose, a DICOM image widget has been designed, capable of displaying 3D DICOM volumes as a set of three perpendicular planes. An example widget and its property panel is shown in Figure 12.1.

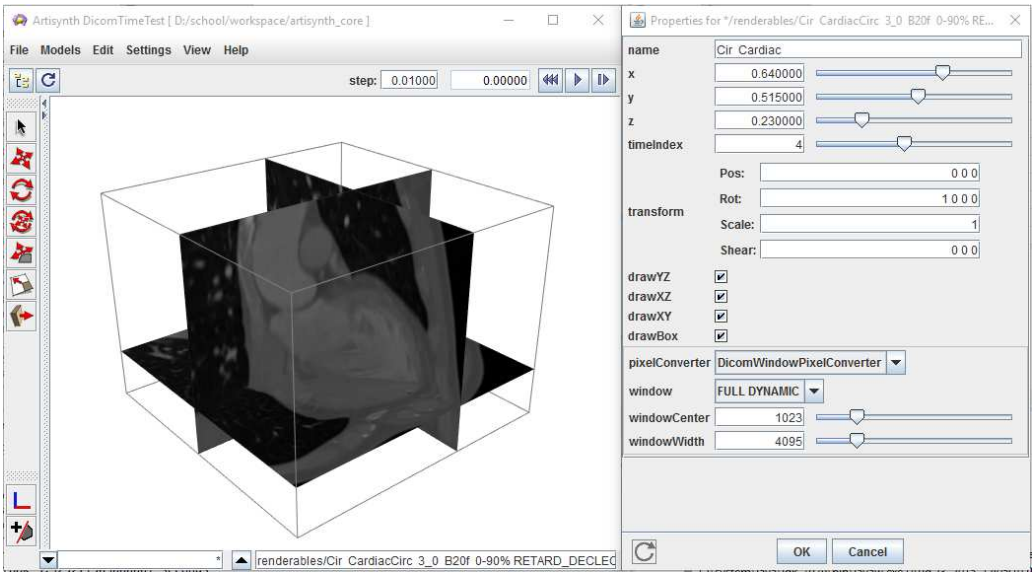



Figure 12.1: DICOM image of the heart, downloaded from http://www.osirix-viewer.com.

The main classes related to the reading and displaying of DICOM images are:

**DicomElement**
: Describes a single attribute in a DICOM file.

**DicomHeader**
: Contains all header attributes (all but the image data) extracted from a DICOM file.

**DicomPixelBuffer**
: Contains the *decoded* image pixels for a single image frame.

**DicomSlice**
: Contains both the header and image information for a single 2D DICOM slice.

**DicomImage**
: Container for DICOM slices, creating a 3D volume (or 3D + time)

**DicomReader**
: Parses DICOM files and folders, appending information to a DicomImage.

**DicomViewer**
: Displays the DicomImage in the viewer.

If the purpose is simply to display a DICOM volume in the ArtiSynth viewer, then only the last three classes will be of interest. Readers who simply want to display a DICOM image in their model can skip to Section 12.3.

## 12.1 The DICOM file format

For a complete description of the DICOM format, see the specification page at

http://medical.nema.org/standard.html

which provides a brief description. Another excellent resource is the blog by Roni Zaharia:

http://dicomiseasy.blogspot.ca/

Each DICOM file contains a number of concatenated attributes (a.k.a. elements), one of which defines the embedded binary image pixel data. The other attributes act as meta-data, which can contain identity information of the subject, equipment settings when the image was acquired, spatial and temporal properties of the acquisition, voxel spacings, etc.... The image data typically represents one or more 2D images, concatenated, representing slices (or 'frames') of a 3D volume whose locations are described by 13.5 the meta-data. This image data can be a set of raw pixel values, or can be encoded using almost any image-encoding scheme (e.g. JPEG, TIFF, PNG). For medical applications, the image data is typically either raw or compressed using a lossless encoding technique. Complete DICOM acquisitions are typically separated into multiple files, each defining one or few frames. The frames can then be assembled into 3D image 'stacks' based on the meta-information, and converted into a form appropriate for display.

Table 12.1: A selection of Value Representations

| VR | Description |
|---|---|
| CS | Code String |
| DS | Decimal String |
| DT | Date Time |
| IS | Integer String |
| OB | Other Byte String |
| OF | Other Float String |
| OW | Other Word String |
| SH | Short String |
| UI | Unique Identifier |
| US | Unsigned Short |
| OX | One of OB, OW, OF |

Each DICOM attribute is composed of:

- a standardized unique integer *tag* in the format (XXXX,XXXX) that defines the *group* and *element* of the attribute
- a *value representation* (VR) that describes the data type and format of the attribute's value (see Table 12.1)
- a *value length* that defines the length in bytes of the attribute's value to follow
- a *value field* that contains the attribute's value

This layout is depicted in Figure 12.2. A list of important attributes are provided in Table 12.2.

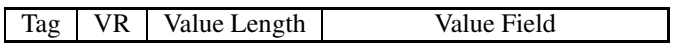



Figure 12.2: DICOM attribute structure

## 12.2 The DICOM classes

Each DicomElement represents a single attribute contained in a DICOM file. The DicomHeader contains the collection of `DicomElements` defined in a file, apart from the pixel data. The image pixels are decoded and stored in a DicomPixelBuffer. Each DicomSlice contains a `DicomHeader`, as well as the decoded `DicomPixelBuffer` for a single slice (or 'frame'). All slices are assembled into a single DicomImage, which can be used to extract 3D voxels and spatial locations from the set of slices. These five classes are described in further detail in the following sections.

Table 12.2: A selection of useful DICOM attributes

| Attribute name | VR | Tag |
|---|---|---|
| Transfer syntax UID | UI | 0x0002, 0x0010 |
| Slice thickness | DS | 0x0018, 0x0050 |
| Spacing between slices | DS | 0x0018, 0x0088 |
| Study ID | SH | 0x0020, 0x0010 |
| Series number | IS | 0x0020, 0x0011 |
| Aquisition number | IS | 0x0020, 0x0012 |
| Image number | IS | 0x0020, 0x0013 |
| Image position patient | DS | 0x0020, 0x0032 |
| Image orientation patient | DS | 0x0020, 0x0037 |
| Temporal position identifier | IS | 0x0020, 0x0100 |
| Number of temporal positions | IS | 0x0020, 0x0105 |
| Slice location | DS | 0x0020, 0x1041 |
| Samples per pixel | US | 0x0028, 0x0002 |
| Photometric interpretation | CS | 0x0028, 0x0004 |
| Planar configuration (color) | US | 0x0028, 0x0006 |
| Number of frames | IS | 0x0028, 0x0008 |
| Rows | US | 0x0028, 0x0010 |
| Columns | US | 0x0028, 0x0011 |
| Pixel spacing | DS | 0x0028, 0x0030 |
| Bits allocated | US | 0x0028, 0x0100 |
| Bits stored | US | 0x0028, 0x0101 |
| High bit | US | 0x0028, 0x0102 |
| Pixel representation | US | 0x0028, 0x0103 |
| Pixel data | OX | 0x7FE0, 0x0010 |

### 12.2.1 `DicomElement`

The DicomElement class is a simple container for DICOM attribute information. It has three main properties:

- an integer *tag*
- a *value representation* (VR)
- a value

These properties can be obtained using the corresponding `get` function: `getTag()`, `getVR()`, `getValue()`. The tag refers to the concatenated group/element tag. For example, the *transfer syntax UID* which corresponds to group 0x0002 and element 0x0010 has a numeric tag of 0x00020010. The VR is represented by an enumerated type, DicomElement.VR. The 'value' is the *raw* value extracted from the DICOM file. In most cases, this will be a `String`. For raw numeric values (i.e. stored in the DICOM file in binary form) such as the unsigned short (US), the 'value' property is exactly the numeric value.

For VRs such as the integer string (IS) or decimal string (DS), the string will still need to be parsed in order to extract the appropriate sequence of numeric values. There are static utility functions for handling this within `DicomElement`. For a 'best-guess' of the desired parsed value based on the VR, one can use the method `getParsedValue()`. Often, however, the desired value is also context-dependent, so the user should know a priori what type of value(s) to expect. Parsing can also be done automatically by querying for values directly through the `DicomHeader` object.

### 12.2.2 `DicomHeader`

When a DICOM file is parsed, all meta-data (attributes apart from the actual pixel data) is assembled into a DicomHeader object. This essentially acts as a map that can be queried for attributes using one of the following methods:

```
DicomElement getElement(int tag);                 // includes VR and data
String getStringValue(int tag);                   // all non-numeric VRs
String[] getMultiStringValue(int tag);            // UT, SH
int getIntValue(int tag, int defaultValue);       // IS, DS, SL, UL, SS, US
int[] getMultiIntValue(int tag);                  // IS, DS, SL, UL, SS, US
```

```
    double getDecimalValue(int tag, double defaultValue);    // DS, FL, FD
    double[] getMultiDecimalValue(int tag);          // DS, FL, FD
    VectorNd getVectorValue(int tag);                // DS, IS, SL, UL, SS, US, FL, FD
    DicomDateTime getDateTime(int tag);              // DT, DA, TM
```

The first method returns the full element as described in the previous section. The remaining methods are used for convenience when the desired value type is known for the given tag. These methods automatically parse or convert the `DicomElement`'s value to the desired form.

If the tag does not exist in the header, then the `getIntValue(...)` and `getDecimalValue(...)` will return the supplied `defaultValue`. All other methods will return `null`.

### 12.2.3 DicomPixelBuffer

The DicomPixelBuffer contains the decoded image information of an image slice. There are three possible pixel types currently supported:

- byte grayscale values (`PixelType.BYTE`)
- short grayscale values (`PixelType.SHORT`)
- byte RGB values, with layout `RGBRGB...RGB` (`PixelType.BYTE_RGB`)

The pixel buffer stores all pixels in one of these types. The pixels can be queried for directly using `getPixel(idx)` to get a single pixel, or `getBuffer()` to get the entire pixel buffer. Alternatively, a DicomPixelInterpolator object can be passed in to convert between pixel types via one of the following methods:

```
  public int getPixelsByte (
     int x, int dx, int nx, byte[] pixels, int offset, DicomPixelInterpolator interp);

  public int getPixelsShort (
     int x, int dx, int nx, short[] pixels, int offset, DicomPixelInterpolator interp) ↩
      ;

  public int getPixelsRGB (
     int x, int dx, int nx, byte[] pixels, int offset, DicomPixelInterpolator interp);

  public int getPixels(
     int x, int dx, int nx, DicomPixelBuffer pixels, int offset,
     DicomPixelInterpolator interp);
```

These methods populate an output array or buffer with converted pixel values, which can later be passed to a renderer. For further details on these methods, refer to the Javadoc documentation.

### 12.2.4 DicomSlice

A single DICOM file contains both header information, and one or more image 'frames' (slices). In ArtiSynth, we separate each frame and attach them to the corresponding header information in a DicomSlice. Thus, each slice contains a single `DicomHeader` and `DicomPixelBuffer`. These can be obtained using the methods: `getHeader()` and `getPixelBuffer()`.

For convenience, the `DicomSlice` also has all the same methods for extracting and converting between pixel types as the `DicomPixelBuffer`.

### 12.2.5 DicomImage

An complete DICOM acquisition typically consists of multiple slices forming a 3D image stack, and potentially contains multiple 3D stacks to form a dynamic 3D+time image. The collection of `DicomSlices` are thus assembled into a DicomImage, which keeps track of the spatial and temporal positions.

The `DicomImage` is the main object to query for pixels in 3D(+time). To access pixels, it has the following methods:

```
public int getPixelsByte (
   int x, int y, int z, int dx, int dy, int dz, int nx, int ny, int nz,
   int time, byte[] pixels, DicomPixelInterpolator interp);

public int getPixelsShort (
   int x, int y, int z, int dx, int dy, int dz, int nx, int ny, int nz,
   int time, short[] pixels, DicomPixelInterpolator interp);

public int getPixelsRGB (
   int x, int y, int z, int dx, int dy, int dz, int nx, int ny, int nz,
   int time, byte[] pixels, DicomPixelInterpolator interp);

public int getPixels (
   int x, int y, int z, int dx, int dy, int dz, int nx, int ny, int nz,
   int time, DicomPixelBuffer pixels, DicomPixelInterpolator interp);
```

The inputs {x, y, z} refer to voxel indices, and time refers to the time instance index, starting at zero. The four voxel dimensions of the image can be queried with: getNumCols() getNumRows(), getNumSlices(), and getNumTimes().

The DicomImage also contains spatial transform information for converting between voxel indices and patient-centered spatial locations. The affine transform can be acquired with the method getPixelTransform(). This allows the image to be placed in the appropriate 3D location, to correspond with any derived data such as segmentations. The spatial transformation is automatically extracted from the DICOM header information embedded in the files.

## 12.3 Loading a DicomImage

DICOM files and folders are read using the DicomReader class. The reader populates a supplied DicomImage with slices, forming the full 3D(+time) image. The basic pattern is as follows:

```
String DICOM_directory = ...                // define directory of interest
DicomReader reader = new DicomReader();     // create a new reader

// read all files in a directory, returning a newly constructed image
DicomImage image = reader.read(null, DICOM_directory);
```

The first argument in the read(...) command is an existing image in which to append slices. In this case, we pass in null to signal that a new image is to be created.

In some cases, we might wish to exclude certain files, such as meta-data files that happen to be in the DICOM folder. By default, the reader attempts to read all files in a given directory, and will print out an error message for those it fails to detect as being in a valid DICOM format. To limit the files to be considered, we allow the specification of a Java Pattern, which will test each filename against a regular expression. Only files with names that match the pattern will be included. For example, in the following, we limit the reader to files ending with the “dcm” extension.

```
String DICOM_directory = ...                // define directory of interest
DicomReader reader = new DicomReader();     // create a new reader
Pattern dcmPattern = Pattern.compile(".*\\.dcm") ;  // files ending with .dcm

// read all files in a directory, returning a newly constructed image
DicomImage image = reader.read(null, DICOM_directory, dcmPattern, /*subdirs*/ false) ↩
    ;
```

The pattern is applied to the absolute filename, with either windows and mac/linux file separators (both are checked against the regular expression). The method also has an option to recursively search for files in subdirectories. If the full list of files is known, then one can use the method:

```
public DicomImage read(DicomImage im, List<File> files);
```

which will load all specified files.

### 12.3.1 Time-dependent images

In most cases, time-dependent images will be properly assembled using the previously mentioned methods in the `DicomReader`. Each slice *should* have a temporal position identifier that allows for the separate image stacks to be separated. However, we have found in practice that at times, the temporal position identifier is omitted. Instead, each stack might be stored in a separate DICOM folder. For this reason, additional read methods have been added that allow manual specification of the time index:

```
public DicomImage read(DicomImage im, List<File> files, int temporalPosition);
public DicomImage read(DicomImage im, String directory, Pattern filePattern,
                       boolean checkSubdirectories, int temporalPosition);
```

If the supplied `temporalPosition` is non-negative, then the temporal position of all included files will be manually set to that value. If negative, then the method will attempt to read the temporal position from the DICOM header information. If no such information is available, then the reader will guess the temporal position to be one past the last temporal position in the original image stack (or 0 if `im == null`). For example, if the original image has temporal positions {0, 1, 2}, then all appended slices will have a temporal position of three.

### 12.3.2 Image formats

The `DicomReader` attempts to automatically decode any pixel information embedded in the DICOM files. Unfortunately, there are virtually an unlimited number of image formats allowed in DICOM, so there is no way to include native support to decode all of them. By default, the reader can handle raw pixels, and any image format supported by Java's `ImageIO` framework, which includes JPEG, PNG, BMP, WBMP, and GIF. Many medical images, however, rely on lossless or near-lossless encoding, such as lossless JPEG, JPEG 2000, or TIFF. For these formats, we provide an interface that interacts with the third-party command-line utilities provided by **ImageMagick** (`http://www.imagemagick.org`). To enable this interface, the **ImageMagick** utilities `identify` and `convert` must be available and exist somewhere on the system's `PATH` environment variable.

> **ImageMagick Installation**
>
> To enable ImageMagick decoding, required for image formats not natively supported by Java (e.g. JPEG 2000, TIFF), download and install the ImageMagick command-line utilities from: `http://www.imagemagick.org/script/binary-releases.php`
>
> The install path must also be added to your system's `PATH` environment variable so that ArtiSynth can locate the `identify` and `convert` utilities.

## 12.4 The `DicomViewer`

Once a `DicomImage` is loaded, it can be displayed in a model by using the DicomViewer component. The viewer has several key properties:

**name**
: the name of the viewer component

**x, y, z**
: the *normalized* slice positions, in the range [0,1], at which to display image planes

**timeIndex**
: the temporal position (image stack) to display

**transform**
: an affine transformation to apply to the image (on top of the voxel-to-spatial transform extracted from the DICOM file)

**drawYZ**
: draw the YZ plane, corresponding to position `x`

**drawXZ**

draw the XZ plane, corresponding to position `y`

**drawXY**

draw the XY plane, corresponding to position `z`

**drawBox**

draw the 3D image's bounding box

**pixelConverter**

the interpolator responsible for converting pixels decoded in the DICOM slices into values appropriate for display. The converter has additional properties:

**window**

name of a preset window for linear interpolation of intensities

**center**

center intensity

**width**

width of window

Each property has a corresponding `getXxx(...)` and `setXxx(...)` method that can adjust the settings in code. They can also be modified directly in the ArtiSynth GUI. The last property, the `pixelConverter` allows for shifting and scaling intensity values for display. By default a set of intensity 'windows' are loaded directly from the DICOM file. Each window has a name, and defines a center and width used for linearly scale the intensity range. In addition to the windows extracted from the DICOM, two new windows are added: `FULL_DYNAMIC`, corresponding to the entire intensity range of the image; and `CUSTOM`, which allows for custom specification of the window center and width properties.

To add a `DicomViewer` to the model, create the viewer by supplying a component name and reference to a `DicomImage`, then add it as a `Renderable` to the `RootModel`:

```
DicomViewer viewer = new DicomViewer("my image", dicomImage);
addRenderable(viewer);
```

The image will automatically be displayed in the patient-centered coordinates loaded from the `DicomImage`. In addition to this basic construction, there are convenience constructors to avoid the need for a `DicomReader` for simple DICOM files:

```
// loads all matching DICOM files to create a new image
public DicomViewer(String name, String imagePath, Pattern filePattern, boolean ↩
    checkSubdirs);
// loads a list of DICOM files to create a new image
public DicomViewer(String name, List<File> files);
```

These constructors generate a new `DicomImage` internal to the viewer. The image can be retrieved from the viewer using the `getImage()` method.

## 12.5 DICOM example

Some examples of DICOM use can be found in the `artisynth.core.demos.dicom` package. The model `DicomTest` loads a partial image of a heart, which is initially downloaded from the ArtiSynth website:

```
1 package artisynth.demos.dicom;
2
3 import java.awt.Color;
4 import java.io.File;
5 import java.io.IOException;
6
7 import artisynth.core.renderables.DicomViewer;
8 import artisynth.core.workspace.DriverInterface;
```

```
9  import artisynth.core.workspace.RootModel;
10 import maspack.fileutil.FileManager;
11 import maspack.util.PathFinder;
12
13 public class DicomTest extends RootModel {
14
15    // Dicom file name and URL from which to load it
16    String dicom_file = "MR-MONO2-8-16x-heart";
17    String dicom_url =
18       "https://www.artisynth.org/files/data/dicom/MR-MONO2-8-16x-heart.gz";
19
20    public void build(String[] args) throws IOException {
21
22       // cache image in a local directory 'data' beneath Java source
23       String localDir = PathFinder.getSourceRelativePath(
24          this, "data/MONO2_HEART");
25       // create a file manager to get the file and download it if necessary
26       FileManager fileManager = new FileManager(localDir, "gz:"+dicom_url+"!/");
27       fileManager.setConsoleProgressPrinting(true);
28       fileManager.setOptions(FileManager.DOWNLOAD_ZIP); // download zip file first
29
30       // get the file from local directory, downloading first if needed
31       File dicomPath = fileManager.get(dicom_file);
32
33       // create a DicomViewer for the file
34       DicomViewer dcp = new DicomViewer("Heart", dicomPath.getAbsolutePath(),
35          null, /*check subdirectories*/false);
36
37       addRenderable(dcp); // add it to root model's list of renderable
38    }
39
```

Lines 23-28 are responsible for downloading and extracting the sample DICOM zip file. In the end, `dicomPath` contains a reference to the desired DICOM file on the local system, which is used to create a viewer on line 34. We then add the viewer to the model for display purposes.

To run this example in ArtiSynth, select All demos > dicom > DicomTest from the Models menu. The model should load and initially appear as in Figure 12.3.

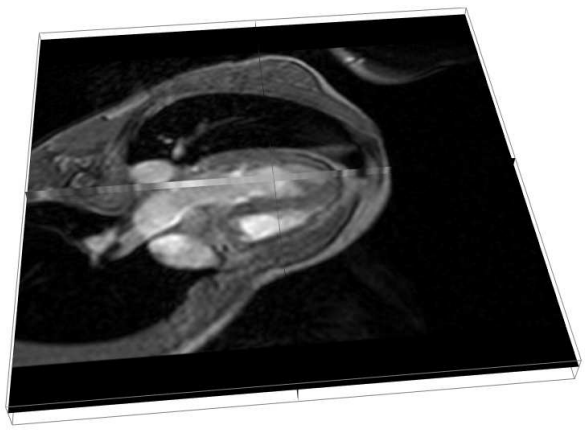


Figure 12.3: DICOM viewer image from `DicomTest`

.

# Appendix A

# Mathematical Review

This appendix reviews some of the mathematical concepts used in this manual.

## A.1 Rotation transforms

Rotation matrices are used to describe the orientation of 3D coordinate frames in space, and to transform vectors between these coordinate frames.

Consider two 3D coordinate frames A and B that are rotated with respect to each other (Figure A.1). The orientation of B with respect to A can be described by a $3 \times 3$ rotation matrix $\mathbf{R}_{BA}$, whose columns are the unit vectors giving the directions of the rotated axes $\mathbf{x}'$, $\mathbf{y}'$, and $\mathbf{z}'$ of B with respect to A.

$\mathbf{R}_{BA}$ is an *orthogonal* matrix, meaning that its columns are both perpendicular and mutually orthogonal, so that

$$\mathbf{R}_{BA}^T \mathbf{R}_{BA} = \mathbf{I} \tag{A.1}$$

where $\mathbf{I}$ is the $3 \times 3$ identity matrix. The inverse of $\mathbf{R}_{BA}$ is hence equal to its transpose:

$$\mathbf{R}_{BA}^{-1} = \mathbf{R}_{BA}^T. \tag{A.2}$$

Because $\mathbf{R}_{BA}$ is orthogonal, $|\det \mathbf{R}_{BA}| = 1$, and because it is a rotation, $\det \mathbf{R}_{BA} = 1$ (the other case, where $\det \mathbf{R}_{BA} = -1$, is not a rotation but a *reflection*). The 6 orthogonality constraints associated with a rotation matrix mean that in spite of having 9 numbers, the matrix only has 3 degrees of freedom.

Now, assume we have a 3D vector $\mathbf{v}$, and consider its coordinates with respect to both frames A and B. Where necessary, we use a preceding superscript to indicate the coordinate frame with respect to which a quantity is described, so that ${}^A\mathbf{v}$ and ${}^B\mathbf{v}$ and denote $\mathbf{v}$ with respect to frames A and B, respectively. Given the definition of $\mathbf{R}_{AB}$ given above, it is fairly straightforward to show that

$${}^A\mathbf{v} = \mathbf{R}_{BA}\, {}^B\mathbf{v} \tag{A.3}$$

and, given (A.2), that

$${}^B\mathbf{v} = \mathbf{R}_{BA}^T\, {}^A\mathbf{v}. \tag{A.4}$$

Hence in addition to describing the orientation of B with respect to A, $\mathbf{R}_{BA}$ is also a transformation matrix that maps vectors in B to vectors in A.

It is straightforward to show that

$$\mathbf{R}_{BA}^{-1} = \mathbf{R}_{BA}^T = \mathbf{R}_{AB}. \tag{A.5}$$

A simple rotation by an angle $\theta$ about one of the basic coordinate axes is known as a *basic* rotation. The three basic rotations about x, y, and z are:

$$\mathbf{R}_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{pmatrix},$$

$$\mathbf{R}_y(\theta) = \begin{pmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{pmatrix},$$

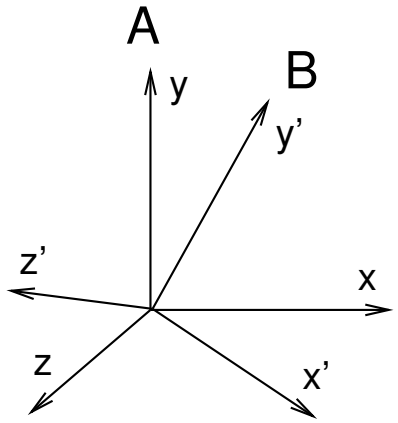



Figure A.1: Two coordinate frames A and B rotated with respect to each other.

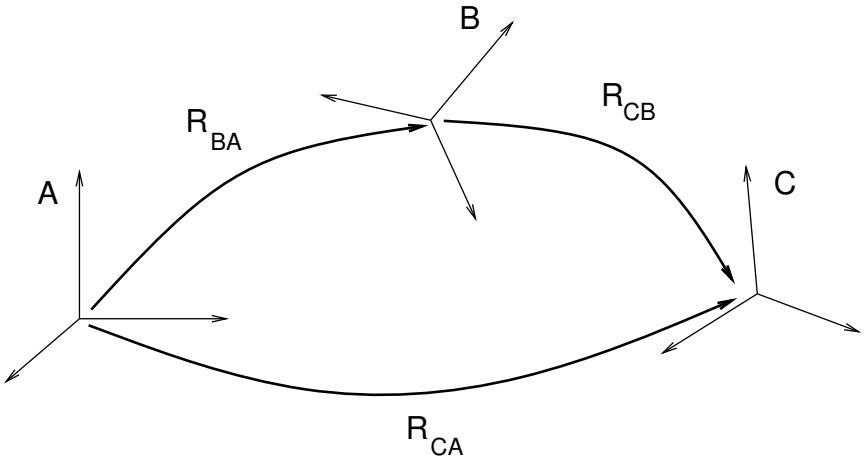



Figure A.2: Schematic illustration of three coordinate frames A, B, and C and the rotational transforms relating them.

$$\mathbf{R}_z(\theta) = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Next, we consider transform composition. Suppose we have three coordinate frames, A, B, and C, whose orientation are related to each other by $\mathbf{R}_{BA}$, $\mathbf{R}_{CB}$, and $\mathbf{R}_{CA}$ (Figure A.6). If we know $\mathbf{R}_{BA}$ and $\mathbf{R}_{CA}$, then we can determine $\mathbf{R}_{CB}$ from

$$\mathbf{R}_{CB} = \mathbf{R}_{BA}^{-1}\mathbf{R}_{CA}. \tag{A.6}$$

This can be understood in terms of vector transforms. $\mathbf{R}_{CB}$ transforms a vector from C to B, which is equivalent to first transforming from C to A,

$${}^{A}\mathbf{v} = \mathbf{R}_{CA}\,{}^{C}\mathbf{v}, \tag{A.7}$$

and then transforming from A to B:

$${}^{B}\mathbf{v} = \mathbf{R}_{BA}^{-1}\,{}^{A}\mathbf{v} = \mathbf{R}_{BA}^{-1}\,\mathbf{R}_{CA}\,{}^{C}\mathbf{v} = \mathbf{R}_{CB}\,{}^{C}\mathbf{v}. \tag{A.8}$$

Note also from (A.5) that $\mathbf{R}_{CB}$ can be expressed as

$$\mathbf{R}_{CB} = \mathbf{R}_{AB}\mathbf{R}_{CA}. \tag{A.9}$$

In addition to specifying rotation matrix components explicitly, there are numerous other ways to describe a rotation. Three of the most common are:

**Roll-pitch-yaw angles**

There are 6 variations of roll-pitch-yaw angles. The one used in ArtiSynth corresponds to older robotics texts (e.g., Paul, Spong) and consists of a roll rotation $r$ about the z axis, followed by a pitch rotation $p$ about the new y axis, followed by a yaw rotation $y$ about the new x axis. The net rotation can be expressed by the following product of basic rotations: $\mathbf{R}_z(r)\,\mathbf{R}_y(p)\,\mathbf{R}_x(y)$.

**Axis-angle**

An axis angle rotation parameterizes a rotation as a rotation by an angle $\theta$ about a specific axis $\mathbf{u}$. Any rotation can be represented in such a way as a consequence of Euler's rotation theorem.

**Euler angles**

There are 6 variations of Euler angles. The one used in ArtiSynth consists of a rotation $\phi$ about the z axis, followed by a rotation $\theta$ about the new y axis, followed by a rotation $\psi$ about the new z axis. The net rotation can be expressed by the following product of basic rotations: $\mathbf{R}_z(\phi)\,\mathbf{R}_y(\theta)\,\mathbf{R}_z(\psi)$.

## A.2 Rigid transforms

Rigid transforms are used to specify both the transformation of points and vectors between coordinate frames, as well as the relative position and orientation between coordinate frames.

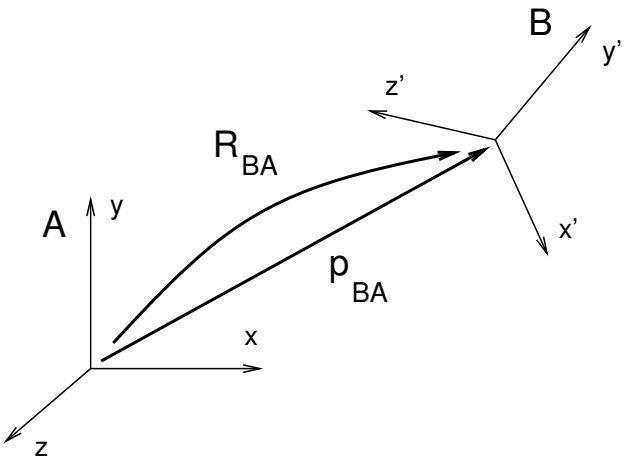



Figure A.3: A position vector $\mathbf{p}_{BA}$ and rotation matrix $\mathbf{R}_{BA}$ describing the position and orientation of frame B with respect to frame A.

Consider two 3D coordinate frames in space, A and B (Figure A.3). The translational position of B with respect to A can be described by a vector $\mathbf{p}_{BA}$ from the origin of A to the origin of B (described with respect to frame A). Meanwhile, the orientation of B with respect to A can be described by the $3 \times 3$ rotation matrix $\mathbf{R}_{BA}$ (Section A.1). The combined position and orientation of B with respect to A is known as the *pose* of B with respect to A.

Now, assume we have a 3D point $\mathbf{q}$, and consider its coordinates with respect to both frames A and B (Figure A.4). Given the pose descriptions given above, it is fairly straightforward to show that

$$ {}^{A}\mathbf{q} = \mathbf{R}_{BA}\,{}^{B}\mathbf{q} + \mathbf{p}_{BA}, \tag{A.10} $$

and, given (A.2), that

$$ {}^{B}\mathbf{q} = \mathbf{R}_{BA}^{T}\,({}^{A}\mathbf{q} - \mathbf{p}_{BA}). \tag{A.11} $$

If we extend our points into a 4D *homogeneous* coordinate space with the fourth coordinate $w$ equal to 1, i.e.,

$$ \mathbf{q}^{*} \equiv \begin{pmatrix} \mathbf{q} \\ 1 \end{pmatrix}, \tag{A.12} $$

then (A.10) and (A.11) can be simplified to

$$ {}^{A}\mathbf{q}^{*} = \mathbf{T}_{BA}\,{}^{B}\mathbf{q}^{*} \quad \text{and} \quad {}^{B}\mathbf{q}^{*} = \mathbf{T}_{BA}^{-1}\,{}^{A}\mathbf{q}^{*} $$

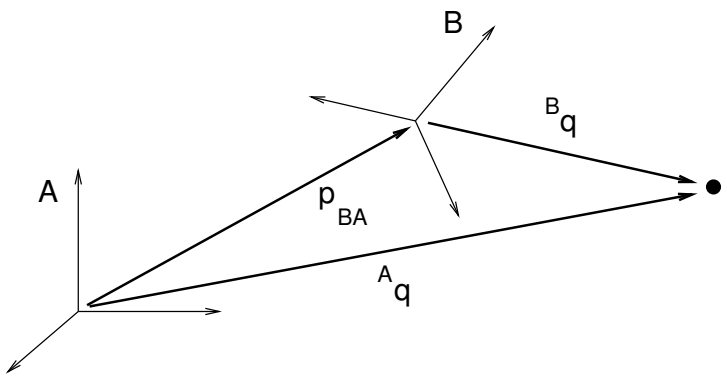



Figure A.4: Point vectors ${}^A\mathbf{q}$ and ${}^B\mathbf{q}$ describing the position of a point $\mathbf{q}$ with respect to frames A and B.

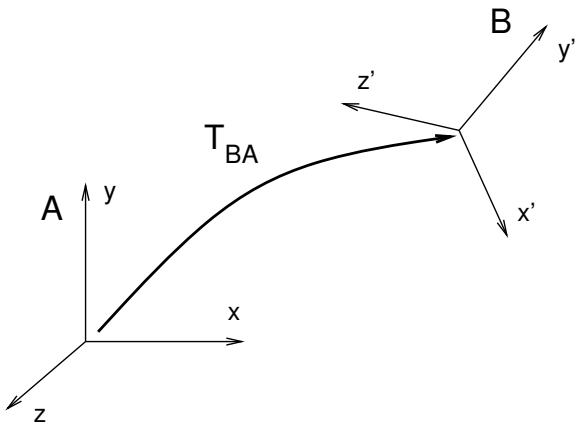



Figure A.5: The transform matrix $\mathbf{T}_{BA}$ from B to A.

where

$$\mathbf{T}_{BA} = \begin{pmatrix} \mathbf{R}_{BA} & \mathbf{p}_{BA} \\ 0 & 1 \end{pmatrix} \tag{A.13}$$

and

$$\mathbf{T}_{BA}^{-1} = \begin{pmatrix} \mathbf{R}_{BA}^T & -\mathbf{R}_{BA}^T\mathbf{p}_{BA} \\ 0 & 1 \end{pmatrix}. \tag{A.14}$$

$\mathbf{T}_{BA}$ is the $4 \times 4$ *rigid transform matrix* that transforms points from B to A and also describes the pose of B with respect to A (Figure A.5).

It is straightforward to show that $\mathbf{R}_{BA}^T$ and $-\mathbf{R}_{BA}^T\mathbf{p}_{BA}$ describe the orientation and position of $A$ with respect to $B$, and so therefore

$$\mathbf{T}_{BA}^{-1} = \mathbf{T}_{AB}. \tag{A.15}$$

Note that if we are transforming a vector $\mathbf{v}$ instead of a point between B and A, then we are only concerned about relative orientation and the vector transforms (A.3) and (A.4) should be used instead. However, we can express these using $\mathbf{T}_{BA}$ if we embed vectors in a homogeneous coordinate space with the fourth coordinate $w$ equal to 0, i.e.,

$$\mathbf{v}^* \equiv \begin{pmatrix} \mathbf{v} \\ 0 \end{pmatrix}, \tag{A.16}$$

so that

$${}^B\mathbf{v}^* = \mathbf{T}_{BA}\,{}^A\mathbf{v}^* \quad \text{and} \quad {}^A\mathbf{v}^* = \mathbf{T}_{BA}^{-1}\,{}^B\mathbf{v}^*.$$

Finally, we consider transform composition. Suppose we have three coordinate frames, A, B, and C, each related to the other by transforms $\mathbf{T}_{BA}$, $\mathbf{T}_{CB}$, and $\mathbf{T}_{CA}$ (Figure A.6). Using the same reasoning used to derive (A.6) and (A.9), it is easy to show that

$$\mathbf{T}_{CB} = \mathbf{T}_{BA}^{-1}\,\mathbf{T}_{CA} = \mathbf{T}_{AB}\,\mathbf{T}_{CA}. \tag{A.17}$$

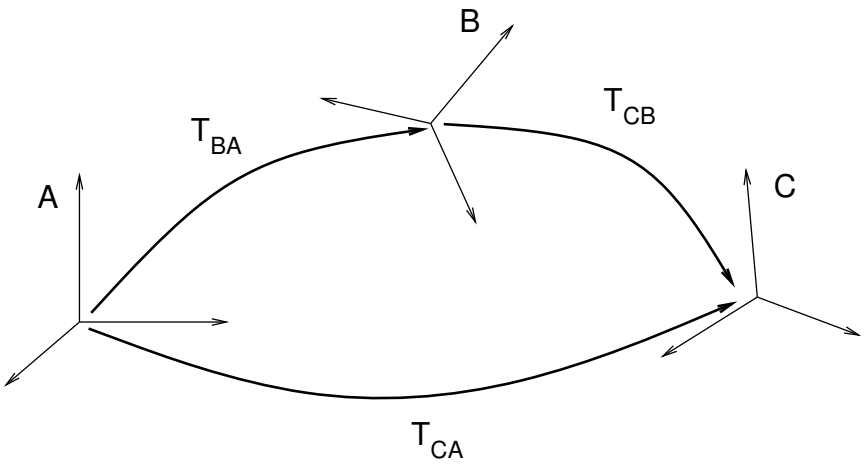



Figure A.6: Three coordinate frames A, B, and C and the transforms relating each one to the other.

## A.3 Affine transforms

An *affine transform* is a generalization of a rigid transform, in which the rotational component **R** is replaced by a general $3 \times 3$ matrix **A**. This means that an affine transform implements a generalized basis transformation combined with an offset of the origin (Figure A.7). As with **R** for rigid transforms, the columns of **A** still describe the transformed basis vectors $\mathbf{x}'$, $\mathbf{y}'$, and $\mathbf{z}'$, but these are generally no longer orthonormal.

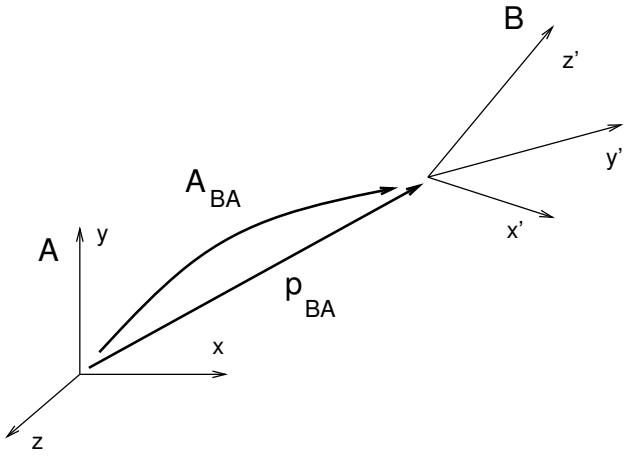



Figure A.7: A position vector $\mathbf{p}_{BA}$ and a general matrix $\mathbf{A}_{BA}$ describing the affine position and basis transform of frame B with respect to frame A.

Expressed in terms of homogeneous coordinates, the affine transform $\mathbf{X}_{AB}$ takes the form

$$\mathbf{X}_{BA} = \begin{pmatrix} \mathbf{A}_{BA} & \mathbf{p}_{BA} \\ 0 & 1 \end{pmatrix} \tag{A.18}$$

with

$$\mathbf{X}_{BA}^{-1} = \begin{pmatrix} \mathbf{A}_{BA}^{-1} & -\mathbf{A}_{BA}^{-1}\mathbf{p}_{BA} \\ 0 & 1 \end{pmatrix}. \tag{A.19}$$

As with rigid transforms, when an affine transform is applied to a vector instead of a point, only the matrix **A** is applied and the translation component **p** is ignored.

Affine transforms are typically used to effect transformations that require stretching and shearing of a coordinate frame. By the polar decomposition theorem, **A** can be factored into a regular rotation **R** plus a symmetric shearing/scaling matrix **P**:

$$\mathbf{A} = \mathbf{R}\mathbf{P} \tag{A.20}$$

Affine transforms can also be used to perform reflections, in which $\mathbf{A}$ is orthogonal (so that $\mathbf{A}^T\mathbf{A} = \mathbf{I}$) but with $\det\mathbf{A} = -1$.

## A.4 Rotational velocity

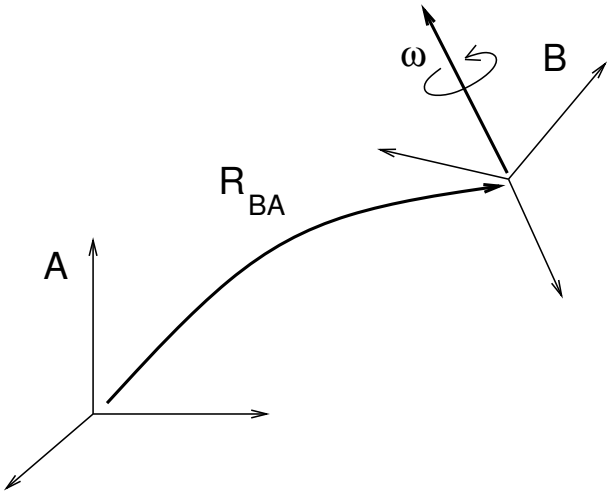



Figure A.8: Frame B rotating with respect to frame A.

Given two 3D coordinate frames A and B, the rotational, or *angular*, velocity of B with respect to A is given by a 3D vector $\omega_{BA}$ (Figure A.8). $\omega_{BA}$ is related to the derivative of $\mathbf{R}_{BA}$ by

$$\dot{\mathbf{R}}_{BA} = [{}^A\omega_{BA}]\mathbf{R}_{BA} = \mathbf{R}_{BA}[{}^B\omega_{BA}] \tag{A.21}$$

where ${}^A\omega_{BA}$ and ${}^B\omega_{BA}$ indicate $\omega_{BA}$ with respect to frames $A$ and $B$ and $[\omega]$ denotes the $3\times 3$ cross product matrix

$$[\omega] \equiv \begin{pmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{pmatrix}. \tag{A.22}$$

If we consider instead the velocity of $A$ with respect to $B$, it is straightforward to show that

$$\omega_{AB} = -\omega_{BA}. \tag{A.23}$$

## A.5 Spatial velocities and forces

Given two 3D coordinate frames A and B, the *spatial velocity*, or *twist*, $\hat{\mathbf{v}}_{BA}$ of B with respect to A is given by the 6D composition of the translational velocity $\mathbf{v}_{BA}$ of the origin of B with respect to A and the angular velocity $\omega_{BA}$:

$$\hat{\mathbf{v}}_{BA} \equiv \begin{pmatrix} \mathbf{v}_{BA} \\ \omega_{BA} \end{pmatrix}. \tag{A.24}$$

Similarly, the *spatial force*, or *wrench*, $\hat{\mathbf{f}}$ acting on a frame B is given by the 6D composition of the translational force $\mathbf{f}_B$ acting on the frame's origin and the moment $\tau$, or torque, acting through the frame's origin:

$$\hat{\mathbf{f}}_B \equiv \begin{pmatrix} \mathbf{f}_B \\ \tau_B \end{pmatrix}. \tag{A.25}$$

If we have two frames $A$ and $B$ rigidly connected within a rigid body (Figure A.9), and we know the spatial velocity $\hat{\mathbf{v}}_{BC}$ of $B$ with respect to some third frame $C$, we may wish to know the spatial velocity $\hat{\mathbf{v}}_{AC}$ of $A$ with respect to $C$. The

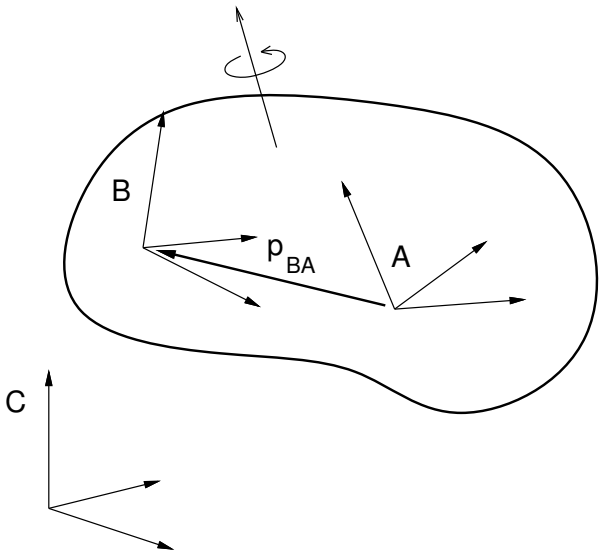



Figure A.9: Two frames A and B rigidly connected within a rigid body and moving with respect to a third frame C.

angular velocity components are the same, but the translational velocity components are coupled by the angular velocity and the offset $\mathbf{p}_{BA}$ between $A$ and $B$, so that

$$\mathbf{v}_{AC} = \mathbf{v}_{BC} + \mathbf{p}_{BA} \times \omega_{BC}.$$

$\hat{\mathbf{v}}_{AC}$ is hence related to $\hat{\mathbf{v}}_{BC}$ via

$$\begin{pmatrix} \mathbf{v}_{AC} \\ \omega_{AC} \end{pmatrix} = \begin{pmatrix} \mathbf{I} & [\mathbf{p}_{BA}] \\ 0 & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{v}_{BC} \\ \omega_{BC} \end{pmatrix}.$$

where $[\mathbf{p}_{BA}]$ is defined by (A.22).

The above equation assumes that all quantities are expressed with respect to the same coordinate frame. If we instead consider $\hat{\mathbf{v}}_{AC}$ and $\hat{\mathbf{v}}_{BC}$ to be represented in frames $A$ and $B$, respectively, then we can show that

$$^{A}\hat{\mathbf{v}}_{AC} = \mathbf{X}_{BA}\, ^{B}\hat{\mathbf{v}}_{BC}, \tag{A.26}$$

where

$$\mathbf{X}_{BA} \equiv \begin{pmatrix} \mathbf{R}_{BA} & [\mathbf{p}_{BA}]\mathbf{R}_{BA} \\ 0 & \mathbf{R}_{BA} \end{pmatrix}. \tag{A.27}$$

The transform $\mathbf{X}_{BA}$ is easily formed from the components of the rigid transform $\mathbf{T}_{BA}$ relating $B$ to $A$.

The spatial forces $\hat{\mathbf{f}}_A$ and $\hat{\mathbf{f}}_B$ acting on frames $A$ and $B$ within a rigid body are related in a similar way, only with spatial forces, it is the moment that is coupled through the moment arm created by $\mathbf{p}_{BA}$, so that

$$\tau_A = \tau_B + \mathbf{p}_{BA} \times \mathbf{f}_B.$$

If we again assume that $\hat{\mathbf{f}}_A$ and $\hat{\mathbf{f}}_B$ are expressed in frames $A$ and $B$, we can show that

$$^{A}\hat{\mathbf{f}}_A = \mathbf{X}^{*}_{BA}\, ^{B}\hat{\mathbf{f}}_B, \tag{A.28}$$

where

$$\mathbf{X}^{*}_{BA} \equiv \begin{pmatrix} \mathbf{R}_{BA} & 0 \\ [\mathbf{p}_{BA}]\mathbf{R}_{BA} & \mathbf{R}_{BA} \end{pmatrix}. \tag{A.29}$$

## A.6 Spatial inertia

Assume we have a rigid body with mass $m$ and a coordinate frame located at the body's center of mass. If $\mathbf{v}$ and $\omega$ give the translational and rotational velocity of the coordinate frame, then the body's linear and angular momentum $\mathbf{p}$ and $\mathbf{L}$ are given by

$$\mathbf{p} = m\mathbf{v} \quad \text{and} \quad \mathbf{L} = \mathbf{J}\omega, \tag{A.30}$$

where $\mathbf{J}$ is the $3 \times 3$ *rotational inertia* with respect to the center of mass. These relationships can be combined into a single equation

$$\hat{\mathbf{p}} = \mathbf{M}\hat{\mathbf{v}}, \tag{A.31}$$

where $\hat{\mathbf{p}}$ is the *spatial momentum* and $\mathbf{M}$ is a $6 \times 6$ matrix representing the *spatial inertia*:

$$\hat{\mathbf{p}} \equiv \begin{pmatrix} \mathbf{p} \\ \mathbf{L} \end{pmatrix}, \qquad \mathbf{M} \equiv \begin{pmatrix} m\mathbf{I} & 0 \\ 0 & \mathbf{J} \end{pmatrix}. \tag{A.32}$$

The spatial momentum satisfies Newton's second law, so that

$$\hat{\mathbf{f}} = \frac{d\hat{\mathbf{p}}}{dt} = \mathbf{M}\frac{d\hat{\mathbf{v}}}{dt} + \dot{\mathbf{M}}\hat{\mathbf{v}}, \tag{A.33}$$

which can be used to find the acceleration of a body in response to a spatial force.

When the body coordinate frame is *not* located at the center of mass, then the spatial inertia assumes the more complicated form

$$\begin{pmatrix} m\mathbf{I} & -m[\mathbf{c}] \\ m[\mathbf{c}] & \mathbf{J} - m[\mathbf{c}][\mathbf{c}] \end{pmatrix}, \tag{A.34}$$

where $\mathbf{c}$ is the center of mass and $[\mathbf{c}]$ is defined by (A.22).

Like the rotational inertia, the spatial inertia is always symmetric positive definite if $m > 0$.

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